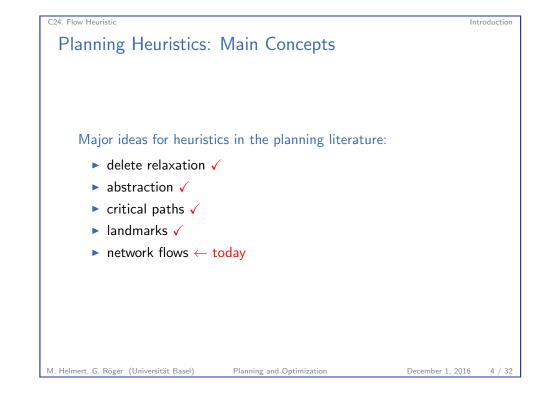


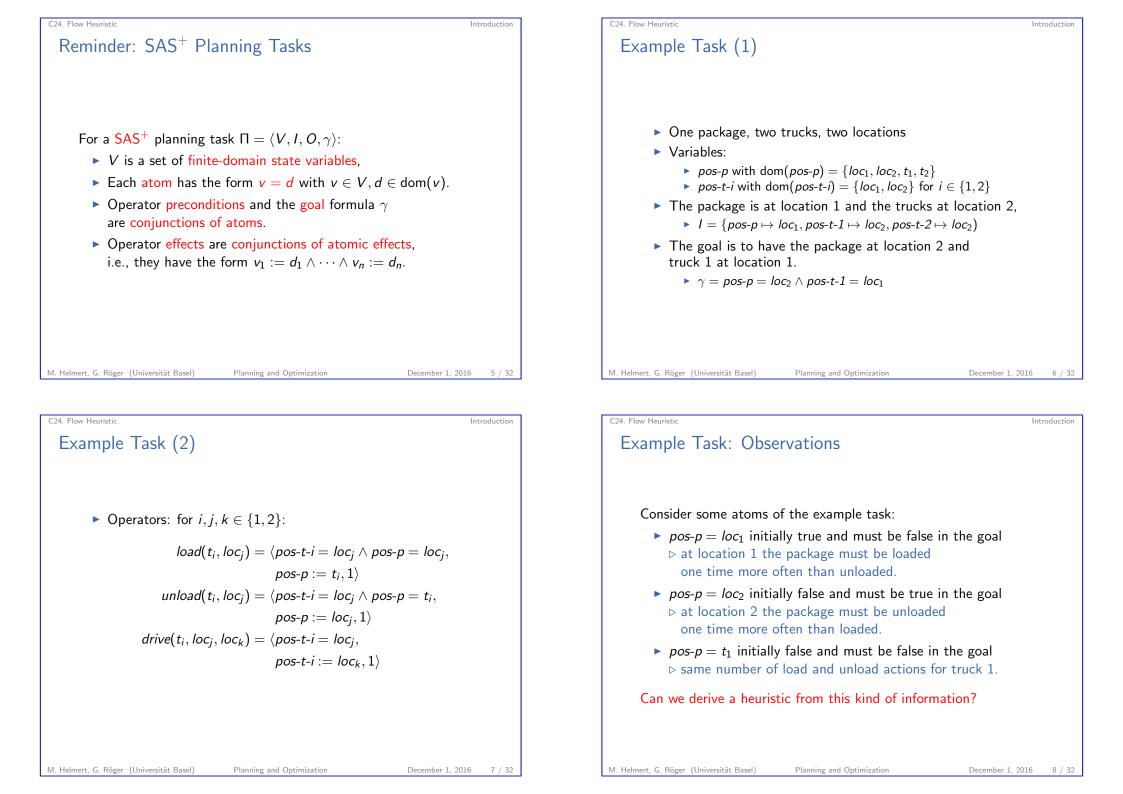
C24.1 Introduction

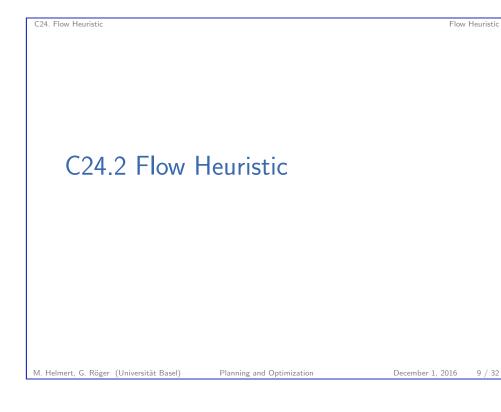
Planning and Optimi December 1, 2016 — C24. Flow			
C24.1 Introduction	ı		
C24.2 Flow Heuris	tic		
C24.3 Summary			
C24.4 Literature			
M. Helmert, G. Röger (Universität Basel)	Planning and Optimization	December 1, 2016	2 / 32



