Planning and Optimization

C13. Merge-and-Shrink Abstractions: Synchronized Product

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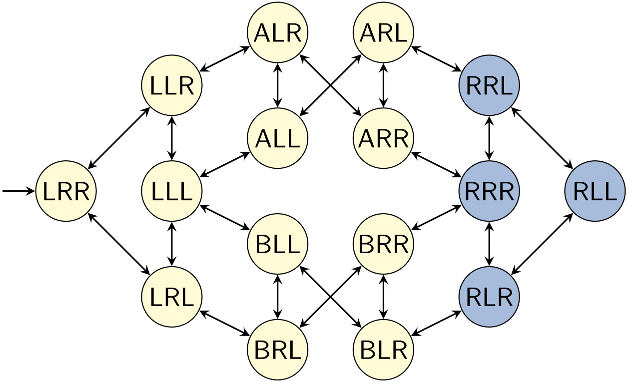
Motivation

Beyond Pattern Databases

- Despite their popularity, pattern databases have some fundamental limitations ($\rightsquigarrow$ example on next slides).
- This week, we study a class of abstractions called merge-and-shrink abstractions.
- Merge-and-shrink abstractions can be seen as a proper generalization of pattern databases.
  - They can do everything that pattern databases can do (modulo polynomial extra effort).
  - They can do some things that pattern databases cannot.

Back to the Running Example

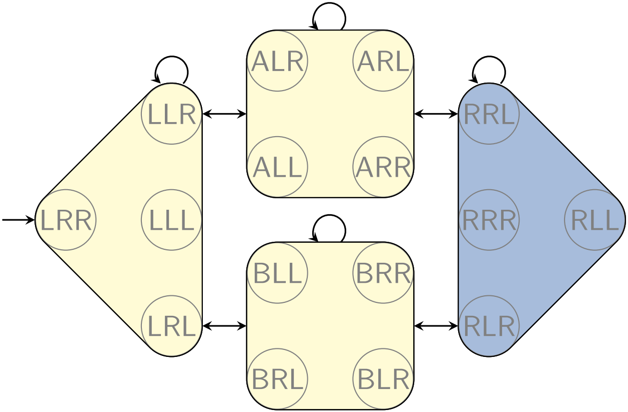


Logistics problem with one package, two trucks, two locations:

- state variable package: $\{L, R, A, B\}$
- state variable truck A: $\{L, R\}$
- state variable truck B: $\{L, R\}$

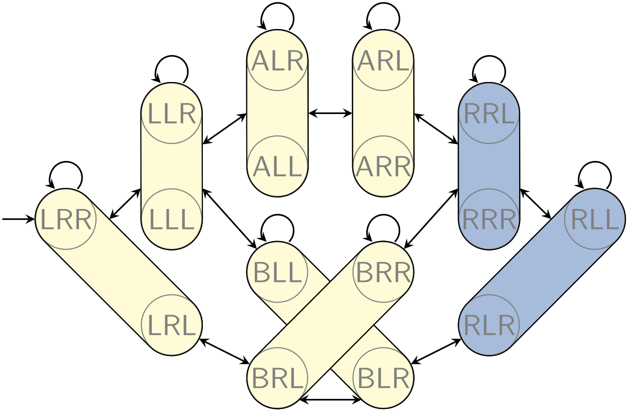
Example: Projection

$\mathcal{T}^{\pi_{\{\text{package}\}}}$:



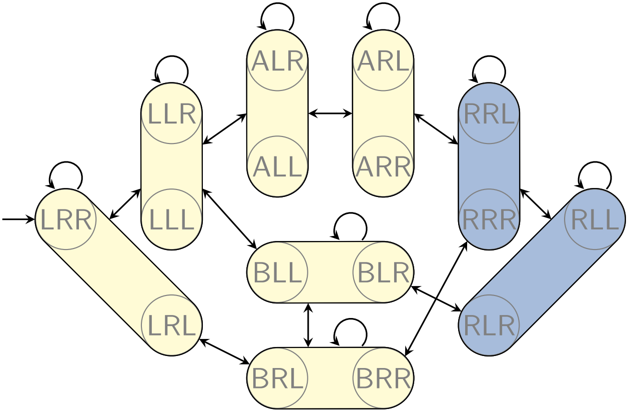
Example: Projection (2)

$\mathcal{T}^{\pi_{\{\text{package},\text{truck A}\}}}$:



Example: Projection (2)

$\mathcal{T}^{\pi_{\{\text{package},\text{truck A}\}}}$:



Limitations of Projections

How accurate is the PDB heuristic?

