Planning and Optimization

B3. General Regression, Part I

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Regression for General Planning Tasks

- With disjunctions and conditional effects, things become more tricky. How to regress $a \lor (b \land c)$ with respect to $\langle q, d \rhd b \rangle$?
- In this chapter, we show how to regress general sets of states through general operators.
- We extensively use the idea of representing sets of states as formulas.

Regressing State Variables

Regressing State Variables: Motivation

Key question for general regression:

- Assume we are applying an operator with effect e.
- What must be true in the predecessor state
  for state variable v to be true in the successor state?

If we can answer this question, a general definition of regression is only a small additional step.

Regressing State Variables: Key Idea

Assume we are in state s and apply effect e to obtain successor state s'.

State variable v is true in s' if

- effect e makes it true, or
- it remains true, i.e., it is true in s and not made false by e.

Regressing a State Variable Through an Effect

Definition (Regressing a State Variable Through an Effect)

Let e be an effect, and let v be a state variable.

The regression of v through e, written $regr_e(v)$,
is defined as the following logical formula:

$$regr_e(v) = effcond_v(\tilde{e}) \lor (v \land \neg effcond_{\neg v}(\tilde{e})),$$

where $\tilde{e}$ is e converted to normal form.

Questions:

- Is this well-defined?
- Why do we require normal form?

Regressing State Variables: Example

Example

Let $e = (b \rhd a) \wedge (c \rhd \neg a) \wedge b \wedge \neg d$.

$\leadsto \tilde{e} = (b \rhd a) \wedge ((c \wedge \neg b) \rhd \neg a) \wedge (\top \rhd b) \wedge (\top \rhd \neg d)$.

v	$regr_e(v)$
a	$b \vee (a \wedge \neg(c \wedge \neg b)) \equiv b \vee (a \wedge \neg c)$
b	$\top \vee (b \wedge \neg\bot) \equiv \top$
c	$\bot \vee (c \wedge \neg\bot) \equiv c$
d	$\bot \vee (d \wedge \neg\top) \equiv \bot$

Reminder: $regr_e(v) = effcond_v(\tilde{e}) \vee (v \wedge \neg effcond_{\neg v}(\tilde{e}))$

Regressing State Variables: Correctness (1)

Lemma (Correctness of $regr_e(v)$)

Let v be a state variable, o an operator and s a state in which o is applicable.

Then $s \models regr_{eff(o)}(v)$ iff $s[\![o]\!] \models v$.

($\Rightarrow$): We know $s \models regr_{eff(o)}(v)$, and hence $s \models effcond_v(\tilde{e}) \lor (v \land \neg effcond_{\neg v}(\tilde{e}))$.

Do a case analysis on the two disjuncts.

Case 1: $s \models effcond_v(\tilde{e})$. Then $v \in [\tilde{e}]_s$ and hence $s[\![o]\!] \models v$.

Case 2: $s \models (v \land \neg effcond_{\neg v}(\tilde{e}))$. Then $s \models v$ and $\neg v \notin [\tilde{e}]_s$, and hence $s[\![o]\!] \models v$. ...

Regressing State Variables: Correctness (1)

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Let v be a state variable, o an operator and s a state in which o is applicable.

Then $s \models regr_{eff(o)}(v)$ iff $s[\![o]\!] \models v$.

Proof.

Let $\tilde{e}$ be $eff(o)$ converted to normal form.

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($\Rightarrow$): We know $s \models regr_{eff(o)}(v)$, and hence $s \models effcond_v(\tilde{e}) \lor (v \land \neg effcond_{\neg v}(\tilde{e}))$.

Do a case analysis on the two disjuncts.

Case 1: $s \models effcond_v(\tilde{e})$. Then $v \in [\tilde{e}]_s$ and hence $s[\![o]\!] \models v$.

Case 2: $s \models (v \land \neg effcond_{\neg v}(\tilde{e}))$. Then $s \models v$ and $\neg v \notin [\tilde{e}]_s$, and hence $s[\![o]\!] \models v$.

...

Regressing State Variables: Correctness (2)

Proof (continued).

($\Leftarrow$): Proof by contraposition.

