Planning and Optimization

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Exercise Sheet 8

Due: November 23, 2016

Exercise 8.1 (3 marks)

In the lecture, the following theorem has been presented (chapter 13, slide 33 in the handout version): Let Π be an SAS$^+$ planning task with variable set V, and let V_1 and V_2 be disjoint subsets of V. For $i \in \{1,2\}$, let α_i be an abstraction of $\mathcal{T}(\Pi)$ such that α_i is a coarsening of π_{V_i}. Then $\mathcal{T}^{\alpha_1 \otimes \alpha_2} = \mathcal{T}^{\alpha_1} \otimes \mathcal{T}^{\alpha_2}$.
This theorem can be shown by extending the proof given in chapter 13 (slides 30 and 31 in the handout version), by also showing that $T_{\alpha_1 \otimes \alpha_2} \supseteq T_\otimes$ and $S_{\star\alpha_1 \otimes \alpha_2} \supseteq S_{\star\otimes}$.
In this exercise, show that $T_{\alpha_1 \otimes \alpha_2} \supseteq T_\otimes$.

Hint: It suffices to show that for any $\langle s, l, t\rangle, \langle s', l, t'\rangle \in T$, there exists $\langle s'', l, t''\rangle \in T$ such that $\alpha_1(s) = \alpha_1(s'')$, $\alpha_1(t) = \alpha_1(t'')$, $\alpha_2(s') = \alpha_2(s'')$, and $\alpha_2(t') = \alpha_2(t'')$. This shows the claim because from $\langle s'', l, t''\rangle \in T$, it follows that $\langle\langle\alpha_1(s''), \alpha_2(s'')\rangle, l, \langle\alpha_1(t''), \alpha_2(t'')\rangle\rangle \in T_{\alpha_1 \otimes \alpha_2}$ and hence, $\langle\langle\alpha_1(s), \alpha_2(s')\rangle, l, \langle\alpha_1(t), \alpha_2(t')\rangle\rangle \in T_{\alpha_1 \otimes \alpha_2}$.

Exercise 8.2 (1+2+2 marks)

Consider the planning task $\Pi = \langle V, I, O, \gamma\rangle$ with

$$
\begin{aligned}
V &= \{v_1, v_2, v_3\} \text{ with } \mathrm{dom}(v_1) = \{A, B, C, D\} \text{ and } \mathrm{dom}(v_2) = \mathrm{dom} v_3 = \{A, B\} \\
I &= \{v_1 \mapsto A, v_2 \mapsto A, v_3 \mapsto A\} \\
O &= \{o_1, o_2, o_3, o_4\} \text{ with} \\
&\quad o_1 = \langle v_3 = A, v_3 := B, 1\rangle \\
&\quad o_2 = \langle v_1 = A \land v_3 = B, v_1 := B \land v_3 := A, 1\rangle \\
&\quad o_3 = \langle v_1 = B \land v_2 = A, v_1 := C \land v_2 := B, 1\rangle \\
&\quad o_4 = \langle (v_1 = B \land v_2 = B) \lor (v_1 = C), v_1 := D, 1\rangle \\
\gamma &= (v_1 = D)
\end{aligned}
$$

(a) Provide $\mathcal{T}^{\pi_{\{v_1\}}}$ and $\mathcal{T}^{\pi_{\{v_2\}}}$ graphically.

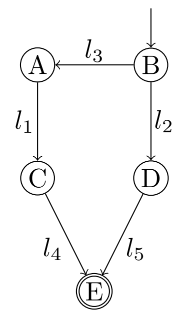
(b) Provide $\mathcal{T}^{\pi_{\{v_1\}}} \otimes \mathcal{T}^{\pi_{\{v_2\}}}$ graphically.

(c) Provide $\mathcal{T}^{\pi_{\{v_1\}} \otimes \pi_{\{v_2\}}}$ graphically.

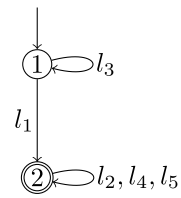
Exercise 8.3 (1+0.5+1+1.5 marks)

Consider a set $X = \{\mathcal{T}_1, \mathcal{T}_2\}$ of abstract transition systems with identical label set $L = \{l_1, \dots, l_5\}$ and cost function c such that $c(l_1) = c(l_2) = c(l_3) = 1$ and $c(l_4) = c(l_5) = 2$. $\mathcal{T}_1$ and $\mathcal{T}_2$ are depicted graphically below. As usual, an incoming arrow indicates the initial state, and goal states are marked by a double circle.

Please turn around



$\mathcal{T}_1$



$\mathcal{T}_2$

(a) Determine all pairs of labels that are $\mathcal{T}_1$-combinable.

(b) Based on the result from a), identify all pairs of labels that are $\mathcal{T}_1$-combinable and also have the same cost. Determine a mapping $\tau : L \mapsto L'$ that maps all $\mathcal{T}_1$-combinable labels with identical cost to the same (new) label and all labels l that are not $\mathcal{T}_1$-combinable with another label to l. Let c' be the cost function that allows exact label reduction with $\langle \tau, c' \rangle$. Graphically provide $\mathcal{T}_1^{\langle \tau, c' \rangle}$ and $\mathcal{T}_2^{\langle \tau, c' \rangle}$.

(c) Graphically provide the transition systems $\mathcal{T}_1'$ and $\mathcal{T}_1''$ that result from shrinking $\mathcal{T}_1^{\langle \tau, c' \rangle}$ with the following shrinking strategies:

- $\mathcal{T}_1'$ results from applying h-preserving shrinking (a simplification of f-preserving shrinking that ignores g values and only considers h values), and
- $\mathcal{T}_1''$ results from applying bisimulation-based shrinking.

(d) Compare $\mathcal{T}_1' \otimes \mathcal{T}_2$, $\mathcal{T}_1'' \otimes \mathcal{T}_2$, and $\mathcal{T}_1 \otimes \mathcal{T}_2$ with respect to size and heuristic value of the initial state. (It is voluntary to provide the transition systems graphically for the discussion, but you can potentially earn partial marks if you do.)

The exercise sheets can be submitted in groups of two students. Please provide both student names on the submission.