3 / 32

Planning and Optimization





C24. Flow Heuristic

Network Flow Heuristics: General Idea

- ► Formulate flow constraints for each atom.
- These are satisfied by every plan of the task.
- The cost of a plan is $\sum_{o \in O} cost(o) \# o$
- The objective value of an integer program that minimizes this cost subject to the flow constraints is a lower bound on the plan cost (i.e., an admissible heuristic estimate).
- As solving the IP is NP-hard, we solve the LP relaxation instead.

How do we get the flow constraints?

Flow Heuristic



Example: Flow Constraints

Let π be some arbitrary plan for the example task and let #o denote the number of occurrences of operator o in π . Then the following holds:

- pos-p = loc₁ initially true and must be false in the goal
 > at location 1 the package must be loaded one time more often than unloaded.
 #load(t₁, loc₁) + #load(t₂, loc₁) = 1 + #unload(t₁, loc₁) + #unload(t₂, loc₁)
- pos-p = t₁ initially false and must be false in the goal
 same number of load and unload actions for truck 1.
 #unload(t₁, loc₁) + #unload(t₁, loc₂) =
 #load(t₁, loc₁) + #load(t₁, loc₂)

Planning and Optimization

M. Helmert, G. Röger (Universität Basel)

M. Helmert, G. Röger (Universität Basel)

December 1, 2016

C24. Flow Heuristic

How to Derive Flow Constraints?

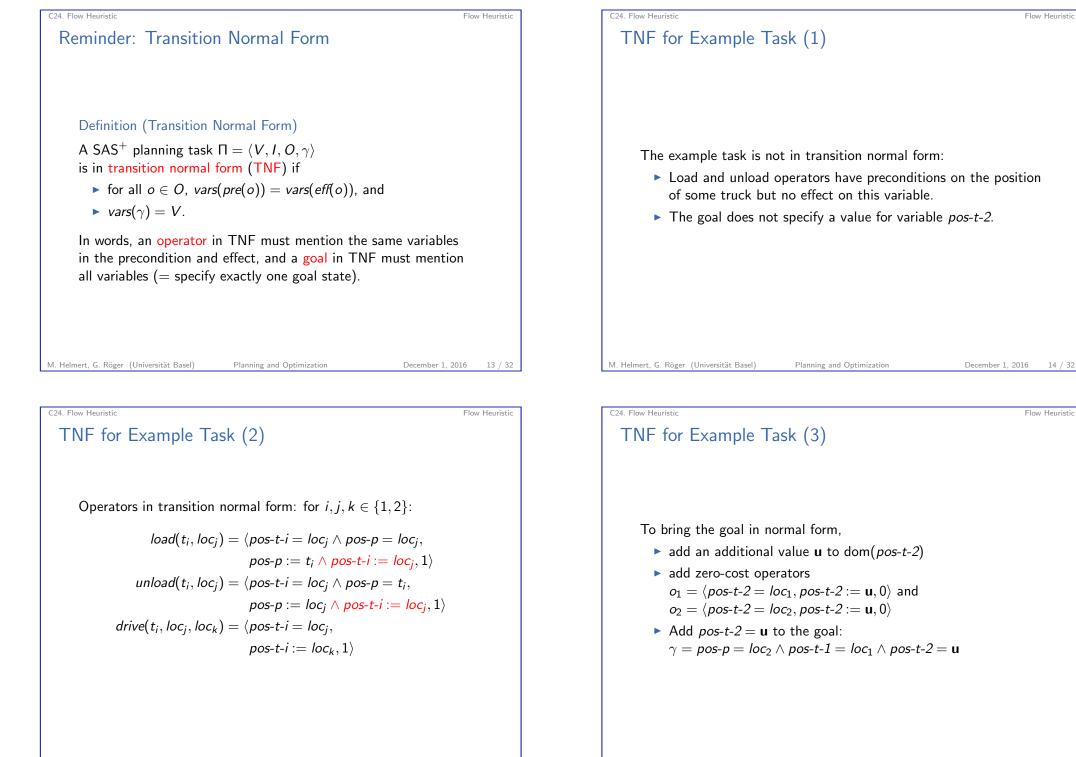
- The constraints formulate how often an atom can be produced or consumed.
- "Produced" (resp. "consumed") means that the atom is false (resp. true) before an operator application and true (resp. false) in the successor state.
- For general SAS⁺ operators, this depends on the state where the operator is applied: effect v := d only produces v = d if the operator is applied in a state s with s(v) ≠ d.
- For general SAS⁺ tasks, the goal does not have to specify an value for every variable.
- All this makes the definition of flow constraints somewhat involved and in general such constraints are inequalitites.