- consider generalization of the example:
  N trucks, M locations (fully connected), still one package
- consider any pattern that is a proper subset of variable set V.
- $h(s_0) \leq 2 \rightsquigarrow$ no better than atomic projection to package

These values cannot be improved by maximizing over several patterns or using additive patterns.

Merge-and-shrink abstractions can represent heuristics with $h(s_0) \geq 3$ for tasks of this kind of any size.
Time and space requirements are polynomial in N and M.

Merge-and-shrink Abstractions: Main Idea

Main Idea of Merge-and-shrink Abstractions

(due to Dräger, Finkbeiner & Podelski, 2006):

Instead of perfectly reflecting a few state variables, reflect all state variables, but in a potentially lossy way.

The Need for Succinct Abstractions

- One major difficulty for non-PDB abstraction heuristics is to succinctly represent the abstraction.
- For pattern databases, this is easy because the abstractions – projections – are very structured.
- For less rigidly structured abstractions, we need another idea.

Merge-and-shrink Abstractions: Idea

- The main idea underlying merge-and-shrink abstractions is that given two abstract transition systems $\mathcal{A}$ and $\mathcal{A}'$, we can merge them into a new abstract product transition system.
- The product transition system captures all information of both transition systems and can be better informed than either.
- It can even be better informed than their sum.
- By merging a set of very simple abstractions, we can in theory represent arbitrary abstractions of an SAS$^+$ task.
- In practice, due to memory limitations, such systems can become too large. In that case, we can shrink them by abstracting them further using any abstraction on an intermediate result, then continue the merging process.

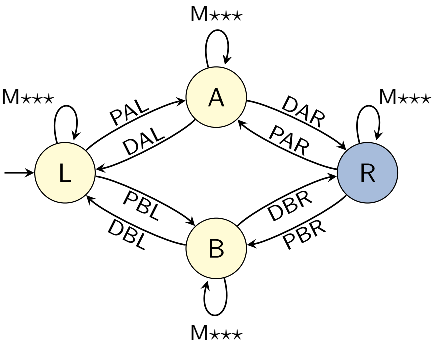
Synchronized Products

Running Example: Explanations

- Atomic projections – projections to a single state variable – play an important role for merge-and-shrink abstractions.
- Unlike previous chapters, transition labels are critically important for this topic.
- Hence we now look at the transition systems for atomic projections of our example task, including transition labels.
- We abbreviate operator names as in these examples:
  - MALR: move truck A from left to right
  - DAR: drop package from truck A at right location
  - PBL: pick up package with truck B at left location
- We abbreviate parallel arcs with commas and wildcards ($\star$) in the labels as in these examples:
  - PAL, DAL: two parallel arcs labeled PAL and DAL
  - MA$\star\star$: two parallel arcs labeled MALR and MARL

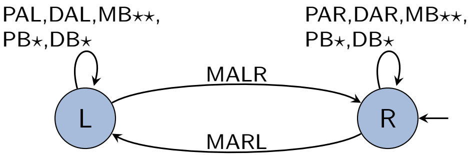
Running Example: Atomic Projection for Package

$\mathcal{T}^{\pi_{\{\text{package}\}}}$:



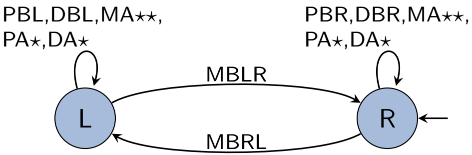
Running Example: Atomic Projection for Truck A

$\mathcal{T}^{\pi_{\{\text{truck A}\}}}$:



Running Example: Atomic Projection for Truck B

$\mathcal{T}^{\pi_{\{\text{truck B}\}}}$:



Synchronized Product of Transition Systems

Definition (Synchronized Product of Transition Systems)

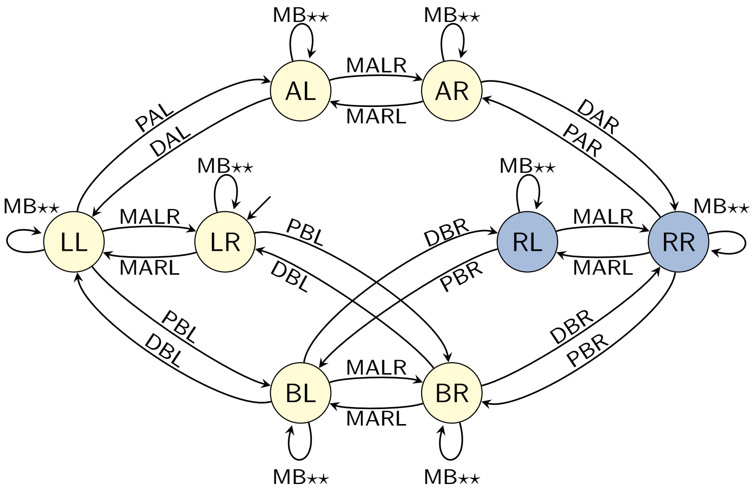
For $i \in \{1, 2\}$, let $\mathcal{T}_i = \langle S_i, L, c, T_i, s_{0i}, S_{\star i} \rangle$ be transition systems with identical label set and identical label cost function.

The synchronized product of $\mathcal{T}_1$ and $\mathcal{T}_2$, in symbols $\mathcal{T}_1 \otimes \mathcal{T}_2$, is the transition system $\mathcal{T}_\otimes = \langle S_\otimes, L, c, T_\otimes, s_{0\otimes}, S_{\star\otimes} \rangle$ with

- $S_\otimes := S_1 \times S_2$
- $T_\otimes := \{\langle\langle s_1, s_2\rangle, l, \langle t_1, t_2\rangle\rangle \mid \langle s_1, l, t_1\rangle \in T_1$ and $\langle s_2, l, t_2\rangle \in T_2\}$
- $s_{0\otimes} := \langle s_{01}, s_{02}\rangle$
- $S_{\star\otimes} := S_{\star 1} \times S_{\star 2}$

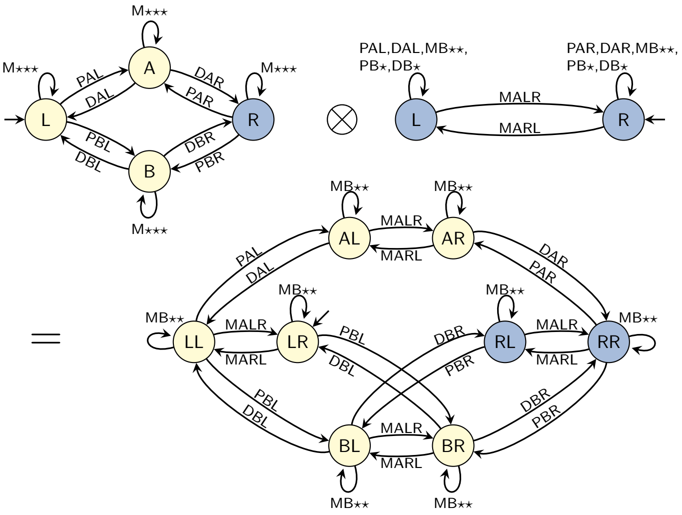
Example: Synchronized Product

$\mathcal{T}^{\pi\{\text{package}\}} \otimes \mathcal{T}^{\pi\{\text{truck A}\}}$:



