

Planning and Optimization

D2. Cost Partitioning: Landmarks and Generalization

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Cost Partitioning for Landmarks

Reminder: Disjunctive Action Landmarks

Disjunctive action landmark

- Set of operators
- Every plan uses at least one of them
- Landmark cost = cost of cheapest operator

Reminder: Cost Partitioning Heuristic for Landmarks

We have already seen a landmark heuristic based on cost partitioning:

Definition (Uniform Cost Partitioning Heuristic for Landmarks)

Let \mathcal{L} be a set of disjunctive action landmarks.

The **uniform cost partitioning heuristic** $h^{\text{UCP}}(\mathcal{L})$ is defined as

$$h^{\text{UCP}}(\mathcal{L}) = \sum_{L \in \mathcal{L}} \min_{o \in L} c'(o) \text{ with}$$

$$c'(o) = \text{cost}(o) / |\{L \in \mathcal{L} \mid o \in L\}|.$$

Reminder: Proof Back Then

Theorem (Uniform Cost Partitioning Heuristic is Admissible)

Let \mathcal{L} be a set of disjunctive action landmarks for state s of Π .
Then $h^{UCP}(\mathcal{L})$ is an admissible heuristic estimate for s .

Proof.

Let $\pi = \langle o_1, \dots, o_n \rangle$ be an optimal plan for s . For $L \in \mathcal{L}$ define a new cost function $cost_L$ as $cost_L(o) = c'(o)$ if $o \in L$ and $cost_L(o) = 0$ otherwise. Let Π_L be a modified version of Π , where for all operators o the cost is replaced with $cost_L(o)$.
(...)

$$\sum_{L \in \mathcal{L}} cost_L(o) = \sum_{L \in \mathcal{L}: o \in L} cost(o) / |\{L \in \mathcal{L} \mid o \in L\}| = cost(o)$$

Heuristic is Based on Cost Partitioning

- For disj. action landmark L of state s in task Π' , let $h_{L,\Pi'}(s)$ be the cost of L in Π' . Then $h_{L,\Pi'}(s)$ is admissible.
- Consider set $\{L_1, \dots, L_n\}$ of disj. action landmarks for state s of task Π .
- Use cost partitioning $\langle cost_{L_1}, \dots, cost_{L_n} \rangle$, where

$$cost_{L_i}(o) = \begin{cases} cost(o)/|\{L \in \mathcal{L} \mid o \in L\}| & \text{if } o \in L_i \\ 0 & \text{otherwise} \end{cases}$$

- Let $\langle \Pi_{L_1}, \dots, \Pi_{L_n} \rangle$ be the tuple of induced tasks.
- $h(s) = \sum_{i=1}^n h_{L_i, \Pi_{L_i}}(s)$ is an admissible estimate for s in Π .
- h is uniform cost partitioning heuristic for landmarks.

Optimal Cost Partitioning for Landmarks

Can we find a better cost partitioning?

- Use again LP that covers heuristic computation and cost partitioning.
- LP variable Cost_L for cost of landmark L in induced task (corresponds to $h_{L_i, \Pi_{L_i}}$)
- Explicit variables for cost partitioning not necessary. Use implicitly $\text{cost}_L(o) = \text{Cost}_L$ for all $o \in L$ and 0 otherwise.

Optimal Cost Partitioning for Landmarks: LP

Variables

$Cost_L$ for each disj. action landmark $L \in \mathcal{L}$

Objective

Maximize $\sum_{L \in \mathcal{L}} Cost_L$

Subject to

$$\sum_{L \in \mathcal{L}: o \in L} Cost_L \leq cost(o) \quad \text{for all operators } o$$

$$Cost_L \geq 0 \quad \text{for all landmarks } L \in \mathcal{L}$$

Optimal Cost Partitioning for Landmarks (Dual view)

Variables

Occurrences_{*o*} for each operator *o*

Objective

Minimize $\sum_o \text{Occurrences}_o \cdot \text{cost}(o)$

Subject to

$$\sum_{o \in L} \text{Occurrences}_o \geq 1 \text{ for all landmarks } L$$

$$\text{Occurrences}_o \geq 0 \text{ for all operators } o$$

Minimize “plan cost” with all landmarks satisfied.

