Planning and Optimization

C19. Landmarks: Introduction & Minimum Hitting Set Heuristic

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Introduction

Planning Heuristics: Main Concepts

Major ideas for heuristics in the planning literature:

- delete relaxation ✓
- abstraction ✓
- critical paths ✓
- landmarks ← starting now
- network flows

Landmarks

Basic Idea: Something that must happen in every solution

For example

- some operator must be applied
- some atom must be true
- some formula must be true

→ Derive heuristic estimate from this kind of information.

Reminder: Terminology

Consider sequence of transitions $s^0 \xrightarrow{\ell_1} s^1, \ldots, s^{n-1} \xrightarrow{\ell_n} s^n$ such that $s^0 = s$ and $s^n = s'$.

- $s^0, \ldots, s^n$ is called (state) path from s to s'
- $\ell_1, \ldots, \ell_n$ is called (label) path from s to s'
- $s^0 \xrightarrow{\ell_1} s^1, \ldots, s^{n-1} \xrightarrow{\ell_n} s^n$ is called trace from s to s'

Landmarks

Disjunctive Action Landmarks

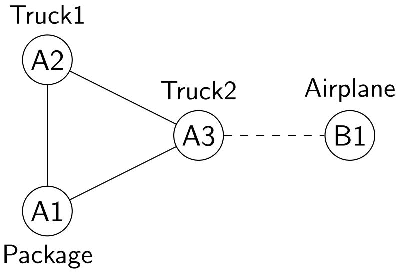
Definition (Disjunctive Action Landmark)

Let s be a state of planning task $\Pi = \langle V, I, O, \gamma \rangle$.
A disjunctive action landmark for s is a set of operators $L \subseteq O$ such that every label path from s to a goal state contains an operator from L.

The cost of landmark L is $cost(L) = \min_{o \in L} cost(o)$.

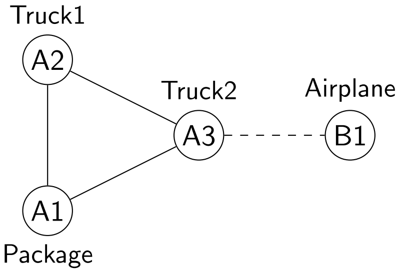
Example Task

- Two trucks, one airplane
- Airplane can fly between locations A3 and B1
- Trucks can drive arbitrarily between locations A1, A2, and A3
- Package to be transported from A1 to B1
- Operators
  - Load(v, l) and Unload(v, l) for vehicle v and location l
  - Drive(t, l, l') for truck t and locations l, l'
  - Fly(l, l') for locations l, l'



Example: Disjunctive Action Landmarks

$L_1 = \{Load(Truck1, A1), Load(Truck2, A1)\}$ and
$L_2 = \{Fly(B1, A3)\}$ are disjunctive action landmarks.



What other disjunctive action landmarks are there?

Fact and Formula Landmarks

Definition (Formula and Fact Landmark)

Let s be a state of planning task $\Pi = \langle V, I, O, \gamma \rangle$.

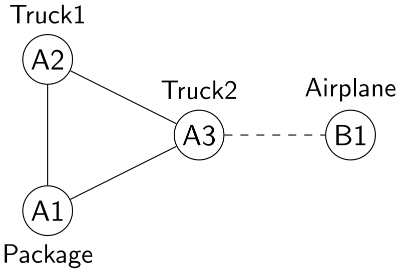
A formula landmark for s is a formula λ over V such that every state path from s to a goal state contains a state s' with $s' \models \lambda$.

If $\lambda \in V$ then λ is a fact landmark.

Example: Formula Landmarks

at(*Package*, *A3*) and *in*(*Package*, *Airplane*) are fact landmarks.

in(*Package*, *Truck1*) $\vee$ *in*(*Package*, *Truck2*) is a formula landmark.



What other formula and fact landmarks are there?

Remarks

- Not every landmark is informative. Some examples:
  - If the initial state is not already a goal state then the set of all operators is a disjunctive action landmark.
  - Every variable that is initially true is a fact landmark.
  - The goal formula is a formula landmark.
- Deciding whether a given variable is a fact landmark is as hard as the planning problem.
- The same is true for operator sets and disjunctive action landmarks.

Relationship

Disjunctive action landmarks and fact/formula landmarks are related:

- Every fact landmark f that is initially false induces a disjunctive action landmark consisting of all operators that possibly make f true.
- A disjunctive action landmark $\{o_1, \dots, o_n\}$ induces a formula landmark $\lambda = pre(o_1) \lor \dots \lor pre(o_n)$ and therefore also a fact landmark v for all $v \in V$ with $\lambda \models v$.

Minimum Hitting Set Heuristic and Uniform Cost Partitioning

Exploiting Disjunctive Action Landmarks

How can we exploit a given set $\mathcal{L}$ of disjunctive action landmarks?

- Sum of costs $\sum_{L \in \mathcal{L}} cost(L)$?
  $\rightsquigarrow$ not admissible!
- Maximize costs $\max_{L \in \mathcal{L}} cost(L)$?
  $\rightsquigarrow$ usually very weak heuristic
- better: hitting sets or cost partitioning

Hitting Sets

Definition (Hitting Set)

Let X be a set, $\mathcal{F} = \{F_1, \ldots, F_n\} \subseteq 2^X$ be a family of subsets of X and $c : X \to \mathbb{R}_0^+$ be a cost function for X.