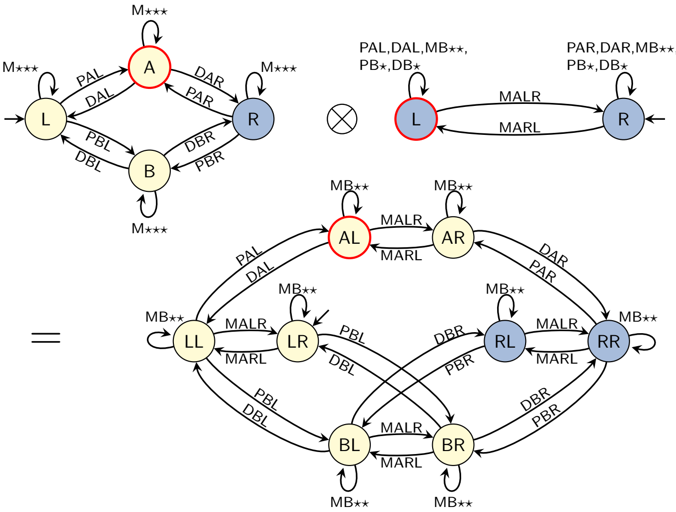
Example: Computation of Synchronized Product

$\mathcal{T}^{\pi\{\text{package}\}} \otimes \mathcal{T}^{\pi\{\text{truck A}\}}$:



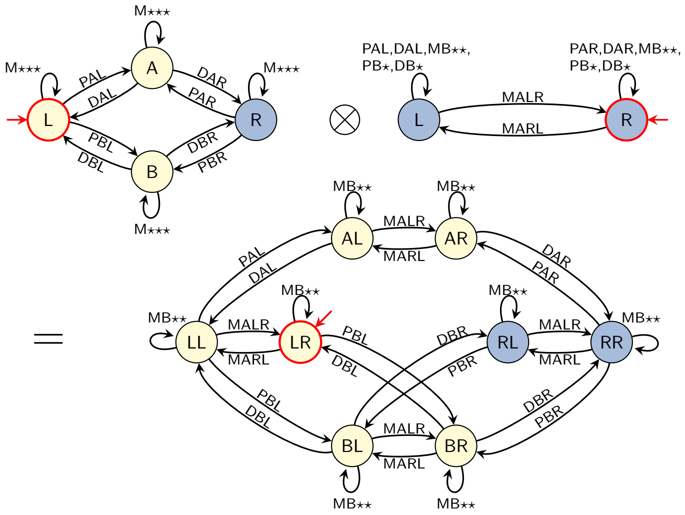
Example: Computation of Synchronized Product

$\mathcal{T}^{\pi\{\text{package}\}} \otimes \mathcal{T}^{\pi\{\text{truck A}\}}$: $S_\otimes = S_1 \times S_2$



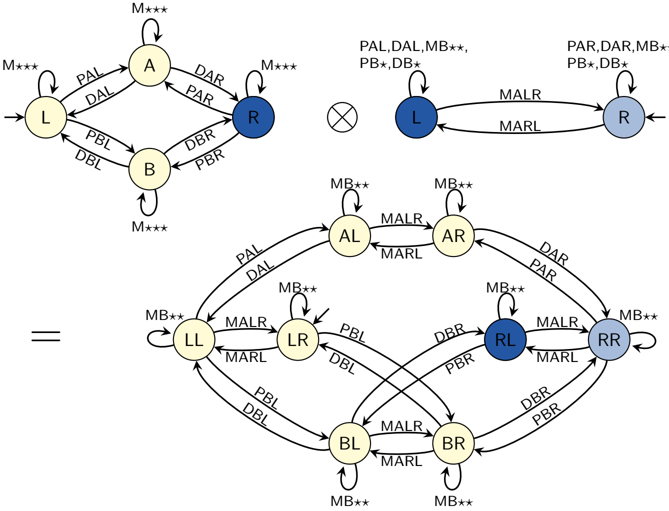
Example: Computation of Synchronized Product

$\mathcal{T}^{\pi\{\text{package}\}} \otimes \mathcal{T}^{\pi\{\text{truck A}\}}$: $s_{0\otimes} = \langle s_{01}, s_{02} \rangle$