Planning and Optimization

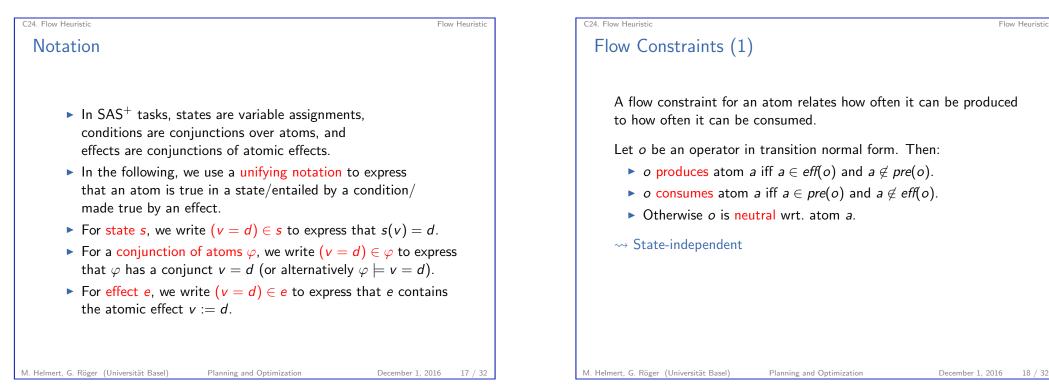
Good news: easy for tasks in transition normal form

10 / 32

Flow Heuristic



Planning and Optimization



C24. Flow Heuristic

Flow Constraints (2)

A flow constraint for an atom relates how often it can be produced to how often it can be consumed.

The constraint depends on the current state *s* and the goal γ . If γ mentions all variables (as in TNF), the following holds:

- If a ∈ s and a ∈ γ then atom a must be equally often produced and consumed.
- Analogously for $a \notin s$ and $a \notin \gamma$.
- If a ∈ s and a ∉ γ then a must be consumed one time more often than it is produced.
- If a ∉ s and a ∈ γ then a must be produced one time more often than it is consumed.

Planning and Optimization

December 1, 2016 19 / 32

Flow Heuristic

C24. Flow Heuristic

Iverson Bracket

The dependency on the current state and the goal can concisely be expressed with lverson brackets:

Definition (Iverson Bracket)

Let P be a logical proposition (= some statement that can be evaluated to true or false). Then

$$[P] = \begin{cases} 1 & \text{if } P \text{ is true} \\ 0 & \text{if } P \text{ is false.} \end{cases}$$

Example:
$$[2 \neq 3] = 1$$

Flow Heuristic

Flow Heuristic

Flow Constraints (3)

Definition (Flow Constraint)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a task in transition normal form. The flow constraint for atom *a* in state *s* is

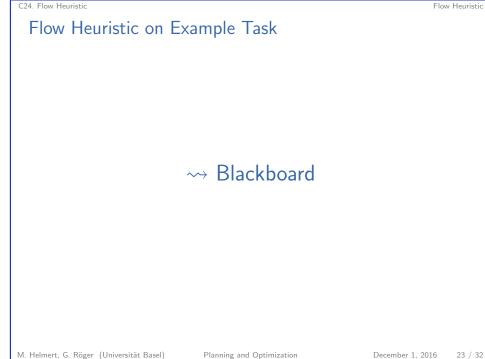
$$[a \in s] + \sum_{o \in O: a \in eff(o)} Y_o = [a \in \gamma] + \sum_{o \in O: a \in pre(o)} Y_o$$

- \triangleright Y_o is an LP variable for the number of occurrences of operator o.
- Neutral operators either appear on both sides or on none.

Planning and Optimization

M. Helmert, G. Röger (Universität Basel)

December 1, 2016 21 / 32



C24. Flow Heuristic

Flow Heuristic

Definition (Flow Heuristic)

Let $\Pi = \langle V, I, O, \gamma \rangle$ be a SAS⁺ task in transition normal form and let $A = \{(v = d) \mid v \in V, d \in dom(v)\}$ be the set of atoms of Π .

