Chapter 7

Stochastic Local Search

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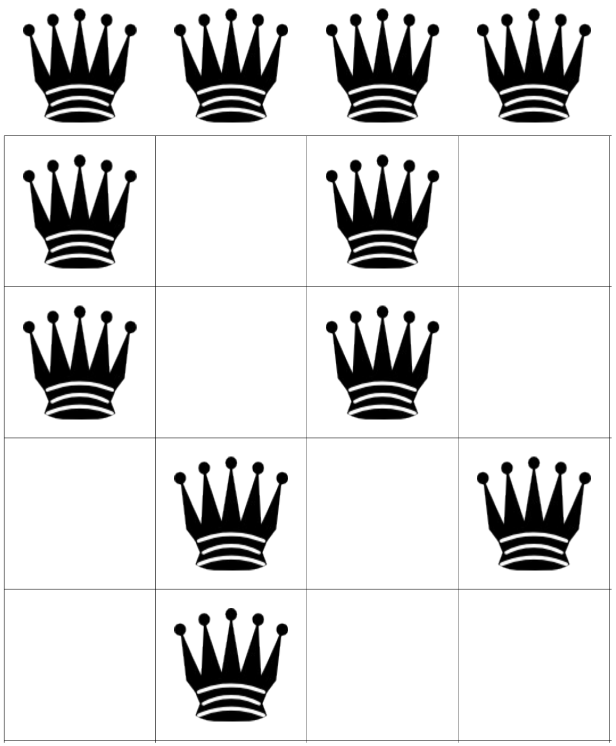
Motivation

n-queens with Backtracking:



- guarantees to find all solutions
- reaches limit for big problems:

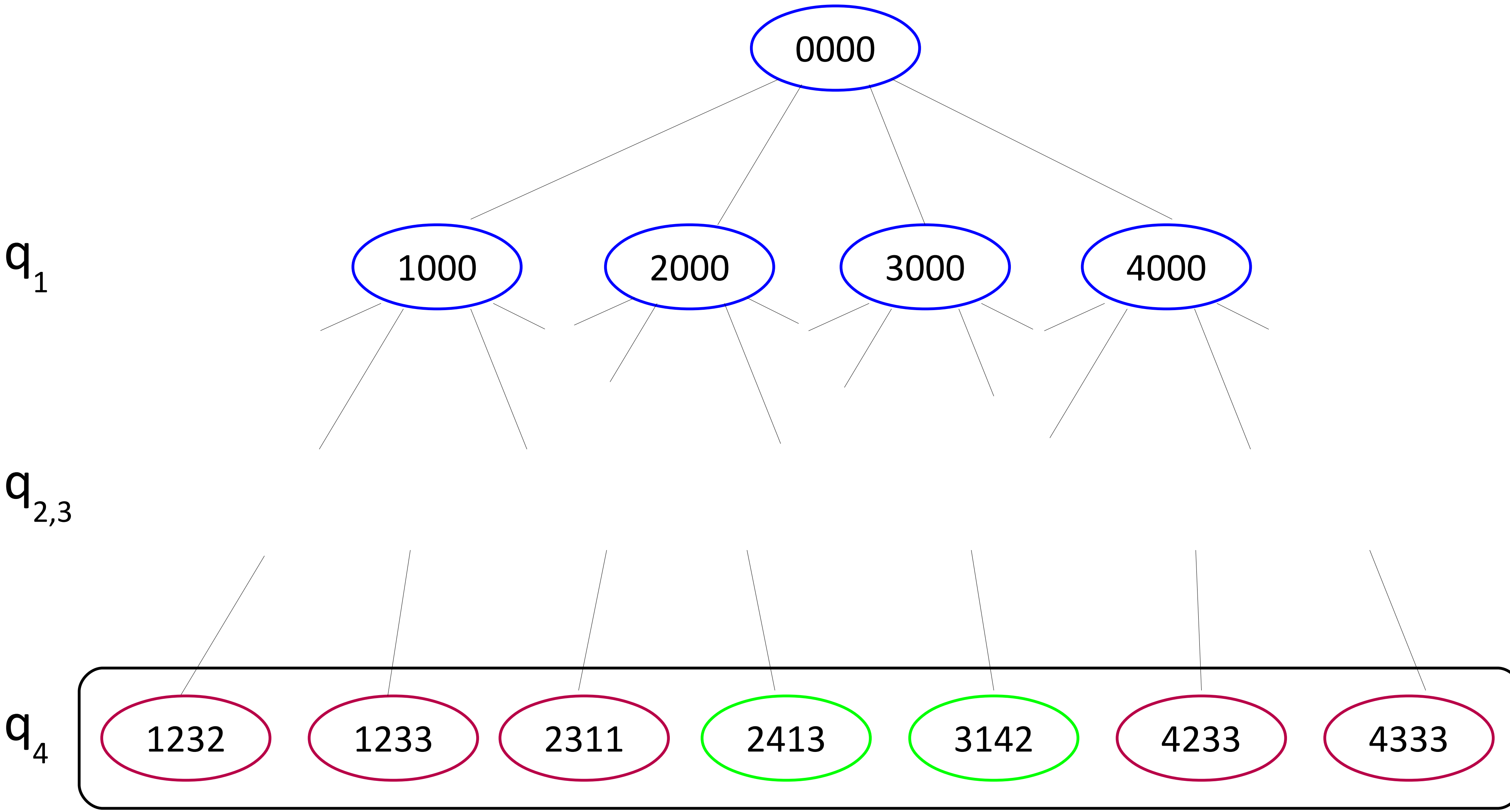
  Best backtracking methods

  solve up to *100*-queens
- Stochastic search:

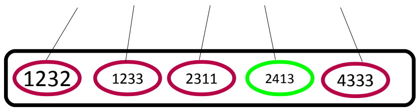
  1 million queens solvable

  in less than a minute

Systematic vs. Stochastic Search



Greedy Local Search



- usually runs on complete instantiations (leaves)
- starts in a randomly chosen instantiation
- assignments aren't necessarily consistent

Progressing:

- Local changes (of one variable assignment)
- *Greedy*, minimizing cost function (#broken constraints)

Stopping Criterion:

- Assignment is consistent (const function = 0)

Greedy SLS: Algorithm

procedure: `SLS`

Input : A constraint network $\mathfrak{R} = (X, D, C)$. A cost function defined on full assignments.

Output: A solution (no guarantee to terminate)

initialization: let $\bar{a} = (a_1, ..., a_n)$ be a random initial assignment to all variables.

while $\bar{a}$ *is not consistent* **do**

let $Y = (x_i, a'_i)$ be the set of variable-value pairs that when x_i is assigned a'_i, give a maximum improvement in the cost of the assignment

pick a pair $x_i, a'_i \in Y$.