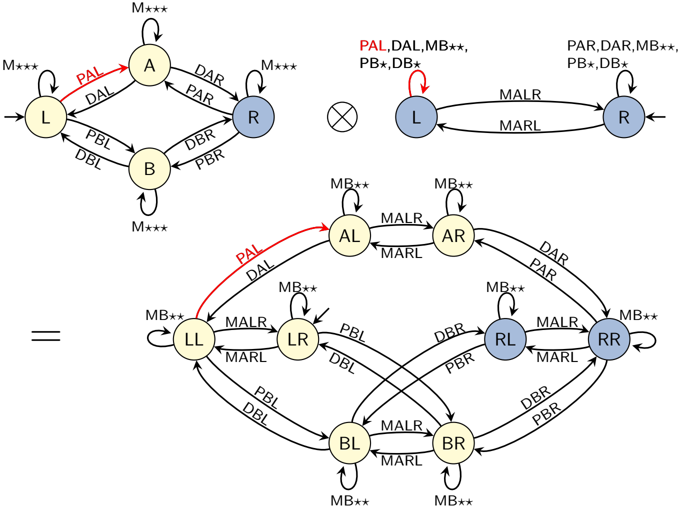
Example: Computation of Synchronized Product

$\mathcal{T}^{\pi_{\{\text{package}\}}} \otimes \mathcal{T}^{\pi_{\{\text{truck A}\}}}$: $S_{\star\otimes} = S_{\star 1} \times S_{\star 2}$



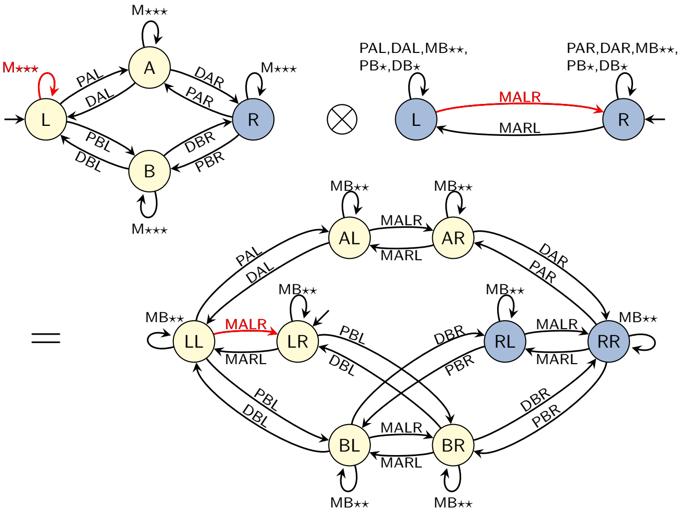
Example: Computation of Synchronized Product

$\mathcal{T}^{\pi\{\text{package}\}} \otimes \mathcal{T}^{\pi\{\text{truck A}\}}$: $T_{\otimes} := \{\langle\langle s_1, s_2\rangle, l, \langle t_1, t_2\rangle\rangle \mid \dots\}$



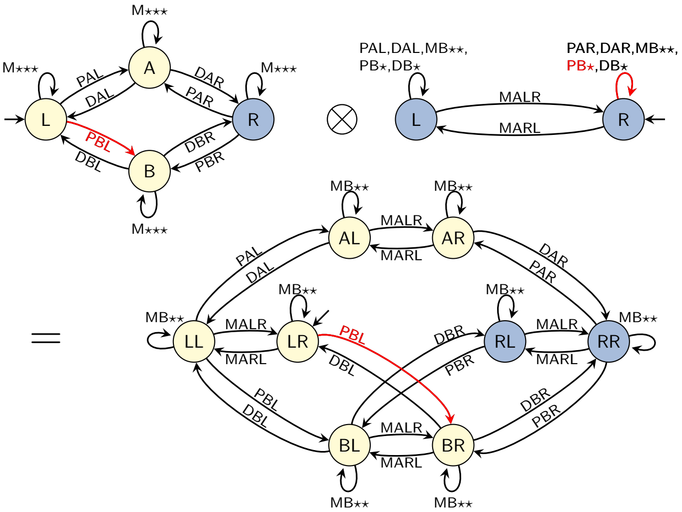
Example: Computation of Synchronized Product

$\mathcal{T}^{\pi\{\text{package}\}} \otimes \mathcal{T}^{\pi\{\text{truck A}\}}$: $T_\otimes := \{\langle\langle s_1, s_2\rangle, l, \langle t_1, t_2\rangle\rangle \mid \dots\}$



