

Planning and Optimization

## Regressing State Variables: Correctness (1)



Let v be a state variable, o an operator and s a state in which o is applicable.

Then  $s \models regr_{eff(o)}(v)$  iff  $s[[o]] \models v$ .

## Proof.

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Let  $\tilde{e}$  be *eff*(o) converted to normal form.

 $(\Rightarrow): We know s \models regr_{eff(o)}(v), and hence$  $s \models effcond_v(\tilde{e}) \lor (v \land \neg effcond_{\neg v}(\tilde{e})).$ Do a case analysis on the two disjuncts. Case 1: s \models effcond\_v(\tilde{e}). Then  $v \in [\tilde{e}]_s$  and hence  $s[\![o]\!] \models v$ . Case 2: s  $\models (v \land \neg effcond_{\neg v}(\tilde{e}))$ . Then  $s \models v$  and  $\neg v \notin [\tilde{e}]_s$ , and hence  $s[\![o]\!] \models v$ . ...

B3.2 Regressing Formulas Through Effects

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Regressing State Variable

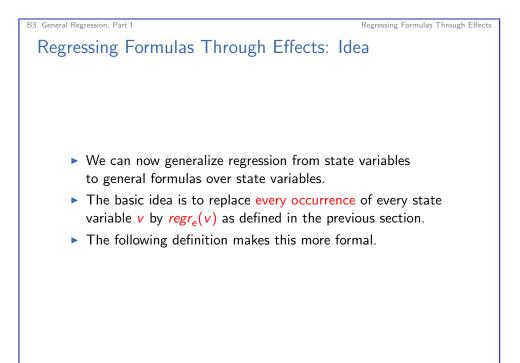
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Regressing Formulas Through Effects

# Regressing State Variables: Correctness (2)

## Proof (continued).

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(⇐): Proof by contraposition.
We show that if regr<sub>eff(o)</sub>(v) is false in s, then v is false in s[[o]].
By prerequisite, s ≠ effcond<sub>v</sub>(ẽ) ∨ (v ∧ ¬effcond<sub>¬v</sub>(ẽ)).
Hence s ⊨ ¬effcond<sub>v</sub>(ẽ) ∧ (¬v ∨ effcond<sub>¬v</sub>(ẽ)).
From the first conjunct, we get s ⊨ ¬effcond<sub>v</sub>(ẽ), which implies s ≠ effcond<sub>v</sub>(ẽ) and therefore v ∉ [ẽ]<sub>s</sub>.
From the second conjunct, we get s ⊨ ¬v ∨ effcond<sub>¬v</sub>(ẽ).
Case 1: s ⊨ ¬v. Then v is false before applying o and remains false, so s[[o]] ≠ v.
Case 2: s ⊨ effcond<sub>¬v</sub>(ẽ). Then v is deleted by o and not simultaneously added, so s[[o]] ≠ v.
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# Regressing Formulas Through Effects: Definition

Definition (Regressing a Formula Through an Effect) Let *e* be an effect, and let  $\varphi$  be a formula over state variables. The regression of  $\varphi$  through *e*, written  $regr_e(\varphi)$ , is defined as the following logical formula:

> $regr_e(\top) = \top$  $regr_{o}(\perp) = \perp$  $regr_{e}(v) = effcond_{v}(\tilde{e}) \lor (v \land \neg effcond_{\neg v}(\tilde{e}))$  $\operatorname{regr}_{a}(\neg\psi) = \neg\operatorname{regr}_{a}(\psi)$  $\operatorname{regr}_{e}(\psi \lor \chi) = \operatorname{regr}_{e}(\psi) \lor \operatorname{regr}_{e}(\chi)$  $\operatorname{regr}_{e}(\psi \wedge \chi) = \operatorname{regr}_{e}(\psi) \wedge \operatorname{regr}_{e}(\chi),$

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where  $\tilde{e}$  is e converted to normal form.

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B3. General Regression, Part I Regressing Formulas Through Effects Regressing Formulas Through Effects: Correctness (1) Lemma (Correctness of  $regr_e(\varphi)$ ) Let  $\varphi$  be a logical formula, o an operator and s a state in which o is applicable. Then  $s \models \operatorname{regr}_{eff(o)}(\varphi)$  iff  $s[[o]] \models \varphi$ . Planning and Optimization

B3. General Regression, Part I

Regressing Formulas Through Effects: Example

## Example

Let  $e = (b \triangleright a) \land (c \triangleright \neg a) \land b \land \neg d$ .

### Recall:

- $regr_e(a) \equiv b \lor (a \land \neg c)$
- ▶  $regr_{e}(b) \equiv \top$
- ▶  $regr_e(c) \equiv c$
- $regr_{e}(d) \equiv \bot$

We get:

$$\begin{aligned} \operatorname{regr}_e((a \lor d) \land (c \lor d)) &\equiv ((b \lor (a \land \neg c)) \lor \bot) \land (c \lor \bot) \\ &\equiv (b \lor (a \land \neg c)) \land c \\ &\equiv b \land c \end{aligned}$$

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B3. General Regression, Part I

Regressing Formulas Through Effects: Correctness (2) Proof. The proof is by structural induction on  $\varphi$ . Induction hypothesis:  $s \models regr_{eff(o)}(\psi)$  iff  $s[o] \models \psi$ . Base case  $\varphi = \top$ :

We have  $regr_{eff(o)}(\top) = \top$ , and  $s \models \top$  iff  $s[[o]] \models \top$  is correct.

Base case  $\varphi = \bot$ :

We have  $regr_{eff(o)}(\bot) = \bot$ , and  $s \models \bot$  iff  $s[[o]] \models \bot$  is correct.

Base case  $\varphi = v$ :

We have  $s \models regr_{eff(o)}(v)$  iff  $s \llbracket o \rrbracket \models v$  from the previous lemma.

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#### B3. General Regression, Part I

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