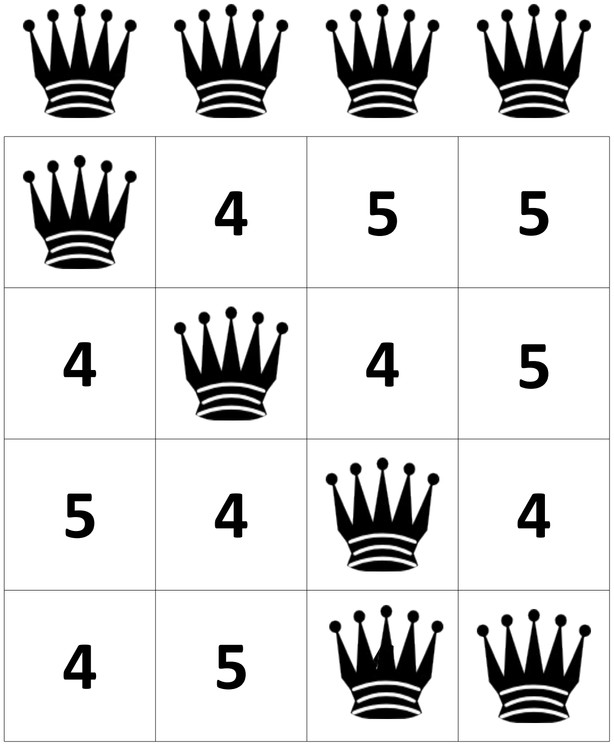
$\bar{a} \leftarrow (a_1, ...a_{i-1}, a'_i, a_{i+1}, ...a_n)$ (just flip a_i to a'_i)

end

return $\bar{a}$

Example

4-queens with SLS:



- starts in a randomly chosen instantiation
- random change of one assignment
- *minimize* #broken constraints
- stop when cost function = 0

Cost function value: **6**

Example

4-queens with SLS:

♛	3	5	2
4	♛	3	♛
3	3	6	2
4	4	♛	♛

- starts in a randomly chosen instantiation
- random change of one assignment
- *minimize* #broken constraints
- stop when cost function = 0

<u>Cost function value</u>: **4**

Example

4-queens with SLS:

♛	♛	4	2
3	♛	3	♛
2	1	5	2
2	2	♛	4

- starts in a randomly chosen instantiation
- random change of one assignment
- *minimize* #broken constraints
- stop when cost function = 0

Cost function value: **2**

Example

4-queens with SLS:

♛	♛	4	3
3	2	3	♛
♛	1	2	2
1	2	♛	3

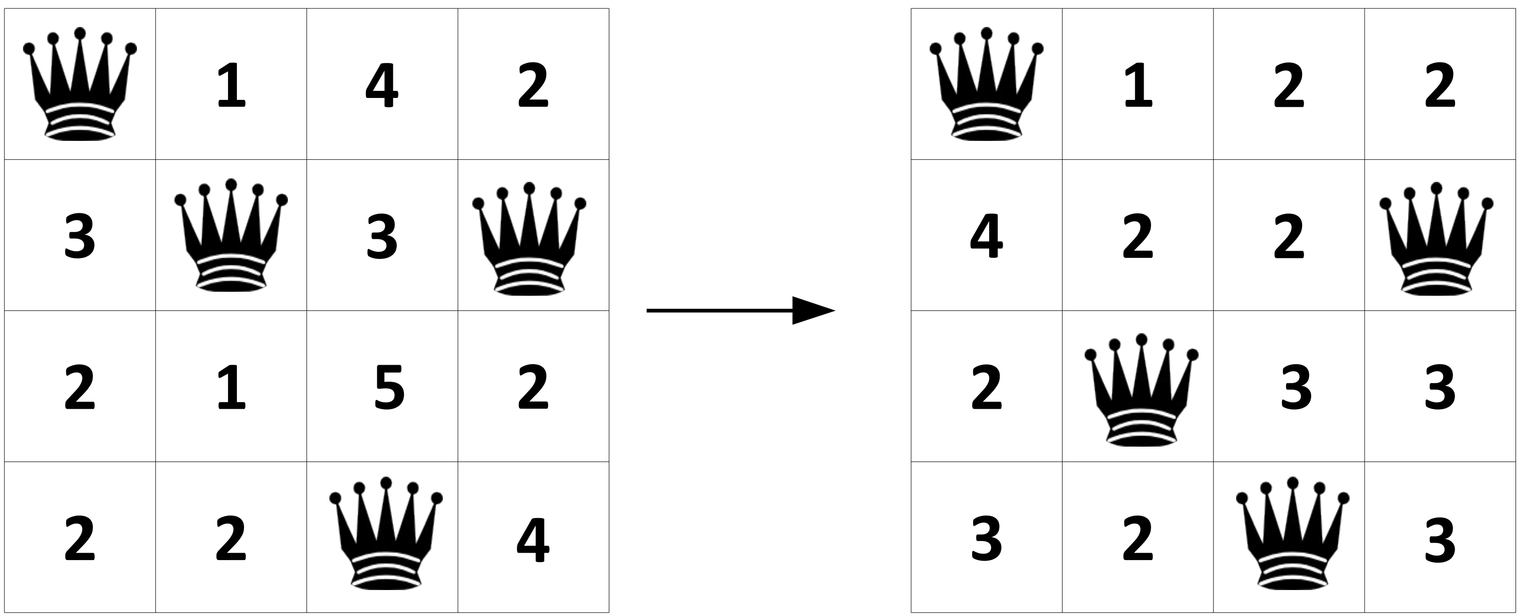
- starts in a randomly chosen instantiation
- random change of one assignment
- *minimize* #broken constraints
- stop when cost function = 0