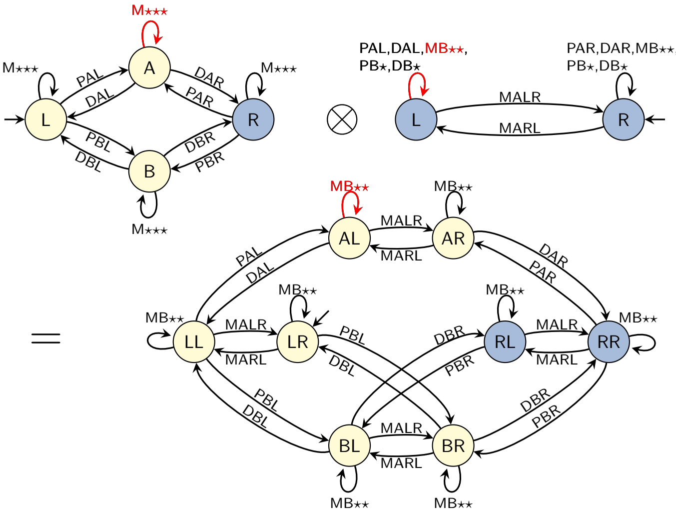
Example: Computation of Synchronized Product

$\mathcal{T}^{\pi\{\text{package}\}} \otimes \mathcal{T}^{\pi\{\text{truck A}\}}$: $T_{\otimes} := \{\langle\langle s_1, s_2\rangle, l, \langle t_1, t_2\rangle\rangle \mid \dots\}$



Example: Computation of Synchronized Product

$\mathcal{T}^{\pi\{\text{package}\}} \otimes \mathcal{T}^{\pi\{\text{truck A}\}}$: $T_\otimes := \{\langle\langle s_1, s_2\rangle, l, \langle t_1, t_2\rangle\rangle \mid \ldots\}$



Synchronized Product of Functions

Definition (Synchronized Product of Functions)

Let $\alpha_1 : S \to S_1$ and $\alpha_2 : S \to S_2$ be functions with identical domain.

The synchronized product of α_1 and α_2, in symbols $\alpha_1 \otimes \alpha_2$, is the function $\alpha_\otimes : S \to S_1 \times S_2$ defined as $\alpha_\otimes(s) = \langle \alpha_1(s), \alpha_2(s) \rangle$.

Synchronized Product of Abstractions

Theorem

Let α_1 and α_2 be abstractions of transition system $\mathcal{T}$ such that $\alpha_\otimes := \alpha_1 \otimes \alpha_2$ is surjective.

Then $\alpha_\otimes$ is an abstraction of $\mathcal{T}$ and a refinement of α_1 and α_2.

Proof.

Abstraction: suitable domain as α_1, α_2 are abstractions of $\mathcal{T}$, surjective by premise

Refinement: For $i \in \{1, 2\}$, $\alpha_i = \beta_i \circ \alpha_\otimes$ with $\beta_i(\langle x_1, x_2 \rangle) = x_i$. □

Relationship of $\mathcal{T}^{\alpha_1 \otimes \alpha_2}$ and $\mathcal{T}^{\alpha_1} \otimes \mathcal{T}^{\alpha_2}$ (1)

Theorem ($\mathcal{T}^{\alpha_1 \otimes \alpha_2}$ is Substructure of $\mathcal{T}^{\alpha_1} \otimes \mathcal{T}^{\alpha_2}$)

Let α_1 and α_2 be abstractions of transition system $\mathcal{T}$ such that $\alpha_\otimes := \alpha_1 \otimes \alpha_2$ is surjective.

