Planning and Optimization

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Exercise Sheet 7

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Exercise 7.1 (1 mark)

Prove the following claim from the lecture: let α_1 and α_2 be abstractions of a transition system $\mathcal{T}$. If no label of $\mathcal{T}$ affects both $\mathcal{T}^{\alpha_1}$ and $\mathcal{T}^{\alpha_2}$, then α_1 and α_2 are orthogonal.

Exercise 7.2 (3+1+1+1 marks)

Let Π be a planning task in finite domain representation, and let P be a pattern for Π. Prove the following:

(a) If Π is a SAS$^+$ planning task that is not trivially unsolvable and does not contain trivially inapplicable operators, then $\mathcal{T}(\Pi|_P) \overset{G}{\sim} \mathcal{T}(\Pi)^{\pi_P}$, i.e., $\mathcal{T}(\Pi|_P)$ is graph-equivalent to $\mathcal{T}(\Pi)^{\pi_P}$.

(b) $\mathcal{T}(\Pi|_P)$ is *not* graph-equivalent to $T(\Pi)^{\pi_P}$ if Π is trivially unsolvable.

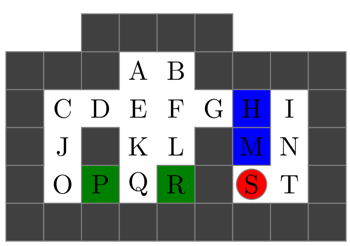
(c) $\mathcal{T}(\Pi|_P)$ is *not* graph-equivalent to $T(\Pi)^{\pi_P}$ if Π contains trivially inapplicable operators.

(d) $\mathcal{T}(\Pi|_P)$ is *not* graph-equivalent to $T(\Pi)^{\pi_P}$ if Π contains operators with conditional effects.

Hint: For part (a), show the properties regarding initial state, goal states, and transitions needed for establishing graph-equivalence (chapter C8, slide 19 in the handout version). For parts (b)–(d), it suffices to provide a counterexample with a justification why graph-equivalence is violated.

Exercise 7.3 (2+2+1 marks)

In the *Sokoban* domain, a worker has to push boxes to goal positions. Consider the *Sokoban* problem given in the figure. The red dot denotes the initial position of the worker, the blue cells denote the initial positions of the boxes, and the green cells denote the goal positions of the boxes, where it does not matter which box is finally located at which goal position. The letters (A – T) are only shown to indicate the cells.



Consider the SAS$^+$ representation of this problem with variables pos_w, pos_{b1}, pos_{b2} (which denote the positions of the worker and the two boxes), $atgoal_{b1}$, $atgoal_{b2}$ (which indicate whether the boxes are at goal positions), and $content_A$, ..., $content_T$ (which denote the content of the individual cells). Formally, the variable domains are defined as follows:

- $\mathrm{dom}(pos_w) = \mathrm{dom}(pos_{b1}) = \mathrm{dom}(pos_{b2}) = \{A, \dots, T\}$
- $\mathrm{dom}(atgoal_{b1}) = \mathrm{dom}(atgoal_{b2}) = \{\mathbf{T}, \mathbf{F}\}$
- $\mathrm{dom}(content_A) = \cdots = \mathrm{dom}(content_T) = \{empty, w, b1, b2\}$

The initial state is defined by the set consisting of the following mappings:

- $pos_w \mapsto S$, $pos_{b1} \mapsto M$, $pos_{b2} \mapsto H$, $atgoal_{b1} \mapsto \mathbf{F}$, $atgoal_{b2} \mapsto \mathbf{F}$
- $content_H \mapsto b2$, $content_M \mapsto b1$, $content_S \mapsto w$
- $content_X \mapsto empty$ for all $X \in \{A, \ldots, T\} \setminus \{H, M, S\}$

The goal is given by the formula $atgoal_{b1} = \mathbf{T} \wedge atgoal_{b2} = \mathbf{T}$. The operators (*move* and *push*) are defined as usual (recall that it is not allowed to pull boxes). We call cells c and c' *adjacent* if c' is located next to c and either above, below, left or right to c (i.e., diagonal cells are *not* adjacent).

- *move* operators: For adjacent cells c and c', the worker can move from c to c' if the worker is currently at c and c' is empty. After moving, c is empty and the worker is at c'.
- *push* operators: For cells c, c', c'' such that c is adjacent to c' in direction X iff c' is adjacent to c'' in direction X for $X \in \{\text{above}, \text{below}, \text{left}, \text{right}\}$, the worker can push a box from c' to c'' if the worker is at c, the box is at c' and c'' is empty. After pushing, c is empty, the worker is at c', and the box is at c''.

Consider the pattern collection $\mathcal{C}$ that consists of exactly the following patterns:

- $P_1 = \{atgoal_{b2}\}$
- $P_2 = \{atgoal_{b1}, pos_{b1}\}$
- $P_3 = \{atgoal_{b2}, pos_{b2}\}$
- $P_4 = \{atgoal_{b1}, pos_{b1}, pos_w\}$
- $P_5 = \{pos_{b1}, pos_w\}$
- $P_6 = \{atgoal_{b1}, content_H\}$
- $P_7 = \{atgoal_{b1}, content_G\}$
- $P_8 = \{atgoal_{b2}, content_D\}$
- $P_9 = \{content_A, content_E\}$
- $P_{10} = \{atgoal_{b1}, content_Q\}$

(a) Construct the compatibility graph for $\mathcal{C}$ and determine the maximal cliques.

(b) Provide the canonical heuristic $h^{\mathcal{C}}$ and simplify it as much as possible.

(c) Not all of the patterns in $\mathcal{C}$ are reasonable. Which patterns could be obviously removed, and why? How does the canonical heuristic look like in case these patterns are removed from $\mathcal{C}$?

The exercise sheets can be submitted in groups of two students. Please provide both student names on the submission.