# Planning and Optimization

## A4. Planning Tasks

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# A4.1 Introduction





## State Variables

How to specify huge transition systems without enumerating the states?

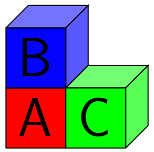
- represent different aspects of the world in terms of different Boolean state variables
- treat state variables as atomic propositions
  $\rightsquigarrow$ a state is a valuation of state variables
- $n$ state variables induce $2^n$ states
  $\rightsquigarrow$ exponentially more compact than "flat" representations

Example: $O(n^2)$ variables suffice for blocks world with $n$ blocks

## Blocks World State with Boolean State Variables

Example

$s(A\text{-}on\text{-}B) = \mathbf{F}$
$s(A\text{-}on\text{-}C) = \mathbf{F}$
$s(A\text{-}on\text{-}table) = \mathbf{T}$
$s(B\text{-}on\text{-}A) = \mathbf{T}$
$s(B\text{-}on\text{-}C) = \mathbf{F}$
$s(B\text{-}on\text{-}table) = \mathbf{F}$
$s(C\text{-}on\text{-}A) = \mathbf{F}$
$s(C\text{-}on\text{-}B) = \mathbf{F}$
$s(C\text{-}on\text{-}table) = \mathbf{T}$








## Boolean State Variables

Problem:

- How to succinctly represent transitions and goal states?

Idea: Use logical formulas to describe sets of states

- state variables: atomic propositions
- states: all valuations of the state variables
- goal states: defined by a logical formula
- transitions: defined by operators (see following section)





# A4.2 Operators





## Syntax of Operators

Definition (Operator)

An operator $o$ over state variables $V$ is an object with three properties:

- a precondition $pre(o)$, a logical formula over $V$
- an effect $eff(o)$ over $V$, defined on the following slides
- a cost $cost(o) \in \mathbb{R}_0^+$

Notes:

- Operators are also called actions.
- Operators are often written as triples $\langle pre(o), eff(o), cost(o)\rangle$.
- This can be abbreviated to pairs $\langle pre(o), eff(o)\rangle$ when the cost of the operator is irrelevant.

## Operators: Intuition

Intuition for operators $o$:

- The operator precondition describes the set of states in which a transition labeled with $o$ can be taken.
- The operator effect describes how taking such a transition changes the state.
- The operator cost describes the cost of taking a transition labeled with $o$.





## Syntax of Effects

Definition (Effect)

Effects over state variables $V$ are inductively defined as follows:

- If $v \in V$ is a state variable, then $v$ and $\neg v$ are effects (atomic effect).
- If $e_1, \dots, e_n$ are effects, then $(e_1 \wedge \dots \wedge e_n)$ is an effect (conjunctive effect).
  The special case with $n = 0$ is the empty effect $\top$.
- If $\chi$ is a logical formula and $e$ is an effect, then $(\chi \rhd e)$ is an effect (conditional effect).

Parentheses can be omitted when this does not cause ambiguity.





## Effects: Intuition

Intuition for effects:

- Atomic effects $v$ and $\neg v$ can be understood as assignments "$v := \mathbf{T}$" and "$v := \mathbf{F}$".
- A conjunctive effect $e = (e_1 \wedge \dots \wedge e_n)$ means that all subeffects $e_1, \dots, e_n$ take place simultaneously.
- A conditional effect $e = (\chi \rhd e')$ means that subeffect $e'$ takes place iff $\chi$ is true in the state where $e$ takes place.





## Semantics of Effects

Definition (Update Set for an Effect)

For all effects $e$ and states $s$, the update set of $e$ in $s$, written $[e]_s$, is defined as the following set of literals:

- $[v]_s = \{v\}$ and $[\neg v]_s = \{\neg v\}$ for atomic effects $v$, $\neg v$
- $[(e_1 \wedge \dots \wedge e_n)]_s = [e_1]_s \cup \dots \cup [e_n]_s$
- $[(\chi \rhd e)]_s = \begin{cases} [e]_s & \text{if } s \models \chi \\ \emptyset & \text{otherwise} \end{cases}$

## Semantics of Operators

### Definition (Applicable, Resulting State)

Let $V$ be a set of state variables.
Let $s$ be a state over $V$, and let $o$ be an operator over $V$.