For $\mathcal{T}^{\alpha_1 \otimes \alpha_2} = \langle S_{\alpha_1 \otimes \alpha_2}, L, c, T_{\alpha_1 \otimes \alpha_2}, s_{0\alpha_1 \otimes \alpha_2}, S_{\star\alpha_1 \otimes \alpha_2} \rangle$ and $\mathcal{T}^{\alpha_1} \otimes \mathcal{T}^{\alpha_2} = \langle S_\otimes, L, c, T_\otimes, s_{0\otimes}, S_{\star\otimes} \rangle$ it holds that

- $S_{\alpha_1 \otimes \alpha_2} = S_\otimes$
- $T_{\alpha_1 \otimes \alpha_2} \subseteq T_\otimes$
- $s_{0\alpha_1 \otimes \alpha_2} = s_{0\otimes}$
- $S_{\star\alpha_1 \otimes \alpha_2} \subseteq S_{\star\otimes}$

Remark: If $\alpha_\otimes$ is not surjective, then the proof also holds if we restrict $\mathcal{T}^{\alpha_1} \otimes \mathcal{T}^{\alpha_2}$ and the codomain of $\alpha_\otimes$ to the states in the image of $\alpha_\otimes$.

Relationship of $\mathcal{T}^{\alpha_1 \otimes \alpha_2}$ and $\mathcal{T}^{\alpha_1} \otimes \mathcal{T}^{\alpha_2}$ (2)

Proof.

Let $\mathcal{T} = \langle S, L, c, T, s_0, S_\star \rangle$ and
for $i \in \{1, 2\}$ let $\mathcal{T}^{\alpha_i} = \langle S_i, L, c, T_i, s_{0i}, S_{\star i} \rangle$ (with $\alpha_i : S \to S_i$).

$$
\begin{aligned}
S_{\alpha_1 \otimes \alpha_2} &= S_1 \times S_2 = S_\otimes \\
s_{0\alpha_1 \otimes \alpha_2} &= \alpha_1 \otimes \alpha_2(s_0) = \langle \alpha_1(s_0), \alpha_2(s_0) \rangle = \langle s_{01}, s_{02} \rangle = s_{0\otimes} \\
S_{\star \alpha_1 \otimes \alpha_2} &= \{\alpha_1 \otimes \alpha_2(s) \mid s \in S_\star\} \\
&= \{\langle \alpha_1(s), \alpha_2(s) \rangle \mid s \in S_\star\} \\
&\subseteq \{\langle \alpha_1(s), \alpha_2(s') \rangle \mid s, s' \in S_\star\} \\
&= \{\langle s_1, s_2 \rangle \mid s_1 \in S_{\star 1}, s_2 \in S_{\star 2}\} \\
&= S_{\star 1} \times S_{\star 2} \\
&= S_{\star \otimes}
\end{aligned}
$$

...

Relationship of $\mathcal{T}^{\alpha_1 \otimes \alpha_2}$ and $\mathcal{T}^{\alpha_1} \otimes \mathcal{T}^{\alpha_2}$ (3)

Proof (continued).

Let $\mathcal{T} = \langle S, L, c, T, s_0, S_\star \rangle$ and
for $i \in \{1, 2\}$ let $\mathcal{T}^{\alpha_i} = \langle S_i, L, c, T_i, s_{0i}, S_{\star i} \rangle$ (with $\alpha_i : S \to S_i$).

$$\begin{aligned}
T_{\alpha_1 \otimes \alpha_2} &= \{\langle \alpha_1 \otimes \alpha_2(s), \ell, \alpha_1 \otimes \alpha_2(t) \rangle \mid \langle s, \ell, t \rangle \in T\} \\
&= \{\langle \langle \alpha_1(s), \alpha_2(s) \rangle, \ell, \langle \alpha_1(t), \alpha_2(t) \rangle \rangle \mid \langle s, \ell, t \rangle \in T\} \\
&\subseteq \{\langle \langle \alpha_1(s), \alpha_2(s') \rangle, \ell, \langle \alpha_1(t), \alpha_2(t') \rangle \rangle \\
&\qquad\qquad \mid \langle s, \ell, t \rangle, \langle s', \ell, t' \rangle \in T\} \\
&= \{\langle \langle s_1, s_2 \rangle, \ell, \langle t_1, t_2 \rangle \rangle \mid \langle s_1, \ell, t_1 \rangle \in T_1, \langle s_2, \ell, t_2 \rangle \in T_2\} \\
&= T_\otimes
\end{aligned}$$

□

Preserving Abstractions

- It would be very nice if we could also prove that if α_1 and α_2 are abstractions of $\mathcal{T}$ then there is an abstraction of $\mathcal{T}$ inducing $\mathcal{T}^{\alpha_1} \otimes \mathcal{T}^{\alpha_2}$.
- However, this is not true in general.
- It is not even true for SAS$^+$ tasks.
- However, there is an important sufficient condition for preserving the abstraction property.

