Planning and Optimization

B2. Regression: Introduction & STRIPS Case

Malte Helmert and Gabriele Röger

Universität Basel

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B2.1 Regression

B2.2 Regression Example

B2.3 Regression for STRIPS Tasks

B2.4 Summary

B2.1 Regression

Forward Search vs. Backward Search

Searching planning tasks in forward vs. backward direction is not symmetric:

- forward search starts from a single initial state; backward search starts from a set of goal states
- when applying an operator o in a state s in forward direction, there is a unique successor state s'; if we just applied operator o and ended up in state s', there can be several possible predecessor states s

$\rightsquigarrow$ in most natural representation for backward search in planning, each search state corresponds to a set of world states

Planning by Backward Search: Regression

Regression: Computing the possible predecessor states $regr_o(S')$ of a set of states S' ("subgoal") given the last operator o that was applied.

$\rightsquigarrow$ formal definition in next chapter

Regression planners find solutions by backward search:

- start from set of goal states
- iteratively pick a previously generated subgoal (state set) and regress it through an operator, generating a new subgoal
- solution found when a generated subgoal includes initial state

pro: can handle many states simultaneously
con: basic operations complicated and expensive

Search Space Representation in Regression Planners

identify state sets with logical formulas (again):

- each search state corresponds to a set of world states ("subgoal")
- each search state is represented by a logical formula: φ represents $\{s \in S \mid s \models \varphi\}$
- many basic search operations like detecting duplicates are NP-complete or coNP-complete

Search Space for Regression

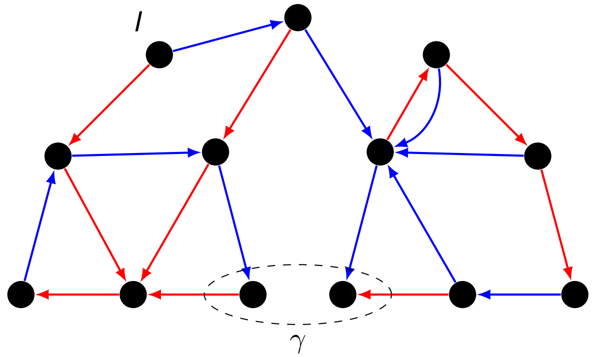
Search Space for Regression

search space for regression in a planning task $\Pi = \langle V, I, O, \gamma\rangle$ (search states are formulas φ describing sets of world states; actions of search space are operators $o \in O$)

- init() $\rightsquigarrow$ returns γ
- is_goal(φ) $\rightsquigarrow$ tests if $I \models \varphi$
- succ(φ) $\rightsquigarrow$ returns all pairs $\langle o, regr_o(\varphi)\rangle$ where $o \in O$ and $regr_o(\varphi)$ is defined
- cost(o) $\rightsquigarrow$ returns $cost(o)$ as defined in Π
- h(φ) $\rightsquigarrow$ estimates cost from I to φ ($\rightsquigarrow$ Parts C and D)

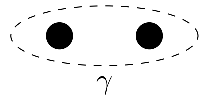
B2.2 Regression Example

Regression Planning Example (Depth-first Search)



Regression Planning Example (Depth-first Search)

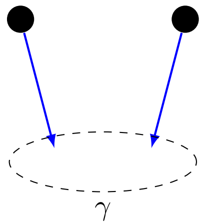
γ



Regression Planning Example (Depth-first Search)

$\varphi_1 = \mathit{regr}_{\longrightarrow}(\gamma)$

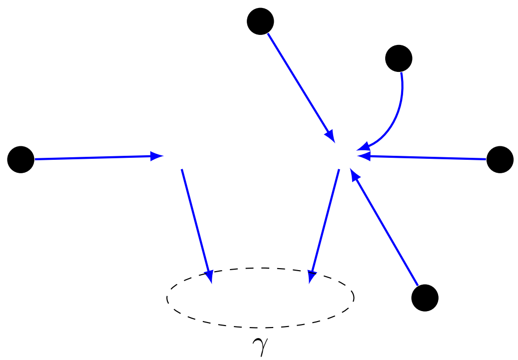
$\varphi_1 \longrightarrow \gamma$



Regression Planning Example (Depth-first Search)

$\varphi_1 = \mathit{regr}_{\longrightarrow}(\gamma)$
$\varphi_2 = \mathit{regr}_{\longrightarrow}(\varphi_1)$

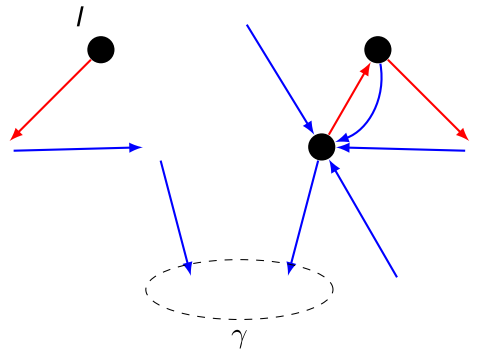
$\varphi_2 \longrightarrow \varphi_1 \longrightarrow \gamma$



Regression Planning Example (Depth-first Search)

$\varphi_1 = regr_{\longrightarrow}(\gamma)$
$\varphi_2 = regr_{\longrightarrow}(\varphi_1)$
$\varphi_3 = regr_{\longrightarrow}(\varphi_2), I \models \varphi_3$

$\varphi_3 \longrightarrow \varphi_2 \longrightarrow \varphi_1 \longrightarrow \gamma$



B2.3 Regression for STRIPS Tasks

Regression for STRIPS Planning Tasks

Regression for STRIPS planning tasks is much simpler than the general case:

- Consider subgoal φ that is conjunction of atoms $a_1 \land \dots \land a_n$ (e.g., the original goal γ of the planning task).
- First step: Choose an operator o that deletes no a_i.
- Second step: Remove any atoms added by o from φ.
- Third step: Conjoin $pre(o)$ to φ.