Operator $o$ is applicable in $s$ if $s \models pre(o)$.

If $o$ is applicable in $s$, the resulting state of applying $o$ in $s$, written $s[\![o]\!]$, is the state $s'$ defined as follows:

$$s'(v) = \begin{cases} \mathbf{T} & \text{for all } v \in V \text{ with } v \in [\mathit{eff}(o)]_s \\ \mathbf{F} & \text{for all } v \in V \text{ with } \neg v \in [\mathit{eff}(o)]_s \text{ and } v \notin [\mathit{eff}(o)]_s \\ s(v) & \text{for all other } v \in V \end{cases}$$





## Add-after-Delete Semantics

### Note:

- The definition implies that if a variable is simultaneously "added" (set to **T**) and "deleted" (set to **F**), the value **T** takes precedence.
- This is called add-after-delete semantics.
- This detail of semantics is somewhat arbitrary, and other definitions are sometimes used.





## Applying Operators: Example

### Example

Consider the operator $o = \langle a, \neg a \wedge (\neg c \rhd \neg b)\rangle$
and the state $s = \{a \mapsto \mathbf{T}, b \mapsto \mathbf{T}, c \mapsto \mathbf{T}, d \mapsto \mathbf{T}\}$.

The operator $o$ is applicable in $s$ because $s \models a$.

The update set of $\mathit{eff}(o)$ in $s$ is
$[\mathit{eff}(o)]_s = [\neg a]_s \cup [\neg c \rhd \neg b]_s = \{\neg a\} \cup \emptyset = \{\neg a\}$.

The resulting state of applying $o$ in $s$ is the state
$\{a \mapsto \mathbf{F}, b \mapsto \mathbf{T}, c \mapsto \mathbf{T}, d \mapsto \mathbf{T}\}$.





## Example Operators: Blocks World

### Example (Blocks World Operators)

To model blocks world operators conveniently,
we use auxiliary state variables *A-clear*, *B-clear*, and *C-clear*
to express that there is nothing on top of a given block.

Then blocks world operators can be modeled as:

- $\langle \mathit{A\text{-}clear} \wedge \mathit{A\text{-}on\text{-}T} \wedge \mathit{B\text{-}clear},\ \mathit{A\text{-}on\text{-}B} \wedge \neg \mathit{A\text{-}on\text{-}T} \wedge \neg \mathit{B\text{-}clear}\rangle$
- $\langle \mathit{A\text{-}clear} \wedge \mathit{A\text{-}on\text{-}T} \wedge \mathit{C\text{-}clear},\ \mathit{A\text{-}on\text{-}C} \wedge \neg \mathit{A\text{-}on\text{-}T} \wedge \neg \mathit{C\text{-}clear}\rangle$
- $\langle \mathit{A\text{-}clear} \wedge \mathit{A\text{-}on\text{-}B},\ \mathit{A\text{-}on\text{-}T} \wedge \neg \mathit{A\text{-}on\text{-}B} \wedge \mathit{B\text{-}clear}\rangle$
- $\langle \mathit{A\text{-}clear} \wedge \mathit{A\text{-}on\text{-}C},\ \mathit{A\text{-}on\text{-}T} \wedge \neg \mathit{A\text{-}on\text{-}C} \wedge \mathit{C\text{-}clear}\rangle$
- $\langle \mathit{A\text{-}clear} \wedge \mathit{A\text{-}on\text{-}B} \wedge \mathit{C\text{-}clear},\ \mathit{A\text{-}on\text{-}C} \wedge \neg \mathit{A\text{-}on\text{-}B} \wedge \mathit{B\text{-}clear} \wedge \neg \mathit{C\text{-}clear}\rangle$
- $\langle \mathit{A\text{-}clear} \wedge \mathit{A\text{-}on\text{-}C} \wedge \mathit{B\text{-}clear},\ \mathit{A\text{-}on\text{-}B} \wedge \neg \mathit{A\text{-}on\text{-}C} \wedge \mathit{C\text{-}clear} \wedge \neg \mathit{B\text{-}clear}\rangle$
- ...