Synchronized Products and Abstractions

Theorem (Synchronized Products and Abstractions)

Let Π *be an* SAS$^+$ *planning task with variable set* V*, and let* V_1 *and* V_2 *be disjoint subsets of* V*.*

For $i \in \{1, 2\}$*, let* α_i *be an abstraction of* $\mathcal{T}(\Pi)$ *such that* α_i *is a coarsening of* π_{V_i}*.*

Then $\alpha_\otimes := \alpha_1 \otimes \alpha_2$ *is surjective and* $\mathcal{T}^{\alpha_1 \otimes \alpha_2} = \mathcal{T}^{\alpha_1} \otimes \mathcal{T}^{\alpha_2}$*.*

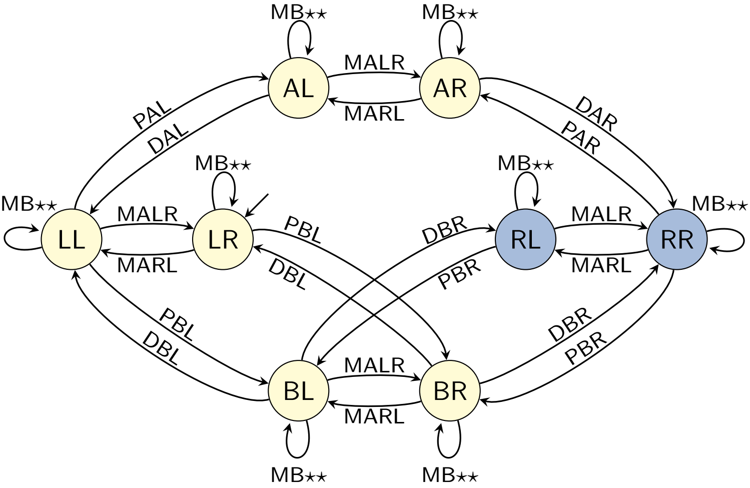
Corollary (Synchronized Products of Projections)

Let Π *be an* SAS$^+$ *planning task with variable set* V*, and let* V_1 *and* V_2 *be disjoint subsets of* V*.*
Then $\mathcal{T}^{\pi_{V_1}} \otimes \mathcal{T}^{\pi_{V_2}} \sim \mathcal{T}^{\pi_{V_1 \cup V_2}}$*.*

(Proofs omitted.)

Example: Product for Disjoint Projections

$\mathcal{T}^{\pi_{\{\text{package}\}}} \otimes \mathcal{T}^{\pi_{\{\text{truck A}\}}} \sim \mathcal{T}^{\pi_{\{\text{package},\text{truck A}\}}}$:



Recovering $\mathcal{T}(\Pi)$ from the Atomic Projections

- By repeated application of the corollary, we can recover all pattern database heuristics of an SAS$^+$ planning task from the abstract transition systems induced by atomic projections.
- In particular, by computing the product of all atomic projections, we can recover the identity abstraction $\mathrm{id} = \pi_V$.

Corollary (Recovering $\mathcal{T}(\Pi)$ from the Atomic Projections)

Let Π be an SAS$^+$ planning task with variable set V. Then $\mathcal{T}(\Pi) \sim \bigotimes_{v \in V} \mathcal{T}^{\pi_{\{v\}}}$.

- This is an important result because it shows that the transition systems induced by atomic projections contain all information of an SAS$^+$ task.

Summary

Summary

- The synchronized product of two transition systems captures "what we can do" in both systems "in parallel".
- With suitable abstractions, the synchronized product of the induced transition systems is induced by the synchronized product of the abstractions.
- We can recover the original transition system from the abstract transition systems induced by the atomic projections.