Cost function value: **1**

Problem with SLS

- Search can get stuck in a *local minimum* or on a *plateau*

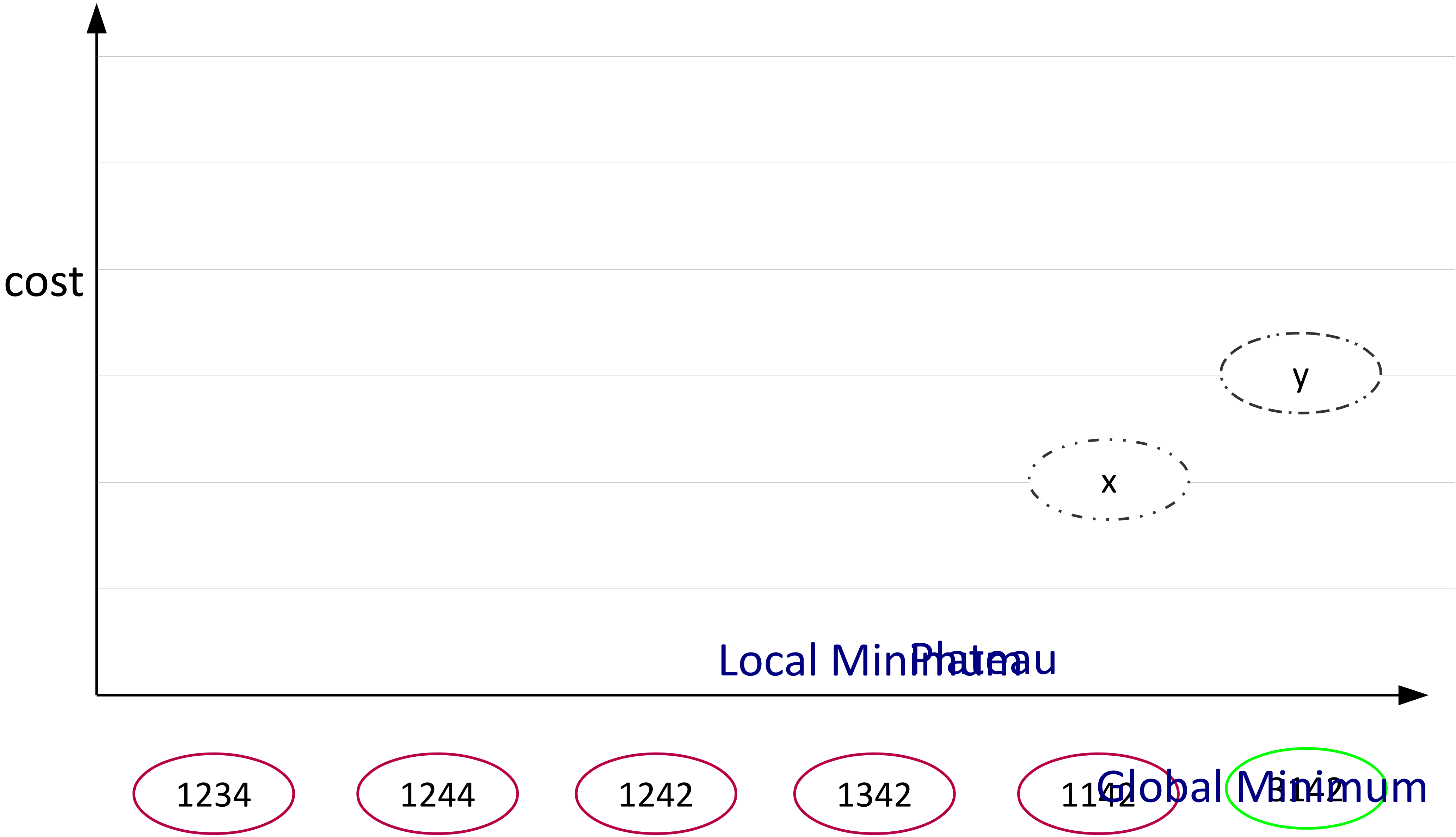
  → Algorithm never terminates



Cost function value: 2

Cost function value: 1

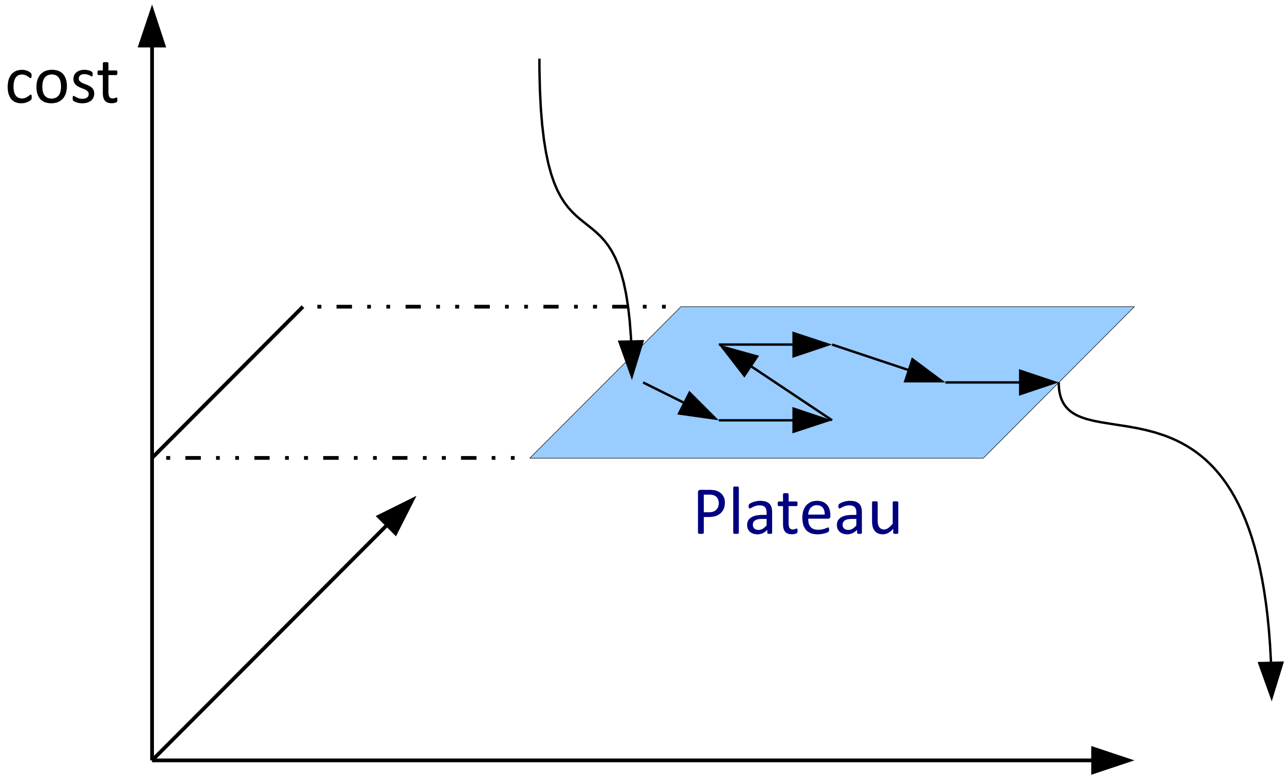
Plateaus & Local Minima



Escaping local minima

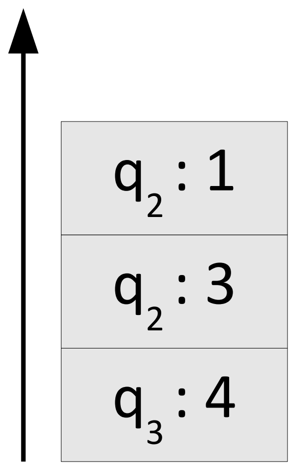
1. Plateau Search

- Allow non-improving sideway steps
- Problem: running in circles



2. Tabu search

- Store last *n* variable-value assignments
- Use list to prevent backward moves



Escaping local minima

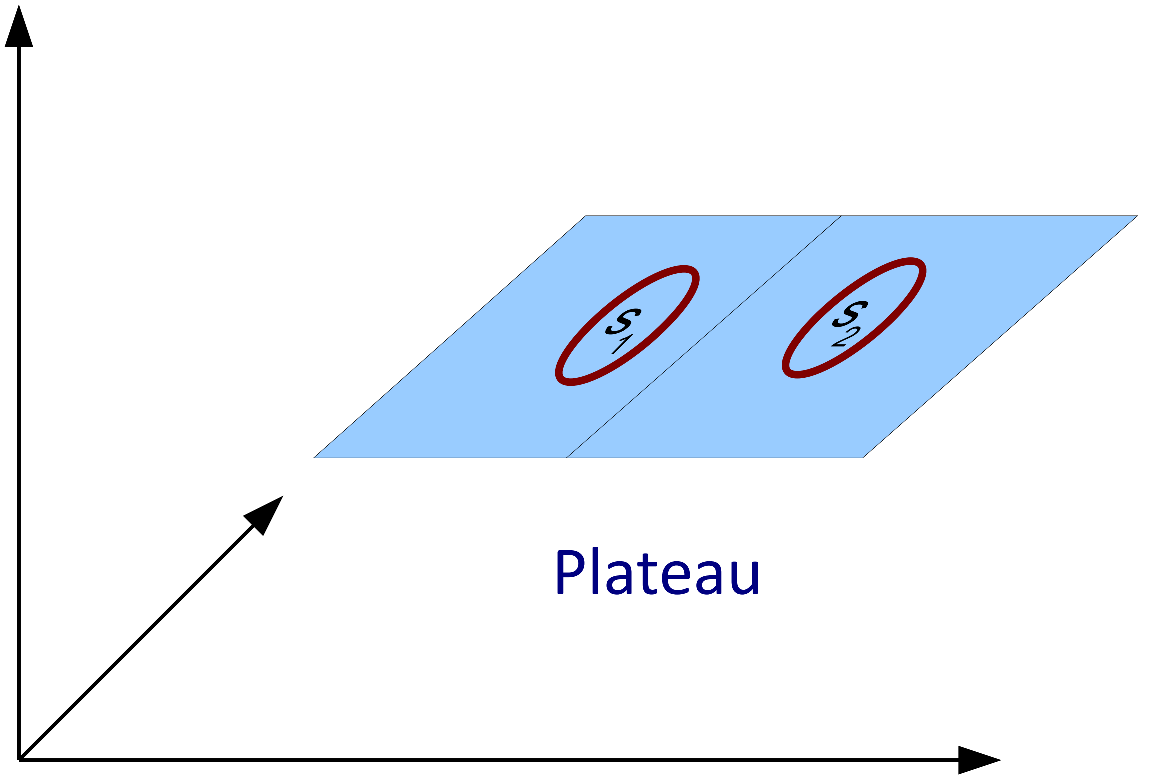
3. Random Restarts

- Restart algorithm in new random initialisation
- Can be combined with other escape-techniques
- Suggestions for restart:
  - when no improvement is possible
  - after *max_flips* steps without improvement (Plateau search)
  - increase *max_flips* after every improvement
- Achieve guarantee to find a solution

Escaping local minima

4. Constraint weighting

- Cost function: $F(\vec{a}) = \sum_i w_i * C_i(\vec{a})$
- Increasing weights of a violated constraint in local minima



Other improvements

Problem: Undetermined Termination

- Set a limit *max_tries* for the algorithm when to stop
- **but**: we lose guarantee to find a solution

Anytime Behaviour

- Store best assignment found so far (minimal #broken constraints)
- Return assignment when we need one (no solution)

Random Walks

procedure: `RandomWalk`

Input : A network $\mathfrak{R} = (X, D, C)$, probability p.

Output: A solution iff the problem is consistent.

start with a random initial assignment $\bar{a}$.

while $\bar{a}$ *is not a solution* **do**

(i) **pick** a violated constraint C, randomly

(ii) **choose** with probability p a variable-value pair $\langle x, a' \rangle$ for $x \in scope(C)$, or, with probability 1 - p, choose a variable-value pair $\langle x, a' \rangle$ that minimizes the value of the cost function when the value of x is changed to a'.

(iii) Change x's value to a'.

end

return $\bar{a}$.

Eventually hits a satisfying assignment (if exists)

p and Simulated Annealing

- Optimal p values for specific problems

Extension: **Simulated Annealing**

- Decrease p over time (by „cooling the temperature“)
  - more random jumps in earlier stages
  - more greedy progress later

