Planning and Optimization

M. Helmert, G. Röger
T. Keller, M. Wehrle

University of Basel
Fall Semester 2016

Exercise Sheet 6

Due: November 9, 2016

Exercise 6.1 (1+1+2 marks)
Consider the planning task $\Pi = \langle V, I, O, \gamma \rangle$ with

$$
\begin{aligned}
V &= \{a, b, c, d, e, f, g\} \\
I &= \{a \mapsto \mathbf{T}, b \mapsto \mathbf{F}, c \mapsto \mathbf{F}, d \mapsto \mathbf{F}, e \mapsto \mathbf{F}, f \mapsto \mathbf{F}, g \mapsto \mathbf{F}\} \\
O &= \{o_1, o_2, o_3, o_4\} \text{ with} \\
&\quad o_1 = \langle a, b, 1 \rangle \\
&\quad o_2 = \langle \top, c, 2 \rangle \\
&\quad o_3 = \langle b \vee c, d \wedge e \wedge (d \rhd g), 1 \rangle \\
&\quad o_4 = \langle e, f, 2 \rangle \\
\gamma &= f \wedge g
\end{aligned}
$$

(a) Compute $h^{\max}(I)$. Justify your answer with the relaxed task graph $RTG(\Pi^+)$.

(b) Compute $h^{\text{add}}(I)$. Justify your answer with the relaxed task graph $RTG(\Pi^+)$.

(c) Compute $h^{\text{FF}}(I)$. Justify your answer with the best achiever graph G^{add}.

Exercise 6.2 (1+1 marks)

(a) Provide a planning task $\Pi = \langle V, I, O, \gamma \rangle$ such that $h^{\max}(I) = h^*(I)$, but $h^{\text{add}}(I) \neq h^*(I)$ (i.e., such that the $h^{\max}$ heuristic is equal to the perfect heuristic in Π's initial state, whereas the h^{add} heuristic is not).

(b) Provide a planning task $\Pi = \langle V, I, O, \gamma \rangle$ such that $h^{\text{add}}(I) = h^*(I)$, but $h^{\max}(I) \neq h^*(I)$ (i.e., such that the h^{add} heuristic is equal to the perfect heuristic in Π's initial state, whereas the $h^{\max}$ heuristic is not).

Exercise 6.3 (2 marks)
Provide a planning task $\langle V, I, O, \gamma \rangle$ which shows that the h^{FF} heuristic is not admissible, i.e., where $h^{\text{FF}}(I) > h^*(I)$ in the initial state I.

Please turn around

Exercise 6.4 (2+2 marks)

A state in the 15-puzzle is given by a permutation $\langle b, t_1, \ldots, t_{15} \rangle$ of $\{1, \ldots, 16\}$, where b denotes the position of the empty tile (the "blank"), and $t_1, \ldots, t_{15}$ denote the positions of the 15 tiles. Let $T^1 = \{t_1^1, \ldots, t_n^1\}, T^2 = \{t_1^2, \ldots, t_m^2\}$ with $1 \leq n, m \leq 14$ be a partitioning of $\{t_1, \ldots, t_{15}\}$, i.e., $T^1 \cup T^2 = \{t_1, \ldots, t_{15}\}$ and $T^1 \cap T^2 = \emptyset$. Consider the following abstraction functions:

(a) $\alpha_1(\langle b, t_1, \ldots, t_{15} \rangle) = \langle b, t_1^1, \ldots, t_n^1 \rangle$

(b) $\alpha_2(\langle b, t_1, \ldots, t_{15} \rangle) = \langle b, t_1^2, \ldots, t_m^2 \rangle$

(c) $\alpha_3(\langle b, t_1, \ldots, t_{15} \rangle) = \langle t_1^1, \ldots, t_n^1 \rangle$

(d) $\alpha_4(\langle b, t_1, \ldots, t_{15} \rangle) = \langle t_1^2, \ldots, t_m^2 \rangle$

For $1 \leq i \leq 4$, the heuristic values of heuristic h_i are defined by the costs to solve the puzzle in the corresponding abstract state space optimally (i.e., $h_i(s) = h^*(\alpha_i(s))$.
Prove the following:

(a) $h_1 + h_2$ is *not* an admissible heuristic.

(b) $h_3 + h_4$ is an admissible heuristic.

Hint: Heuristics that are goal-aware and consistent are also admissible.

The exercise sheets can be submitted in groups of two students. Please provide both student names on the submission.