Planning and Optimization

E3. Symbolic Search: Uniform-cost and A* search

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December 19, 2016

E3.1 Introduction

Introduction

- Previous chapter: Symbolic breadth-first search
- Optimal plans only guaranteed for unit-cost tasks (= all operators same cost)
- Optimal planning in explicit-state forward search:
  - (uninformed) uniform-cost search
  - (informed) A* search
  - ...

Analogous algorithms for symbolic (BDD-based) search?

E3.2 Symbolic Uniform-Cost Search

Cost-separated Transition Relations

- Previously: one transition relation $T_V(O)$ for all operators
- Now: several transition relations for operators of same cost
- Set $\mathcal{T}$ of pairs (T, c), where T is a transition relation for one/some/all operators of cost c
  - All operators must be covered (and nothing else): $\bigcup_{(T,c)\in\mathcal{T}} r(T) = r(T_V(O))$
  - The cost must be correct: For $(T, c') \in \mathcal{T}$: if $a \in r(T)$ then $a \models \bigvee_{o\in O:cost(o)=c'} \tau_V(o)$

Many possibilities to split up $T_V(O)$ (discussed later)

Image Computation

- The apply function (previous chapter) computes the set of states S' that can be reached from a set of states S by applying one operator.
- This is called the image of S wrt. transition relation $T_V(O)$.
- Now: image computation for arbitrary transition relations.

def image(B, T):
    $B := \mathit{bdd\text{-}intersection}(B, T)$
    for each $v \in V$:
        $B := \mathit{bdd\text{-}forget}(B, v)$
    for each $v \in V$:
        $B := \mathit{bdd\text{-}rename}(B, v', v)$
    return B

Exactly like apply but gets transition relation as argument.

Symbolic Uniform-Cost Search (Positive Operator Costs)

def symbolic-uniform-cost(V, I, O, γ):
    $\mathit{goal} := \mathit{build\text{-}BDD}(\gamma)$
    $\mathcal{T} := \mathit{make\text{-}transition\text{-}relations}(V, O)$
    $\mathit{open}_0 := \mathit{bdd\text{-}state}(I)$
    while $\exists g : \mathit{open}_g \neq \mathbf{0}$:
        $g := \min\{g \mid \mathit{open}_g \neq \mathbf{0}\}$
        $\mathit{closed}_g := \mathit{open}_g$
        if $\mathit{bdd\text{-}intersection}(\mathit{open}_g, \mathit{goal}) \neq \mathbf{0}$:
            return $\mathit{construct\text{-}plan}(I, O, \mathit{goal}, \mathit{closed}_*, g)$
        for all $(T, c) \in \mathcal{T}$:
            $\mathit{open}_{g+c} := \mathit{bdd\text{-}union}(\mathit{open}_{g+c}, \mathit{image}(\mathit{open}_g, T))$
        $\mathit{open}_g := \mathbf{0}$
    return unsolvable

Pre-image Computation

- The image of S wrt. transition relation T computes the set of states that can be reached from S by applying a transition represented by T.
- The pre-image of S wrt. T is the set of states from which we can reach S by applying a transition represented by T.

def pre-image(B, T):
  for each $v \in V$:
    $B := \mathit{bdd\text{-}rename}(B, v, v')$
  $B := \mathit{bdd\text{-}intersection}(B, T)$
  for each $v \in V$:
    $B := \mathit{bdd\text{-}forget}(B, v')$
  return B

Plan Extraction (Positive Operator Costs)

def construct-plan(I, O, goal, closed_*, g):
  $\mathit{cut} := \mathit{bdd\text{-}intersection}(\mathit{goal}, \mathit{closed}_g)$
  $\mathit{init} := \mathit{bdd\text{-}state}(I)$
  $\pi := \langle\rangle$
  while $\mathit{bdd\text{-}intersection}(\mathit{cut}, \mathit{init}) = \mathbf{0}$:
    for $o \in O$:
      $\mathit{pre} := \mathit{pre\text{-}image}(\mathit{cut}, \tau_V(o))$
      if $c := \mathit{bdd\text{-}intersection}(\mathit{pre}, \mathit{closed}_{g-\mathit{cost}(o)}) \neq \mathbf{0}$:
        $\mathit{cut} := c$
        $g := g - \mathit{cost}(o)$
        $\pi := \langle o \rangle \pi$
        break
  return π

Zero-cost Operators

What is the problem with zero-cost operators?