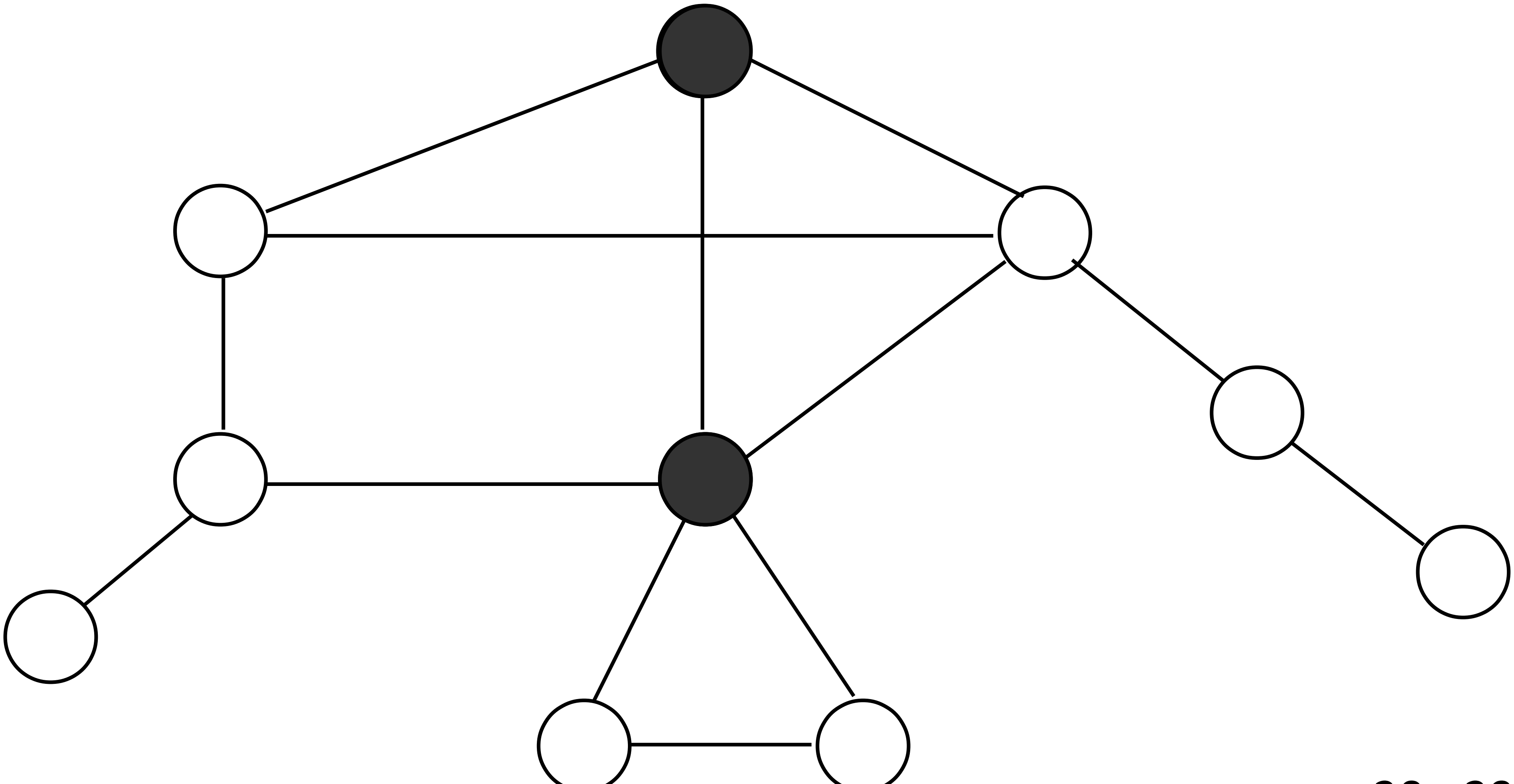
SLS + Inference

Goal: Smaller search space

- use Inference methods as with systematic search
- constraint propagation: performance varies
  - very helpful for removing many near-solutions
  - not good for uniform problem structures

SLS with Cycle-Cutset

Recap: Cycle-cutset decomposition



SLS with Cycle-Cutset

Idea: Replace systematic search on cutset with SLS

- Start with random cutset assignment

Repeat:

- calculate minimal cost in trees:

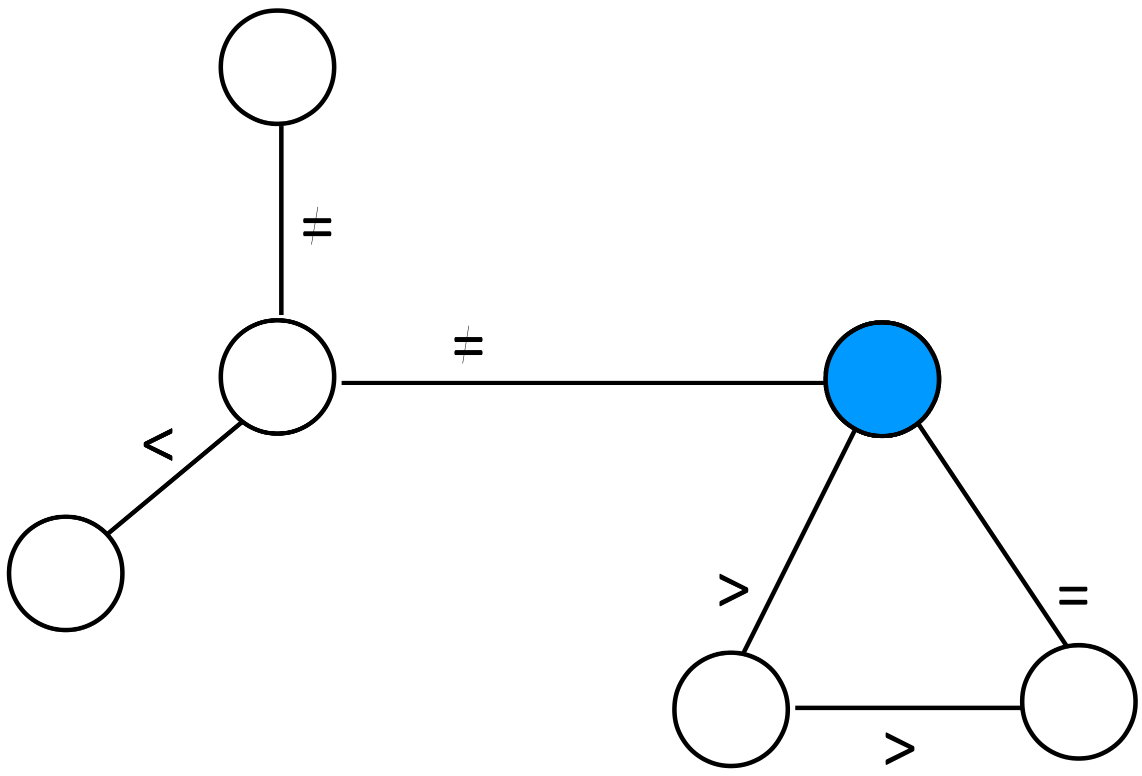
$$C(z_i \rightarrow a_i) = \sum_{children\ z_j} min_{a_j \in D_{z_j}} (C(z_j \rightarrow a_j) + R(z_i \rightarrow a_i, z_j \rightarrow a_j))$$

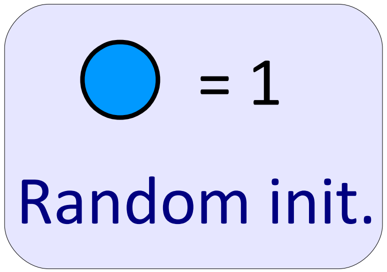
- assign values with minimal cost to tree variables
- greedily optimize cutset assignment (Local Search)