We show that if $regr_{eff(o)}(v)$ is false in s, then v is false in $s[\![o]\!]$.

Regressing State Variables: Correctness (2)

Proof (continued).

($\Leftarrow$): Proof by contraposition.

We show that if $regr_{eff(o)}(v)$ is false in s, then v is false in $s[\![o]\!]$.

- By prerequisite, $s \not\models effcond_v(\tilde{e}) \lor (v \land \neg effcond_{\neg v}(\tilde{e}))$.

Regressing State Variables: Correctness (2)

Proof (continued).

($\Leftarrow$): Proof by contraposition.

We show that if $regr_{eff(o)}(v)$ is false in s, then v is false in $s[\![o]\!]$.

- By prerequisite, $s \not\models effcond_v(\tilde{e}) \lor (v \land \neg effcond_{\neg v}(\tilde{e}))$.
- Hence $s \models \neg effcond_v(\tilde{e}) \land (\neg v \lor effcond_{\neg v}(\tilde{e}))$.

Regressing State Variables: Correctness (2)

Proof (continued).

($\Leftarrow$): Proof by contraposition.

We show that if $regr_{eff(o)}(v)$ is false in s, then v is false in $s[\![o]\!]$.

- By prerequisite, $s \not\models effcond_v(\tilde{e}) \vee (v \wedge \neg effcond_{\neg v}(\tilde{e}))$.
- Hence $s \models \neg effcond_v(\tilde{e}) \wedge (\neg v \vee effcond_{\neg v}(\tilde{e}))$.
- From the first conjunct, we get $s \models \neg effcond_v(\tilde{e})$, which implies $s \not\models effcond_v(\tilde{e})$ and therefore $v \notin [\tilde{e}]_s$.

Regressing State Variables: Correctness (2)

Proof (continued).

($\Leftarrow$): Proof by contraposition.

We show that if $regr_{eff(o)}(v)$ is false in s, then v is false in $s[\![o]\!]$.

- By prerequisite, $s \not\models effcond_v(\tilde{e}) \lor (v \land \lnot effcond_{\lnot v}(\tilde{e}))$.
- Hence $s \models \lnot effcond_v(\tilde{e}) \land (\lnot v \lor effcond_{\lnot v}(\tilde{e}))$.
- From the first conjunct, we get $s \models \lnot effcond_v(\tilde{e})$, which implies $s \not\models effcond_v(\tilde{e})$ and therefore $v \notin [\tilde{e}]_s$.
- From the second conjunct, we get $s \models \lnot v \lor effcond_{\lnot v}(\tilde{e})$.

Regressing State Variables: Correctness (2)

Proof (continued).

($\Leftarrow$): Proof by contraposition.

We show that if $regr_{eff(o)}(v)$ is false in s, then v is false in $s[\![o]\!]$.

- By prerequisite, $s \not\models effcond_v(\tilde{e}) \lor (v \land \neg effcond_{\neg v}(\tilde{e}))$.
- Hence $s \models \neg effcond_v(\tilde{e}) \land (\neg v \lor effcond_{\neg v}(\tilde{e}))$.
- From the first conjunct, we get $s \models \neg effcond_v(\tilde{e})$, which implies $s \not\models effcond_v(\tilde{e})$ and therefore $v \notin [\tilde{e}]_s$.
- From the second conjunct, we get $s \models \neg v \lor effcond_{\neg v}(\tilde{e})$.
- Case 1: $s \models \neg v$. Then v is false before applying o and remains false, so $s[\![o]\!] \not\models v$.

Regressing State Variables: Correctness (2)

Proof (continued).

(⇐): Proof by contraposition.

We show that if $regr_{eff(o)}(v)$ is false in s, then v is false in $s[\![o]\!]$.

- By prerequisite, $s \not\models effcond_v(\tilde{e}) \lor (v \land \neg effcond_{\neg v}(\tilde{e}))$.
- Hence $s \models \neg effcond_v(\tilde{e}) \land (\neg v \lor effcond_{\neg v}(\tilde{e}))$.
- From the first conjunct, we get $s \models \neg effcond_v(\tilde{e})$, which implies $s \not\models effcond_v(\tilde{e})$ and therefore $v \notin [\tilde{e}]_s$.
- From the second conjunct, we get $s \models \neg v \lor effcond_{\neg v}(\tilde{e})$.
- Case 1: $s \models \neg v$. Then v is false before applying o and remains false, so $s[\![o]\!] \not\models v$.
- Case 2: $s \models effcond_{\neg v}(\tilde{e})$. Then v is deleted by o and not simultaneously added, so $s[\![o]\!] \not\models v$.