A hitting set is a subset $H \subseteq X$ that "hits" all subsets in $\mathcal{F}$, i.e., $H \cap F \neq \emptyset$ for all $F \in \mathcal{F}$. The cost of H is $\sum_{x \in H} c(x)$.

A minimum hitting set (MHS) is a hitting set with minimal cost.

MHS is a "classical" NP-complete problem (Karp, 1972)

Example: Hitting Sets

Example

$X = \{o_1, o_2, o_3, o_4\}$

$\mathcal{F} = \{A, B, C, D\}$
with $A = \{o_4\}$, $B = \{o_1, o_2\}$, $C = \{o_1, o_3\}$, $D = \{o_2, o_3\}$

$c(o_1) = 3$, $c(o_2) = 4$, $c(o_3) = 5$, $c(o_4) = 0$

minimum hitting set:

Example: Hitting Sets

Example

$X = \{o_1, o_2, o_3, o_4\}$

$\mathcal{F} = \{A, B, C, D\}$
with $A = \{o_4\}$, $B = \{o_1, o_2\}$, $C = \{o_1, o_3\}$, $D = \{o_2, o_3\}$

$c(o_1) = 3$, $c(o_2) = 4$, $c(o_3) = 5$, $c(o_4) = 0$

minimum hitting set: $\{o_1, o_2, o_4\}$ with cost $3 + 4 + 0 = 7$

Hitting Sets for Disjunctive Action Landmarks

Idea: disjunctive action landmarks are interpreted as instance of minimum hitting set

Definition (Hitting Set Heuristic)

Let $\mathcal{L}$ be a set of disjunctive action landmarks. The hitting set heuristic $h^{\mathrm{MHS}}(\mathcal{L})$ is defined as the cost of a minimum hitting set for $\mathcal{L}$ with $c(o) = cost(o)$.

Proposition (Hitting Set Heuristic is Admissible)

Let $\mathcal{L}$ be a set of disjunctive action landmarks for state s. Then $h^{MHS}(\mathcal{L})$ is an admissible estimate for s.

Why?

Hitting Set Heuristic: Discussion

- The hitting set heuristic is the best possible heuristic that only uses the given information. . .
- . . . but is NP-hard to compute.
- $\rightsquigarrow$ Use approximations that can be efficiently computed.
- Now: uniform cost partitioning
- Later (part D): optimal cost partitioning

Uniform Cost Partitioning (1)

Idea: Distribute cost of operators uniformly among the landmarks.

Definition (Uniform Cost Partitioning Heuristic for Landmarks)

Let $\mathcal{L}$ be a set of disjunctive action landmarks.

The uniform cost partitioning heuristic $h^{\mathrm{UCP}}(\mathcal{L})$ is defined as

$$h^{\mathrm{UCP}}(\mathcal{L}) = \sum_{L \in \mathcal{L}} \min_{o \in L} c'(o) \text{ with}$$

$$c'(o) = cost(o) / |\{L \in \mathcal{L} \mid o \in L\}|$$

.

Uniform Cost Partitioning (2)

Theorem (Uniform Cost Paritioning Heuristic is Admissible)

Let $\mathcal{L}$ be a set of disjunctive action landmarks for state s of Π. Then $h^{UCP}(\mathcal{L})$ is an admissible heuristic estimate for s.

Proof.

Let $\pi = \langle o_1, \dots, o_n \rangle$ be an optimal plan for s. For $L \in \mathcal{L}$ define a new cost function $cost_L$ as $cost_L(o) = c'(o)$ if $o \in L$ and $cost_L(o) = 0$ otherwise. Let Π_L be a modified version of Π, where for all operators o the cost is replaced with $cost_L(o)$. We make three independent observations:

1. For $L \in \mathcal{L}$ the value $cost'(L) := \min_{o \in L} c'(o)$ is an admissible estimate for s in Π_L.
2. π is also a plan for s in Π_L, so $h^*_{\Pi_L}(s) \leq \sum_{i=1}^{n} cost_L(o_i)$.
3. $\sum_{L \in \mathcal{L}} cost_L(o) = cost(o)$ for each operator o.

...

Uniform Cost Partitioning (3)

Proof (continued).

Together, this leads to the following inequality (subscripts indicate for which task the heuristic is computed):

$$h_{\Pi}^{\mathrm{UCP}}(\mathcal{L}) = \sum_{L \in \mathcal{L}} cost'(L) \overset{(1)}{\leq} \sum_{L \in \mathcal{L}} h^*_{\Pi_L}(s)$$

$$\overset{(2)}{\leq} \sum_{L \in \mathcal{L}} \sum_{i=1}^{n} cost_L(o_i) = \sum_{i=1}^{n} \sum_{L \in \mathcal{L}} cost_L(o_i)$$

$$\overset{(3)}{=} \sum_{i=1}^{n} cost(o) = h^*_{\Pi}(s)$$

□

Relationship

Theorem

Let $\mathcal{L}$ be a set of disjunctive action landmarks for state s.

Then $h^{UCP}(\mathcal{L}) \leq h^{MHS}(\mathcal{L}) \leq h^*(s)$.

(Proof omitted.)

Summary

- Landmarks describe properties that are shared by all plans of a task.
- Hitting sets yield the most accurate heuristic for a given set of disjunctive action landmarks, but the computation is NP-hard.
- Uniform cost partitioning is a polynomial approach for the computation of informative heuristics from disjunctive action landmarks.