SLS with Cycle-Cutset

Example: Binary domains

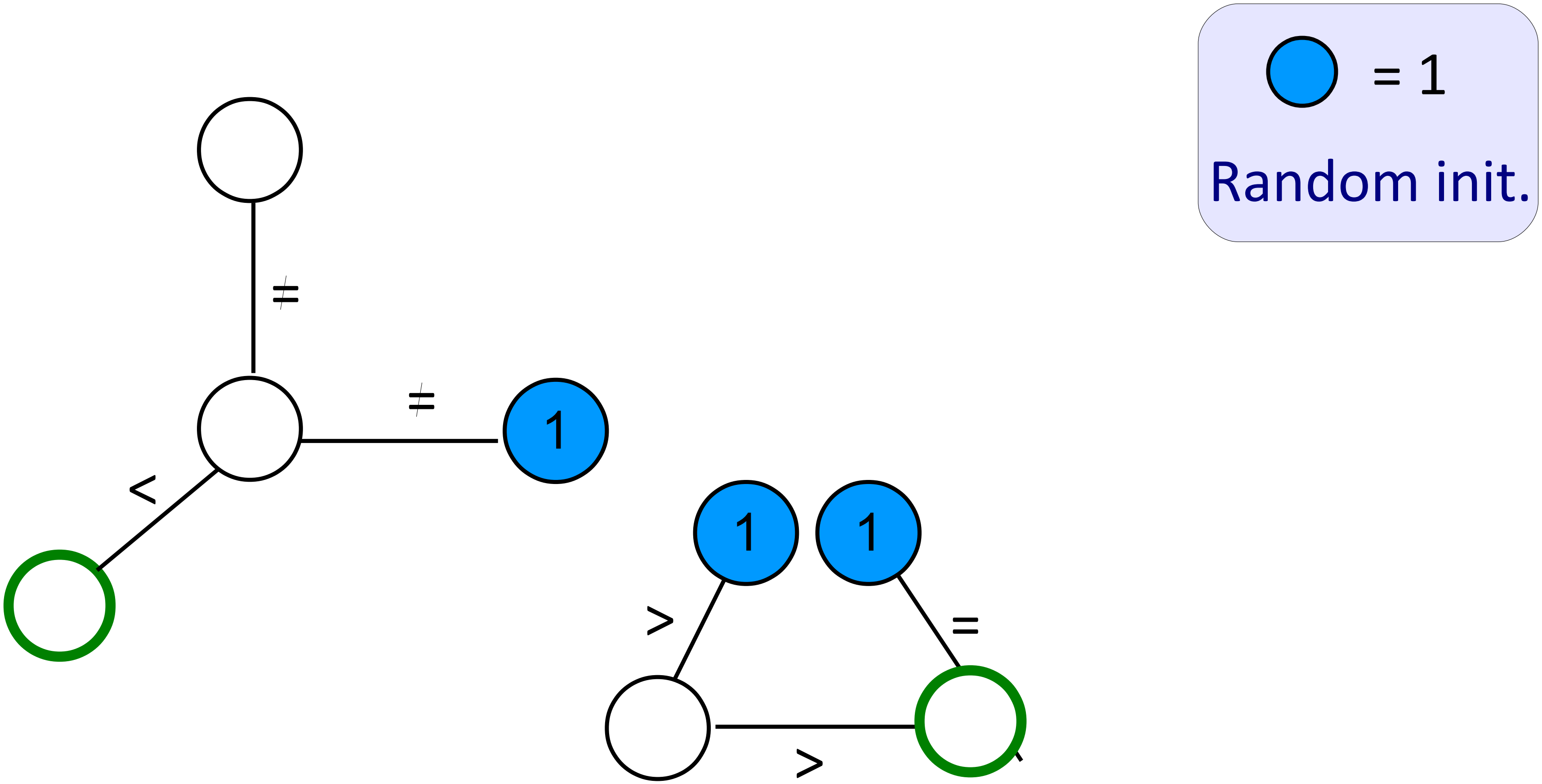
1. Assign values to cutset variables





SLS with Cycle-Cutset

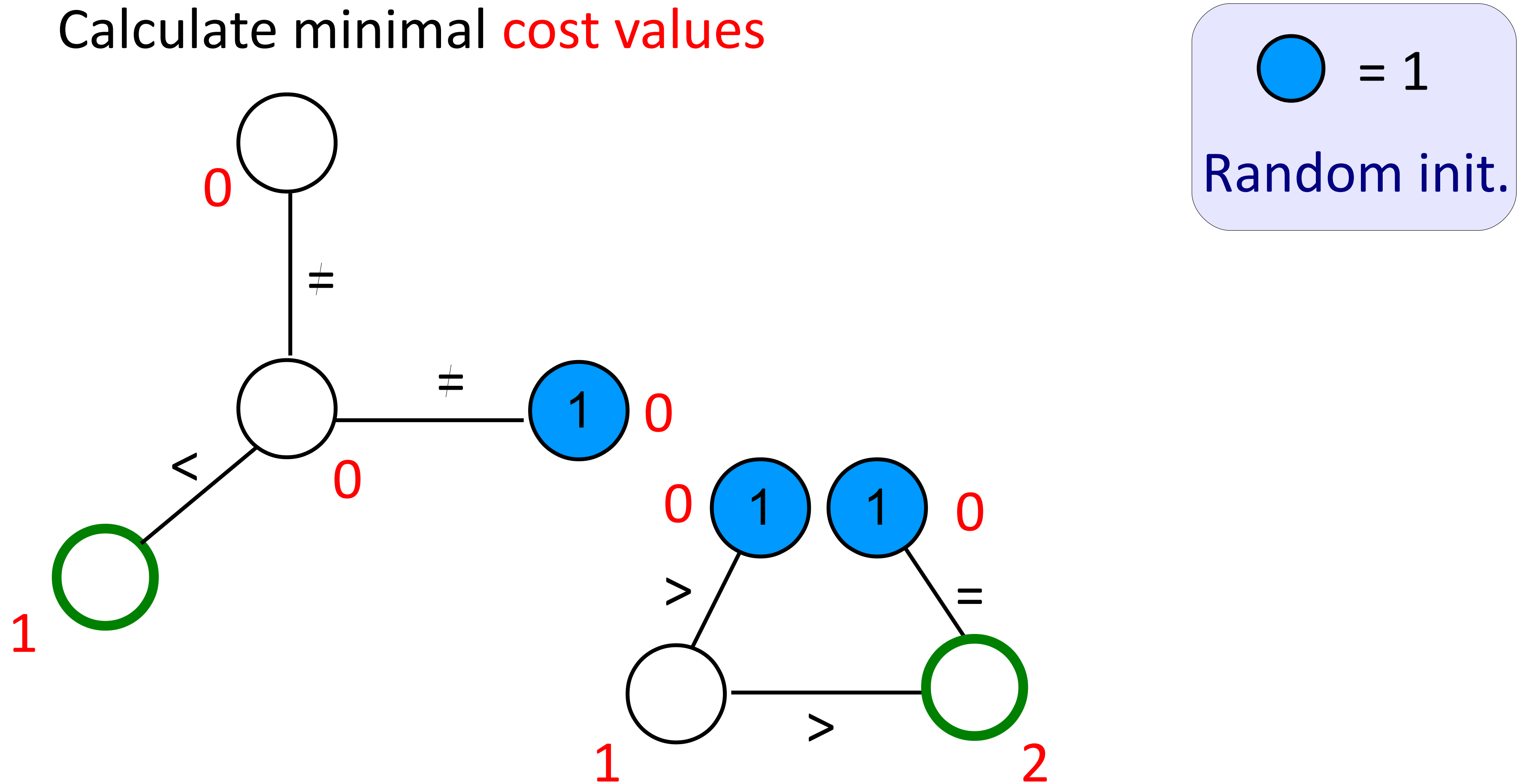
Set a **Root** for each tree



SLS with Cycle-Cutset

2. From leaves to root:

Calculate minimal cost values

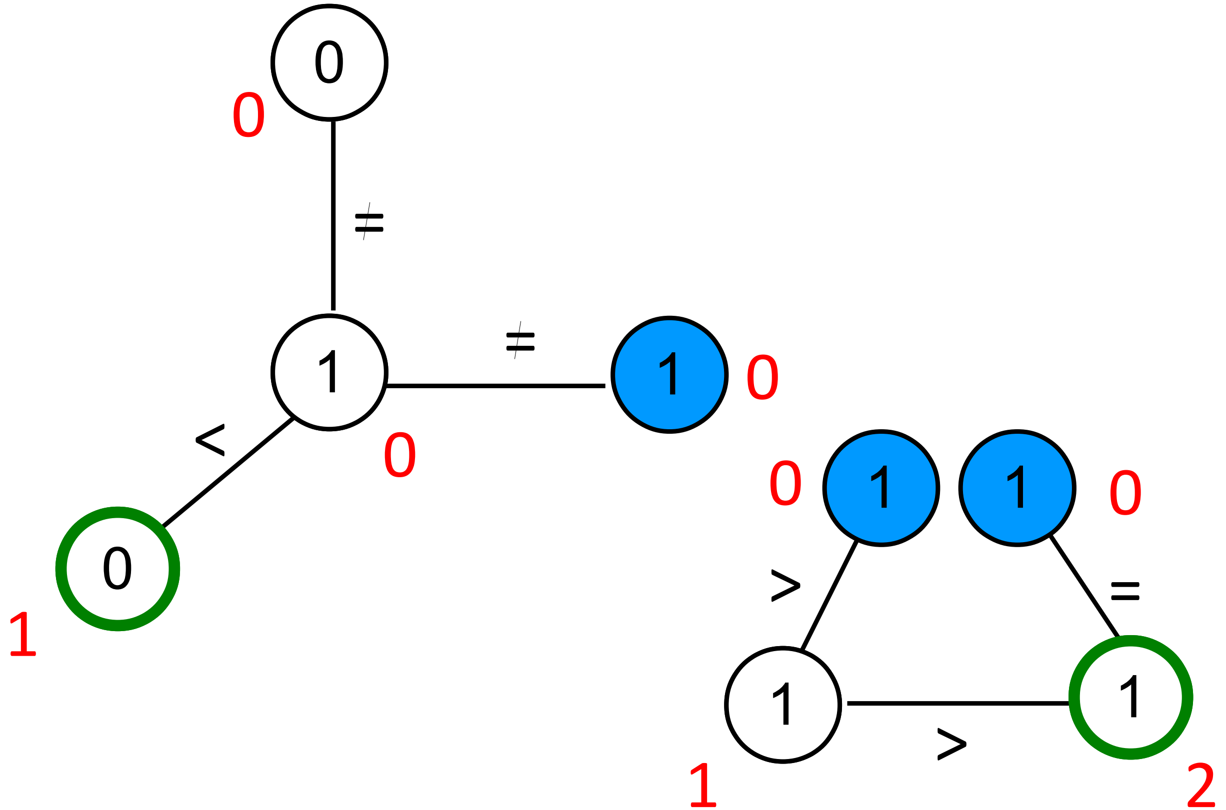


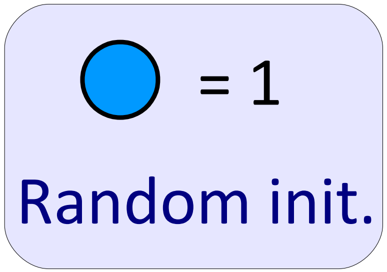
$$C(z_i \rightarrow a_i) = \sum_{children\ z_j} min_{a_j \in D_{z_j}} (C(z_j \rightarrow a_j) + R(z_i \rightarrow a_i, z_j \rightarrow a_j))$$