The flow heuristic $h^{\text{flow}}(s)$ is the objective value of the following LP or ∞ if the LP is infeasible:

minimize $\sum_{o \in O} cost(o) Y_o$ subject to $[a \in s] + \sum_{o \in O: a \in eff(o)} Y_o = [a \in \gamma] + \sum_{o \in O: a \in pre(o)} Y_o \text{ for all } a \in A$ $Y_o > 0$ for all $o \in O$ M. Helmert, G. Röger (Universität Basel) Planning and Optimization December 1, 2016 22 / 32



Theorem

The flow heuristic h^{flow} is goal-aware, safe, consistent and admissible.

Proof

We prove goal-awareness and consistency, the other properties follow from these two.

Goal-awareness: If $s \models \gamma$ then $Y_o = 0$ for all $o \in O$ is feasible and the objective function has value 0. As $Y_o \ge 0$ for all variables and operator costs are nonnegative, the objective value cannot be smaller. . . .

Planning and Optimization

M. Helmert, G. Röger (Universität Basel)

24 / 32

Flow Heuristic

Flow Heuristic: Properties (2)

Proof (continued).

Consistency: Let *o* be an operator that is applicable in state *s* and let s' = s[o]. Consider an optimal feasible vector **v**' for the LP for s' and let $v_{\alpha'}$ denote the value of $Y_{\alpha'}$ in this vector. Let **v** be the vector that assigns Y_o the value $y_o + 1$ and all other variables $Y_{o'}$ $(o' \neq o)$ the value $y_{o'}$. We show that **y** is feasible for the LP for *s*. Let a = (v = d) be an atom. The flow constraint for a in state s is

$$[a \in s] + \sum_{o \in O: a \in eff(o)} Y_o = [a \in \gamma] + \sum_{o \in O: a \in pre(o)} Y_o$$

We consider how the flow constraint for *a* is affected by a change from s' to s and from \mathbf{v}' to \mathbf{v}

Planning and Optimization

M. Helmert, G. Röger (Universität Basel)

December 1, 2016 25 / 32

Flow Heuristic

Flow Heuristic

C24. Flow Heuristic

Flow Heuristic: Properties (4)

Proof (continued).

As $\mathbf{y} \ge \mathbf{y}' \ge \mathbf{0}$, also the constraints that require the LP variables to be non-negative are satisfied.

The value of the objective function with **y** is $h^{\text{flow}}(s') + cost(o)$. Since \mathbf{y} is feasible for the LP for state s, this is an upper bound on $h^{\text{flow}}(s)$, so in total $h^{\text{flow}}(s) < h^{\text{flow}}(s') + cost(o)$.

Flow Heuristic: Properties (3)

Proof (continued).

If $v \notin vars(pre(o))$, the constraint is not affected and stays satisfied as it is satisfied by \mathbf{y}' . Otherwise, we distinguish four cases:

- ▶ $a \in pre(o), a \notin eff(o)$: Then $a \in s$ and $a \notin s'$, increasing the left-hand side by one. Y_o only occurs on the right-hand side, so the change is balanced by the increase of the value for Y_o .
- ▶ $a \notin pre(o), a \in eff(o)$: Then $a \notin s$ and $a \in s'$, decreasing the left-hand side by one. Y_o only occurs on the left-hand side, so the change is balanced by the increase of the value for Y_o .
- $a \in pre(o), a \in eff(o)$: Then $a \in s$ and $a \in s'$ and Y_o occurs on both sides, so the equation stays balanced.
- ▶ $a \notin pre(o), a \notin eff(o)$: Then $a \notin s$ and $a \notin s'$ and Y_o does not occur on either side of the equation.

```
M. Helmert, G. Röger (Universität Basel)
```

Planning and Optimization

. . . December 1, 2016 26 / 32



Planning and Optimization

C24. Flow Heuristic

Summary

29 / 32

C24. Flow Heuristic

Summary

- A flow constraint for an atom describes how the number of producing operator applications is linked to the number of consuming operator applications.
- The flow heuristic computes a lower bound on the cost of each operator sequence that satisfies these constraints for all atoms.
- The heuristic only considers the number of occurrences of each operator, but ignores their order.

M. Helmert, G. Röger (Universität Basel)

Planning and Optimization December 1, 2016



C24.4 Literat	ture		
M. Helmert, G. Röger (Universität Basel)	Planning and Optimization	December 1, 2016	30 / 32