General Cost Partitioning

General Cost Partitioning

Cost functions **usually non-negative**

- We tacitly also required this for task copies
- Makes intuitively sense: original costs are non-negative
- But: not necessary for cost-partitioning!

General Cost Partitioning

Definition (General Cost Partitioning)

Let Π be a planning task with operators O .

A **general cost partitioning** for Π is a tuple $\langle cost_1, \dots, cost_n \rangle$, where

- $cost_i : O \rightarrow \mathbb{R}$ for $1 \leq i \leq n$ and
- $\sum_{i=1}^n cost_i(o) \leq cost(o)$ for all $o \in O$.

General Cost Partitioning: Admissibility

Theorem (Sum of Solution Costs is Admissible)

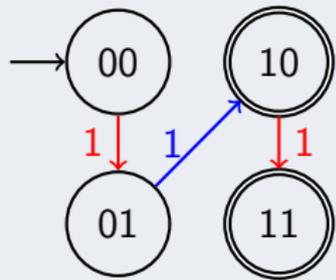
Let Π be a planning task, $\langle cost_1, \dots, cost_n \rangle$ be a **general** cost partitioning and $\langle \Pi_1, \dots, \Pi_n \rangle$ be the tuple of induced tasks.

Then the sum of the solution costs of the induced tasks is an **admissible heuristic** for Π , i.e., $\sum_{i=1}^n h_{\Pi_i}^* \leq h_{\Pi}^*$.

(Proof omitted.)