- Search: could re-open open_g after it was moved to closed_g, possibly running into an infinite loop
  → Apply all zero-cost operators before closing
- Plan extraction: could loop in zero-cost cycles
  → special treatment

Breadth-first Exploration with Zero-cost Operators

def bfs-zero(B, g, $\mathcal{T}$, goal):
  $i := 0$
  $\mathit{closed}_{g,i} := B$
  while $B \neq \mathbf{0}$ and $\mathit{bdd\text{-}intersection}(B, \mathit{goal}) = \mathbf{0}$:
    $B' := \mathbf{0}$
    for $(T, c) \in \mathcal{T}, c = 0$:
      $B' := \mathit{bdd\text{-}union}(B', \mathit{image}(B, T))$
    $B := \mathit{bdd\text{-}intersection}(B', \mathit{bdd\text{-}complement}(\mathit{closed}_{g,i}))$
    $i := i + 1$
    $\mathit{closed}_{g,i} := \mathit{bdd\text{-}union}(B, \mathit{closed}_{g,i-1})$
  return $\mathit{closed}_{g,i}$

Symbolic Uniform-Cost Search

def symbolic-uniform-cost(V, I, O, γ):
    $goal := \textit{build-BDD}(\gamma)$
    $\mathcal{T} := \textit{make-transition-relations}(V, O)$
    $open_0 := \textit{bdd-state}(I)$
    while $\exists g : open_g \neq \mathbf{0}$:
        $g := \min\{g \mid open_g \neq \mathbf{0}\}$
        $open_g := \textit{bfs-zero}(open_g, g, \mathcal{T}, goal)$
        $closed_g := open_g$
        if $\textit{bdd-intersection}(open_g, goal) \neq \mathbf{0}$:
            return $\textit{construct-plan}(I, O, goal, closed_*, g)$
        for all $(T, c) \in \mathcal{T}$ with $c > 0$:
            $open_{g+c} := \textit{bdd-union}(open_{g+c}, \textit{image}(open_g, T))$
        $open_g := \mathbf{0}$
    return unsolvable

Plan Extraction with Zero-cost Operators

Needs all closed sets form bfs-zero and symbolic-uniform-cost.

def construct-plan(I, O, $goal$, $closed_{*,*}$, g):
    $cut := \textit{bdd-intersection}(goal, closed_g)$
    $init := \textit{bdd-state}(I)$; $\pi := \langle\rangle$
    while $\textit{bdd-intersection}(cut, init) = \mathbf{0}$:
        $cut, \pi := \textit{get-to-bfs-level-0}(cut, g, closed_{g,*}, \pi, O)$
        if $g = 0$:
            return π
        for $o \in O$ with $cost(o) > 0$:
            $pre := \textit{pre-image}(cut, \tau_V(o))$
            if $c := \textit{bdd-intersection}(pre, closed_{g-cost(o)}) \neq \mathbf{0}$:
                $cut := c$; $\pi := \langle o \rangle \pi$
                $g := g - cost(o)$
                break
    return π

Plan Extraction: Zero-Cost Plan Fragment

def $\textit{get-to-bfs-level-0}(cut, g, closed_{g,*}, \pi, O)$:
    $level := 0$
    while $\textit{bdd-intersection}(cut, closed_{g,level}) = \mathbf{0}$:
        $level := level + 1$
    while $level \neq 0$:
        for $o \in O$ with $cost(o) = 0$:
            $pre := \textit{pre-image}(cut, \tau_V(o))$
            if $c := \textit{bdd-intersection}(pre, closed_{g,level-1}) \neq \mathbf{0}$:
                $cut := c$
                $level := level - 1$
                $\pi := \langle o \rangle \pi$
                break
    return cut, π

Pruning of Closed States

- In explicit-state uniform-cost search, we never re-expand closed states.
- We can easily introduce such pruning in symbolic uniform-cost search.

Uniform-Cost Search with Pruning of Closed States

def symbolic-uniform-cost(V, I, O, γ):
  $goal := build\text{-}BDD(\gamma)$
  $\mathcal{T} := make\text{-}transition\text{-}relations(V, O)$
  $open_0 := bdd\text{-}state(I)$
  while $\exists g : open_g \neq \mathbf{0}$:
    $g := \min\{g \mid open_g \neq \mathbf{0}\}$
    $open_g := bfs\text{-}zero(open_g, g, \mathcal{T}, goal, closed_*)$
    $closed_g := open_g$
    if $bdd\text{-}intersection(open_g, goal) \neq \mathbf{0}$:
      return $construct\text{-}plan(I, O, goal, closed_*, g)$
    for all $(T, c) \in \mathcal{T}$ with $c > 0$:
      $open_{g+c} := bdd\text{-}union(open_{g+c}, image(open_g, T))$
    $open_g := \mathbf{0}$
  return unsolvable

bfs-zero with Pruning of Closed States

def bfs-zero(B, g, $\mathcal{T}$, $goal$, $prune$):
  for $P \in prune$:
    $B := bdd\text{-}intersection(B, bdd\text{-}complement(P))$
  $i := 0$
  $closed_{g,i} := B$
  while $B \neq \mathbf{0}$ and $bdd\text{-}intersection(B, goal) = \mathbf{0}$:
    $B' := \mathbf{0}$
    for $(T, c) \in \mathcal{T}, c = 0$:
      $B' := bdd\text{-}union(B', image(B, T))$
    $B := bdd\text{-}intersection(B', bdd\text{-}complement(closed_{g,i}))$
    for $P \in prune$:
      $B := bdd\text{-}intersection(B, bdd\text{-}complement(P))$
    $i := i + 1$
    $closed_{g,i} := bdd\text{-}union(B, closed_{g,i-1})$
  return $closed_{g,i}$