$\leadsto$ Outcome of this is regression of φ w.r.t. o.
It is again a conjunction of atoms.

optimization: only consider operators adding at least one a_i

STRIPS Regression

Definition (STRIPS Regression)

Let $\varphi = \varphi_1 \land \dots \land \varphi_n$ be a conjunction of atoms, and let o be a STRIPS operator which adds the atoms $a_1, \dots, a_k$ and deletes the atoms $d_1, \dots, d_l$. (W.l.o.g., $a_i \neq d_j$ for all i, j.)

The STRIPS regression of φ with respect to o is

$$sregr_o(\varphi) := \begin{cases} \bot & \text{if } \varphi_i = d_j \text{ for some } i, j \\ pre(o) \land \bigwedge(\{\varphi_1, \dots, \varphi_n\} \setminus \{a_1, \dots, a_k\}) & \text{otherwise} \end{cases}$$

Note: $sregr_o(\varphi)$ is again a conjunction of atoms, or $\bot$.

Does this Capture the Idea of Regression?

For our definition to capture the concept of regression, it should satisfy the following property:

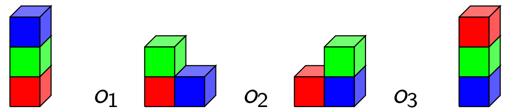
Regression Property

For all sets of states described by a conjunction of atoms φ, all states s and all STRIPS operators o,

$$s \models sregr_o(\varphi) \quad \text{iff} \quad s[\![o]\!] \models \varphi.$$

This is indeed true. We do not prove it now because we prove this property for general regression (not just STRIPS) later.

STRIPS Regression Example



Note: Predecessor states are in general not unique.
This picture is just for illustration purposes.

$$\begin{aligned}
o_1 &= \langle \textcolor{blue}{\blacksquare} on \textcolor{green}{\blacksquare} \land \textcolor{blue}{\blacksquare} clr, \quad \neg \textcolor{blue}{\blacksquare} on \textcolor{green}{\blacksquare} \land \textcolor{blue}{\blacksquare} onT \land \textcolor{green}{\blacksquare} clr \rangle \\
o_2 &= \langle \textcolor{green}{\blacksquare} on \textcolor{red}{\blacksquare} \land \textcolor{green}{\blacksquare} clr \land \textcolor{blue}{\blacksquare} clr, \quad \neg \textcolor{blue}{\blacksquare} clr \land \neg \textcolor{green}{\blacksquare} on \textcolor{red}{\blacksquare} \land \textcolor{green}{\blacksquare} on \textcolor{blue}{\blacksquare} \land \textcolor{red}{\blacksquare} clr \rangle \\
o_3 &= \langle \textcolor{red}{\blacksquare} onT \land \textcolor{red}{\blacksquare} clr \land \textcolor{green}{\blacksquare} clr, \quad \neg \textcolor{green}{\blacksquare} clr \land \neg \textcolor{red}{\blacksquare} onT \land \textcolor{red}{\blacksquare} on \textcolor{green}{\blacksquare} \rangle
\end{aligned}$$

$$\begin{aligned}
\gamma &= \textcolor{red}{\blacksquare} on \textcolor{green}{\blacksquare} \land \textcolor{green}{\blacksquare} on \textcolor{blue}{\blacksquare} \\
\varphi_1 &= sregr_{o_3}(\gamma) = \textcolor{red}{\blacksquare} onT \land \textcolor{red}{\blacksquare} clr \land \textcolor{green}{\blacksquare} clr \land \textcolor{green}{\blacksquare} on \textcolor{blue}{\blacksquare} \\
\varphi_2 &= sregr_{o_2}(\varphi_1) = \textcolor{green}{\blacksquare} on \textcolor{red}{\blacksquare} \land \textcolor{green}{\blacksquare} clr \land \textcolor{blue}{\blacksquare} clr \land \textcolor{red}{\blacksquare} onT \\
\varphi_3 &= sregr_{o_1}(\varphi_2) = \textcolor{blue}{\blacksquare} on \textcolor{green}{\blacksquare} \land \textcolor{blue}{\blacksquare} clr \land \textcolor{green}{\blacksquare} on \textcolor{red}{\blacksquare} \land \textcolor{red}{\blacksquare} onT
\end{aligned}$$

B2.4 Summary

Summary

- Regression search proceeds backwards from the goal.
- Each search state corresponds to a set of world states, for example represented by a formula.
- Regression is simple for STRIPS operators.
- The theory for general regression is more complex. This is the topic of the following chapters.