□

Regressing Formulas Through Effects

Regressing Formulas Through Effects: Idea

- We can now generalize regression from state variables to general formulas over state variables.
- The basic idea is to replace every occurrence of every state variable v by $regr_e(v)$ as defined in the previous section.
- The following definition makes this more formal.

Regressing Formulas Through Effects: Definition

Definition (Regressing a Formula Through an Effect)

Let e be an effect, and let φ be a formula over state variables.

The regression of φ through e, written $regr_e(\varphi)$, is defined as the following logical formula:

$$
\begin{aligned}
regr_e(\top) &= \top \\
regr_e(\bot) &= \bot \\
regr_e(v) &= effcond_v(\tilde{e}) \lor (v \land \lnot effcond_{\lnot v}(\tilde{e})) \\
regr_e(\lnot\psi) &= \lnot regr_e(\psi) \\
regr_e(\psi \lor \chi) &= regr_e(\psi) \lor regr_e(\chi) \\
regr_e(\psi \land \chi) &= regr_e(\psi) \land regr_e(\chi),
\end{aligned}
$$

where $\tilde{e}$ is e converted to normal form.

Regressing Formulas Through Effects: Example

Example

Let $e = (b \rhd a) \wedge (c \rhd \neg a) \wedge b \wedge \neg d$.

Recall:

- $regr_e(a) \equiv b \vee (a \wedge \neg c)$
- $regr_e(b) \equiv \top$
- $regr_e(c) \equiv c$
- $regr_e(d) \equiv \bot$

We get:

$$\begin{aligned} regr_e((a \vee d) \wedge (c \vee d)) &\equiv ((b \vee (a \wedge \neg c)) \vee \bot) \wedge (c \vee \bot) \\ &\equiv (b \vee (a \wedge \neg c)) \wedge c \\ &\equiv b \wedge c \end{aligned}$$

Regressing Formulas Through Effects: Correctness (1)

Lemma (Correctness of $regr_e(\varphi)$)

Let φ be a logical formula, o an operator and s a state in which o is applicable.

Then $s \models regr_{eff(o)}(\varphi)$ iff $s[\![o]\!] \models \varphi$.

Regressing Formulas Through Effects: Correctness (2)

Proof.

The proof is by structural induction on φ.

Regressing Formulas Through Effects: Correctness (2)

Proof.

The proof is by structural induction on φ.

Induction hypothesis: $s \models \mathit{regr}_{\mathit{eff}(o)}(\psi)$ iff $s[\![o]\!] \models \psi$.

Regressing Formulas Through Effects: Correctness (2)

Proof.

The proof is by structural induction on φ.

Induction hypothesis: $s \models regr_{eff(o)}(\psi)$ iff $s[\![o]\!] \models \psi$.

Base case $\varphi = \top$:

We have $regr_{eff(o)}(\top) = \top$, and $s \models \top$ iff $s[\![o]\!] \models \top$ is correct.

Regressing Formulas Through Effects: Correctness (2)

Proof.

The proof is by structural induction on φ.

Induction hypothesis: $s \models regr_{eff(o)}(\psi)$ iff $s[\![o]\!] \models \psi$.

Base case $\varphi = \top$:

We have $regr_{eff(o)}(\top) = \top$, and $s \models \top$ iff $s[\![o]\!] \models \top$ is correct.

Base case $\varphi = \bot$:

We have $regr_{eff(o)}(\bot) = \bot$, and $s \models \bot$ iff $s[\![o]\!] \models \bot$ is correct.

Regressing Formulas Through Effects: Correctness (2)

Proof.

The proof is by structural induction on φ.

Induction hypothesis: $s \models regr_{eff(o)}(\psi)$ iff $s[\![o]\!] \models \psi$.