## Example Operator: 4-Bit Counter

Example (Incrementing a 4-Bit Counter)

Operator to increment a 4-bit number $b_3 b_2 b_1 b_0$ represented by 4 state variables $b_0, \ldots, b_3$:

precondition:

$$\neg b_0 \vee \neg b_1 \vee \neg b_2 \vee \neg b_3$$

effect:

$$\begin{gathered}(\neg b_0 \rhd b_0) \wedge \\ ((\neg b_1 \wedge b_0) \rhd (b_1 \wedge \neg b_0)) \wedge \\ ((\neg b_2 \wedge b_1 \wedge b_0) \rhd (b_2 \wedge \neg b_1 \wedge \neg b_0)) \wedge \\ ((\neg b_3 \wedge b_2 \wedge b_1 \wedge b_0) \rhd (b_3 \wedge \neg b_2 \wedge \neg b_1 \wedge \neg b_0))\end{gathered}$$





# A4.3 Planning Tasks





## Planning Tasks

Definition (Planning Task)

A planning task is a 4-tuple $\Pi = \langle V, I, O, \gamma \rangle$ where

- $V$ is a finite set of state variables (propositions),
- $I$ is a valuation over $V$ called the initial state,
- $O$ is a finite set of operators over $V$, and
- $\gamma$ is a formula over $V$ called the goal.





## Mapping Planning Tasks to Transition Systems

Definition (Transition System Induced by a Planning Task)

The planning task $\Pi = \langle V, I, O, \gamma \rangle$ induces the transition system $\mathcal{T}(\Pi) = \langle S, L, c, T, s_0, S_\star \rangle$, where

- $S$ is the set of all valuations of $V$,
- $L$ is the set of operators $O$,
- $c(o) = cost(o)$ for all operators $o \in O$,
- $T = \{\langle s, o, s' \rangle \mid s \in S,\ o \text{ applicable in } s,\ s' = s[\![o]\!]\}$,
- $s_0 = I$, and
- $S_\star = \{s \in S \mid s \models \gamma\}$.

## Planning Tasks: Terminology

- Terminology for transitions systems is also applied to the planning tasks $\Pi$ that induce them.
- For example, when we speak of the states of $\Pi$, we mean the states of $\mathcal{T}(\Pi)$.
- A sequence of operators that forms a solution of $\mathcal{T}(\Pi)$ is called a plan of $\Pi$.





## Satisficing and Optimal Planning

By planning, we mean the following two algorithmic problems:

Definition (Satisficing Planning)

Given: a planning task $\Pi$
Output: a plan for $\Pi$, or **unsolvable** if no plan for $\Pi$ exists

Definition (Optimal Planning)

Given: a planning task $\Pi$
Output: a plan for $\Pi$ with minimal cost among all plans for $\Pi$, or **unsolvable** if no plan for $\Pi$ exists





# A4.4 Summary





## Summary

- Planning tasks compactly represent transition systems and are suitable as inputs for planning algorithms.
- Planning tasks are based on concepts from propositional logic, enhanced to model state change.
- States of planning tasks are propositional valuations.
- Operators of planning tasks describe in which situations (precondition) and how (effect) the state of the world can be changed, and at which cost.
- In satisficing planning, we must find a solution for a planning task (or show that no solution exists).
- In optimal planning, we must additionally guarantee that generated solutions are of minimal cost.