Planning and Optimization
D7. Comparison of Heuristic Families II

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D7.1 Landmarks vs. $h^{\max}$

Landmarks to $h^{\max}$

Theorem

Elementary landmark heuristics can be compiled into additive h^{max} heuristics in polynomial time.

Proof.

Let L be a subset of the operators. If L^+ is not a landmark for s in Π^+ then $h_L(s) = 0$ and therefore trivially $h^{\max}(s) \geq h_L(s)$. Otherwise no goal state of Π^+ is reachable from s without an operator from L^+. So if $h^{\max}(s) \neq \infty$ then the cost computation of $h^{\max}$ must use an operator from L^+ and therefore $h^{\max}(s) \geq \min_{o \in L} cost(o) = h_L$. $\square$

$h^{\max}$ to Landmarks

Theorem

For states with finite $h^{\max}$ value, the $h^{\max}$ heuristic can be compiled into additive elementary landmark heuristics in polynomial time.

Proof sketch:

The LM-Cut heuristic computes in each step a cut landmark and adapts the operator costs. Let $cost_i, cost_{i+1}$ be the operator costs before and after an iteration that discovered landmark L. Then $h^{\max}_{cost_i}(s) \leq h_{L,cost_i}(s) + h^{\max}_{cost_{i+1}}(s)$. The core argument is that every "reasonable" path in the justification graph enters the goal zone only once and therefore uses only one operator from L. So reducing the cost of each operator in L by $h_{L,cost_i}(s)$ cannot reduce $h^{\max}$ by more than this value. The overall result of the theorem follows from a recursive application of the proof while $h^{\max}_{cost_{i+1}}(s) > 0$. □

D7.2 Abstractions vs. Critical Path

h^m to PDBs

Theorem

There is no polynmial-time compilation from h^m heuristics into additive PDB heuristics.

Proof.

We know that elementary landmarks are in polynomial time compilable into additive $h^{\max}$ but not into additive PDB heuristics. So there is no polynomial-time compilation from $h^{\max} = h^1$ into additive PDB heuristics.
As $h^m \geq h^1$ for $m \geq 1$, this holds for any m. □

PDBs to h^m

Theorem

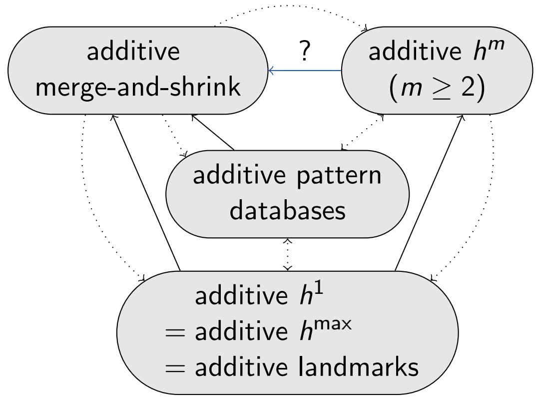
There is no polynmial-time compilation of PDB heuristics into additive h^m heuristics.

Proof.

Consider family $(\Pi_n)_{n \in \mathbb{N}_1}$ of STRIPS tasks, where $\Pi_n = \langle V_n, I_n, O_n, \gamma \rangle$ with $V_n = \{v_1, \dots, v_n\}$, $I_n(v) = \mathbf{F}$ for $v \in V_n$, $O = \{\langle \bigwedge_{i=1}^{j-1} v_i, v_j \wedge \bigwedge_{i=1}^{j-1} \neg v_i, 1 \rangle \mid 1 \leq j \leq n\}$ and $\gamma = \bigwedge_{i: i\text{-th bit in } bin(n) \text{ is } 1} v_i$.
A PDB on pattern $\{v_1, \dots, v_{\lceil \log n \rceil}\}$ has $O(n)$ states and encodes the perfect goal distance $h^*(I) = n$.
For a perfect initial estimate, the h^m heuristic needs to consider variable subsets up to size $\lceil \log n \rceil$. As m must be fixed due to the polynomial-time requirement, we can thus find for any such m a large enough n that proves the theorem. □

D7.3 Overview

Overview



Solid arc: poly-time compilation exists
Dotted arc: compilation not possible

What else?

Post-hoc optimization

- For PDBs it computes state-specific additive set of PDB heuristics. → Covered by results on PDB heuristics.
- Analogously for other classes of heuristics.

So far no results for

- landmarks not based on delete relaxation (Π^m landmarks),
- flow heuristics, and
- compilability from h^m heuristics into additive merge-and-shrink heuristics.

D7.4 Summary

Summary

- Relaxation-based landmark heuristics dominate additive $h^{\max}$ heuristics and vice versa.
- Additive critical path heuristics with $m \geq 2$ strictly dominate relaxation-based landmark heuristics and additive $h^{\max}$ heuristics.
- Merge-and-shrink heuristics strictly dominate relaxation-based landmark heuristics and additive $h^{\max}$ heuristics.
- PDB heuristics are incomparable with relaxation-based landmark heuristics and additive $h^{\max}$ heuristics.

Literature

Malte Helmert and Carmel Domshlak.
Landmarks, Critical Paths and Abstractions: What's the Difference Anyway?
Proc. ICAPS 2009, pp. 162–169, 2009.