Base case $\varphi = \top$:

We have $regr_{eff(o)}(\top) = \top$, and $s \models \top$ iff $s[\![o]\!] \models \top$ is correct.

Base case $\varphi = \bot$:

We have $regr_{eff(o)}(\bot) = \bot$, and $s \models \bot$ iff $s[\![o]\!] \models \bot$ is correct.

Base case $\varphi = v$:

We have $s \models regr_{eff(o)}(v)$ iff $s[\![o]\!] \models v$ from the previous lemma.

...

Regressing Formulas Through Effects: Correctness (3)

Proof (continued).

Inductive case $\varphi = \neg\psi$:

$$\begin{aligned} s \models \mathit{regr}_{\mathit{eff}(o)}(\neg\psi) \quad & \text{iff} \quad s \models \neg\mathit{regr}_{\mathit{eff}(o)}(\psi) \\ & \text{iff} \quad s \not\models \mathit{regr}_{\mathit{eff}(o)}(\psi) \\ & \text{iff} \quad s[\![o]\!] \not\models \psi \\ & \text{iff} \quad s[\![o]\!] \models \neg\psi \end{aligned}$$

Regressing Formulas Through Effects: Correctness (3)

Proof (continued).

Inductive case $\varphi = \neg\psi$:

$$\begin{aligned} s \models \mathit{regr}_{\mathit{eff}(o)}(\neg\psi) \quad &\text{iff} \quad s \models \neg\mathit{regr}_{\mathit{eff}(o)}(\psi) \\ &\text{iff} \quad s \not\models \mathit{regr}_{\mathit{eff}(o)}(\psi) \\ &\text{iff} \quad s[\![o]\!] \not\models \psi \\ &\text{iff} \quad s[\![o]\!] \models \neg\psi \end{aligned}$$

Inductive case $\varphi = \psi \vee \chi$:

$$\begin{aligned} s \models \mathit{regr}_{\mathit{eff}(o)}(\psi \vee \chi) \quad &\text{iff} \quad s \models \mathit{regr}_{\mathit{eff}(o)}(\psi) \vee \mathit{regr}_{\mathit{eff}(o)}(\chi) \\ &\text{iff} \quad s \models \mathit{regr}_{\mathit{eff}(o)}(\psi) \text{ or } s \models \mathit{regr}_{\mathit{eff}(o)}(\chi) \\ &\text{iff} \quad s[\![o]\!] \models \psi \text{ or } s[\![o]\!] \models \chi \\ &\text{iff} \quad s[\![o]\!] \models \psi \vee \chi \end{aligned}$$

Regressing Formulas Through Effects: Correctness (3)

Proof (continued).

Inductive case $\varphi = \neg\psi$:

$$\begin{aligned} s \models regr_{eff(o)}(\neg\psi) \quad & \text{iff} \quad s \models \neg regr_{eff(o)}(\psi) \\ & \text{iff} \quad s \not\models regr_{eff(o)}(\psi) \\ & \text{iff} \quad s[\![o]\!] \not\models \psi \\ & \text{iff} \quad s[\![o]\!] \models \neg\psi \end{aligned}$$

Inductive case $\varphi = \psi \vee \chi$:

$$\begin{aligned} s \models regr_{eff(o)}(\psi \vee \chi) \quad & \text{iff} \quad s \models regr_{eff(o)}(\psi) \vee regr_{eff(o)}(\chi) \\ & \text{iff} \quad s \models regr_{eff(o)}(\psi) \text{ or } s \models regr_{eff(o)}(\chi) \\ & \text{iff} \quad s[\![o]\!] \models \psi \text{ or } s[\![o]\!] \models \chi \\ & \text{iff} \quad s[\![o]\!] \models \psi \vee \chi \end{aligned}$$

Inductive case $\varphi = \psi \wedge \chi$:

Like previous case, replacing "$\vee$" by "$\wedge$"
and replacing "or" by "and". □

Summary

Summary

- Regressing a state variable through an (arbitrary) operator must consider two cases:
  - state variables made true (by add effects)
  - state variables remaining true (by absence of delete effects)
- Regression of state variables can be generalized to arbitrary formulas φ by replacing each occurrence of a state variable in φ by its regression.