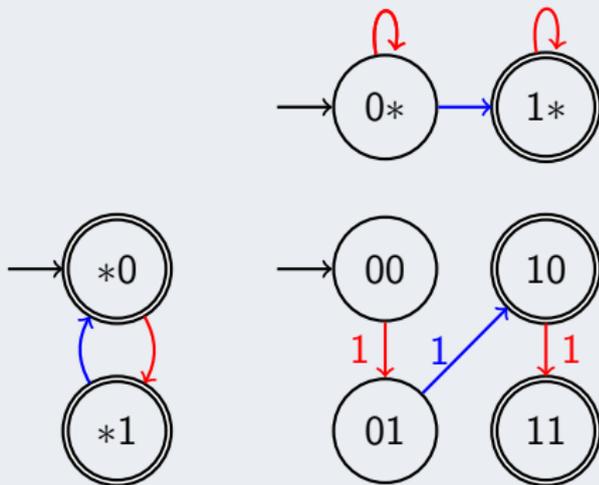
General Cost Partitioning: Example

Example



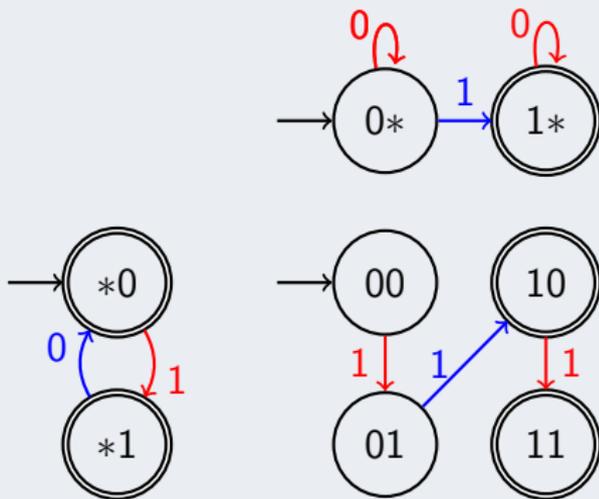
General Cost Partitioning: Example

Example



General Cost Partitioning: Example

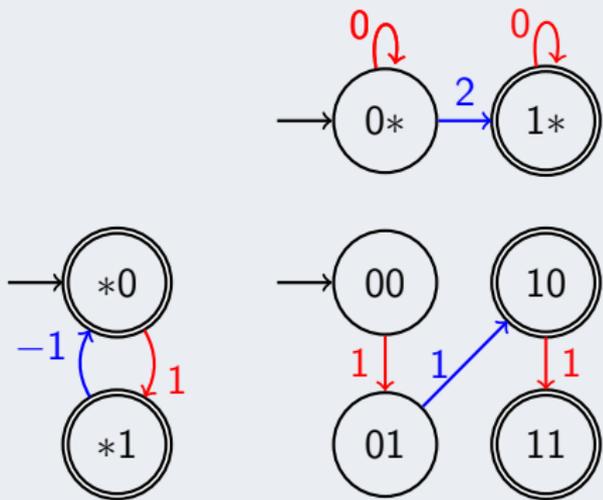
Example



Heuristic value: $0 + 1 = 1$

General Cost Partitioning: Example

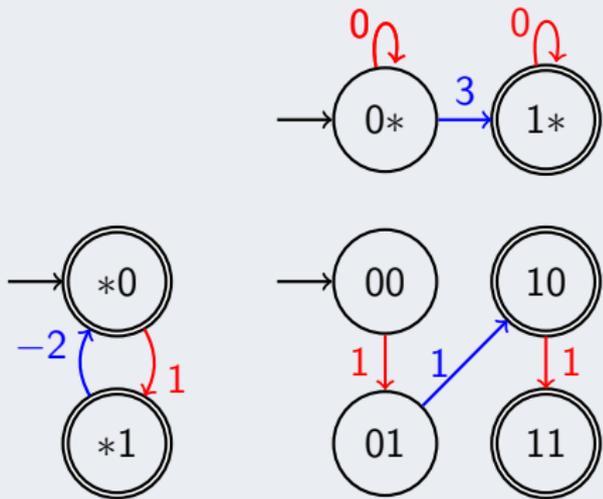
Example



Heuristic value: $0 + 2 = 2$

General Cost Partitioning: Example

Example



Heuristic value: $-\infty + 3 = -\infty$

General Cost Partitioning: Remarks

- **More powerful** than non-negative cost partitioning
- **Optimal** general cost partitioning:
 - omit constraints to non-negative cost variables
 - optimal cost partitioning maximizes objective value
 - removing constraints can only increase heuristic value
- Optimal general cost partitioning is never worse than an optimal non-negative cost partitioning.

Summary

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- We can compute an **optimal cost partitioning** for a given set of disjunctive action **landmarks** in polynomial time.
- In contrast to standard (non-negative) cost partitioning, **general cost partitioning** allows negative operators costs.
- General cost partitioning has the same relevant properties as non-negative cost partitioning but is more powerful.

Literature (1)

References on cost partitioning:



Stefan Edelkamp.

Automated Creation of Pattern Database Search Heuristic.

Proc. MoChArt 2006, pp. 35–50, 2006.

Introduces **0-1 cost partitioning for abstraction heuristics**.



Michael Katz and Carmel Domshlak.

Structural Patterns Heuristics: Basic Idea and Concrete Instance.

ICAPS 2007 HDIP Workshop, 2007.

Introduces **arbitrary non-negative cost partitioning**.

Literature (2)



Fan Yang, Joseph C. Culberson, Robert Holte, Uzi Zahavi and Ariel Felner.

A General Theory of Additive State Space Abstractions.

JAIR 32, pp. 631–662, 2008.

Introduces **arbitrary non-negative cost partitioning**.



Erez Karpas and Carmel Domshlak.

Cost-optimal Planning with Landmarks.

Proc. IJCAI 2009, pp. 1728–1733, 2009.

Introduces **optimal** cost partitioning for **landmarks**
(different formulation than in the slides).

Literature (3)



Emil Keyder, Silvia Richter and Malte Helmert.

Sound and Complete Landmarks for And/Or Graphs.

Proc. ECAI 2010 , pp. 335–340, 2010.

Smaller LP for optimal cost partitioning for landmarks.



Blai Bonet and Malte Helmert.

Strengthening Landmark Heuristics via Hitting Set.

Proc. ECAI 2010 , pp. 329–334, 2010.

Dual LP for optimal cost partitioning for landmarks.

Literature (4)



Michael Katz and Carmel Domshlak.

Optimal Admissible Composition of Abstraction Heuristics.

Artificial Intelligence 174 (12–13), pp. 767–798, 2010.

Introduces **optimal** cost partitioning for **abstraction heuristics**.



Florian Pommerening, Malte Helmert, Gabriele Röger and Jendrik Seipp.

From Non-Negative to General Operator Cost Partitioning.

Proc. AAAI 2015, pp. 3335–3341, 2015.

Introduces **general** cost partitioning.