SLS with Cycle-Cutset

3. From root to leaves:

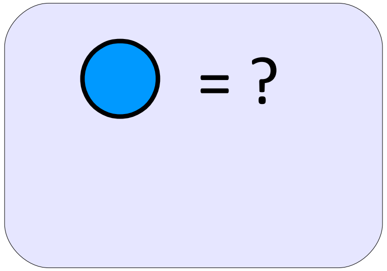
Assign values with minimal cost

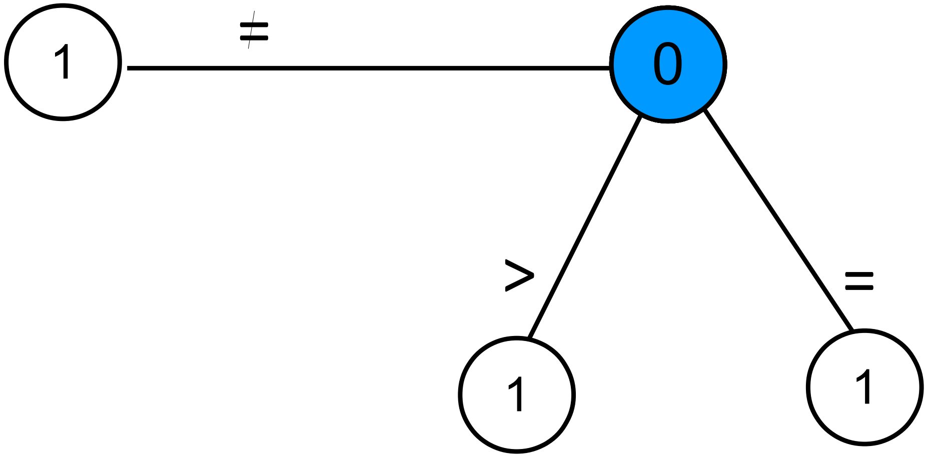




SLS with Cycle-Cutset

1. Assign values to cutset variables





SLS with Cycle-Cutset

2. From leaves to root:

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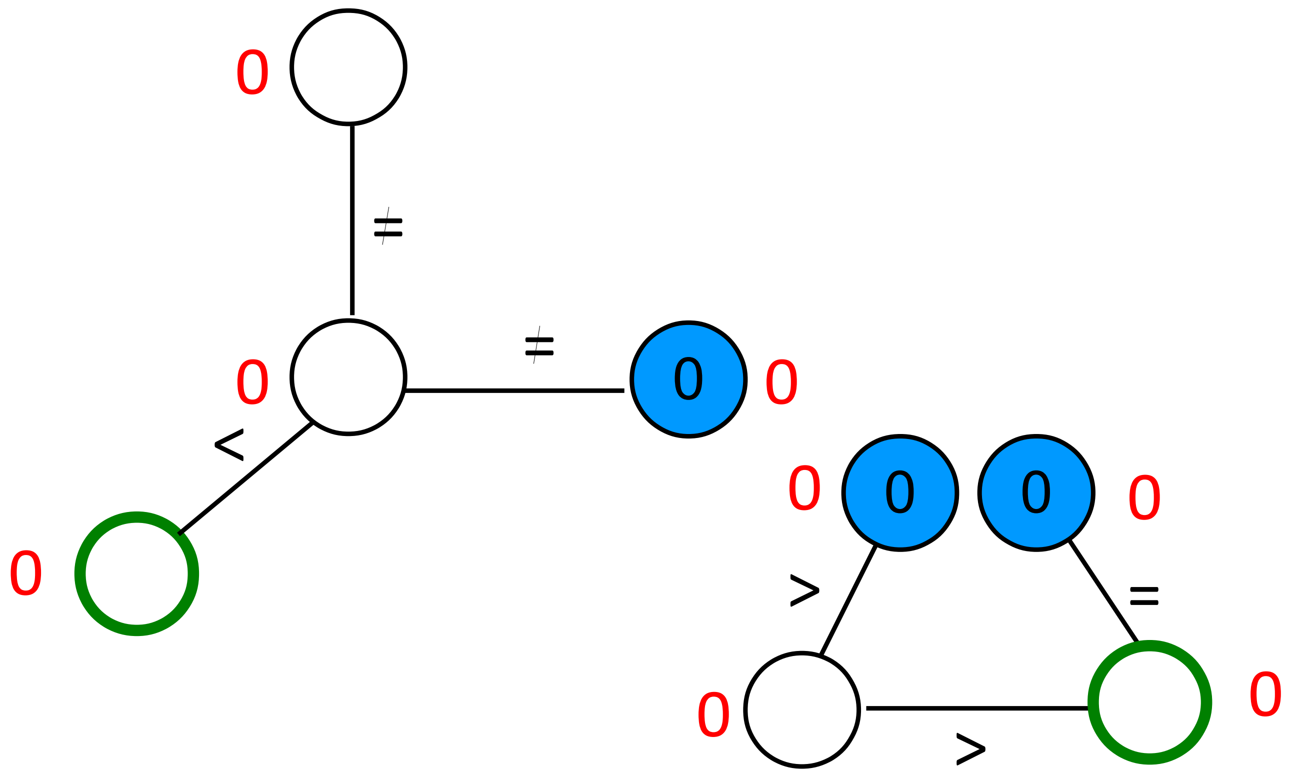

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SLS with Cycle-Cutset

3. From root to leaves:

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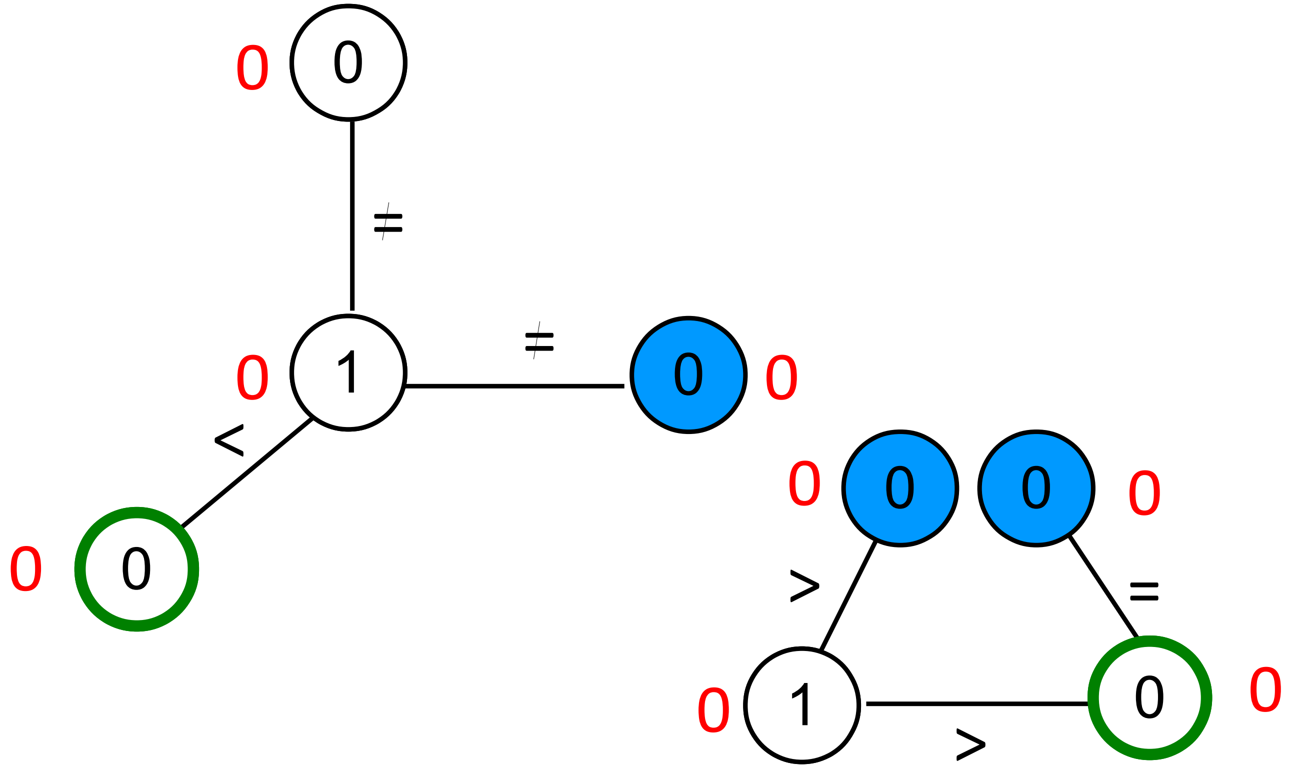

Summary

Stochastic Local Search

- Approximates systematic search
- Greedy algorithms: Techniques to escape local minima
- Random Walk: combines greedy + random choices
- Combination with Inference methods can help

- Can work very well
- but no guarantee of termination AND finding a solution