E3.3 Symbolic A*

Symbolic A*

- Difference between explicit-state uniform-cost search and A*: heuristic to guide search
- $f = g + h$
- Analogously in symbolic search
- Heuristic given as set $heur$ of BDDs $heur_h$ for each heuristic estimate h

Symbolic A* (with Consistent Heuristic)

def symbolic-AStar(V, I, O, γ, *heur*):
  $goal := build\text{-}BDD(\gamma)$
  $\mathcal{T} := make\text{-}transition\text{-}relations(V, O)$
  $open_{0,h(I)} := bdd\text{-}state(I)$
  while $\exists g, h : open_{g,h} \neq \mathbf{0}$:
    $f := \min\{f \mid \exists g, h : open_{g,h} \neq \mathbf{0}, f = g + h\}$
    $g := \min\{g \mid \exists h : open_{g,h} \neq \mathbf{0}, f = g + h\}$
    $open_{g,*} := expand_0(open_{*,*}, g, h, \mathcal{T}, goal, heur, closed_*)$
    $closed_g := bdd\text{-}union(closed_g, open_{g,h})$
    if $bdd\text{-}intersection(open_{g,h}, goal) \neq \mathbf{0}$:
      return $construct\text{-}plan(I, O, goal, closed_*, g)$
    $open_{*,*} := expand_{>0}(open_{*,*}, g, h, \mathcal{T}, heur)$
    $open_{g,h} := \mathbf{0}$
  return unsolvable

For performance it is important to expand the minimum g value.

Expand States and Update Open Lists

def $expand_0(open_{*,*}, g, h, \mathcal{T}, goal, heur, prune)$:
  $B := bfs\text{-}zero(open_{g,h}, (g, h), \mathcal{T}, goal, prune)$
  for $heur_{h'} \in heur, h \leq h' < \infty$:
    $B' := bdd\text{-}intersection(heur_{h'}, open\text{-}zero)$
    $open_{g,h'} := bdd\text{-}union(open_{g,h'}, B')$
  return $open_{g,*}$

def $expand_{>0}(open_{*,*}, g, h, \mathcal{T}, heur)$:
  for all $(T, c) \in \mathcal{T}, c > 0$:
    $B := image(open_{g,h}, T)$
  for $heur_{h'} \in heur, h - c \leq h' < \infty$:
    $B' := bdd\text{-}intersection(heur_{h'}, open\text{-}zero)$
    $open_{g+c,h'} := bdd\text{-}union(open_{g+c,h'}, B')$
  return $open_{*,*}$

Heuristics

How can we generate symbolic heuristics?

- Symbolic Pattern Databases
  - Uniform-cost search can easily be adapted to regression search.
  - Can search backwards in abstract transition systems
  - BDD for closed states with (backwards-) g-value i is heuristic BDD for $h = i$.
- Merge-and-Shrink Abstractions
  - Algebraic Decision Diagrams are like BDDs but sink nodes are labeled with arbitrary numbers.
  - Can map states to numbers.
  - Cascading tables of merge-and-shrink heuristics with linear merge strategy can efficiently be transformed into an ADD.
  - Result can be used in symbolic search instead of BDD set.

E3.4 Discussion

Importance of Variable Ordering

- For good performance, we need a good variable ordering.
  - Variables that refer to the same state variable before and after operator application (v and v') should be neighbors in the transition relation BDD.
- This is important for the performance of *BDD-rename* in the *image* and *pre-image* computation.

Transition Relations in $\mathcal{T}$

- We only required that all operators are represented by some $(T, c) \in \mathcal{T}$ and that the costs are correct.
- Extreme cases:
  - One element $(\tau_V(o), \mathit{cost}(o))$ for each operator o
  - Only one element for each operator cost, covering all operators of that cost.
- Trade-off:
  - Large number of entries leads to large number of image computations.
  - Size of T can grow exponentially with number of covered operators.
- There exist different aggregation strategies.

Performance

- In symbolic planning, blind search is often better than informed search.
- Practical implementations also perform regression or bidirectional search.
- This is only a minor modification of uniform-cost search.

E3.5 Summary

Summary

- Symbolic search operates on sets of states instead of individual states as in explicit-state search.
- State sets and transition relations can be represented as BDDs.
- A good variable ordering and an efficient image computation are crucial for performance.

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