Presentation of the paper
Common Misconceptions Concerning Heuristic Search
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BASEL

Statements

1. More accurate heuristics result in fewer node expansions by A*.
2. A* does fewer node expansions than any other equally informed algorithm that finds optimal solutions.
3. Any admissible heuristic can be turned into a consistent heuristic by a simple technique called pathmax.
4. In search spaces whose operators all have the same cost, A* with the constant heuristic function $h = 0$ is the same as breadth-first search.

Theory: Properties of Heuristic Functions

<u>Definition</u>: Let $\mathcal{S} = \langle S, A, \mathrm{cost}, T, s_0, S_\star \rangle$ be a state space. A **heuristic function** over S is of the following form:

$$h\colon S \to \mathbb{R}_0^+$$

<u>Definition</u>: Let h be a heuristic function with $h = 0$ for all goal states.

- h **admissible** if for all states n:
  $h(n) \leq C(n)$ [1]
- h **consistent** if for all states s and t:
  $h(s) \leq \mathrm{cost}(s, t) + h(t)$ [2]

h is consistent $\Rightarrow$ h is admissible

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[1] $C(n)$ denotes the actual cost of the cheapest path from n to a goal node.
[2] $\mathrm{cost}(s, t)$ denotes the cost of the cheapest path between s and t.
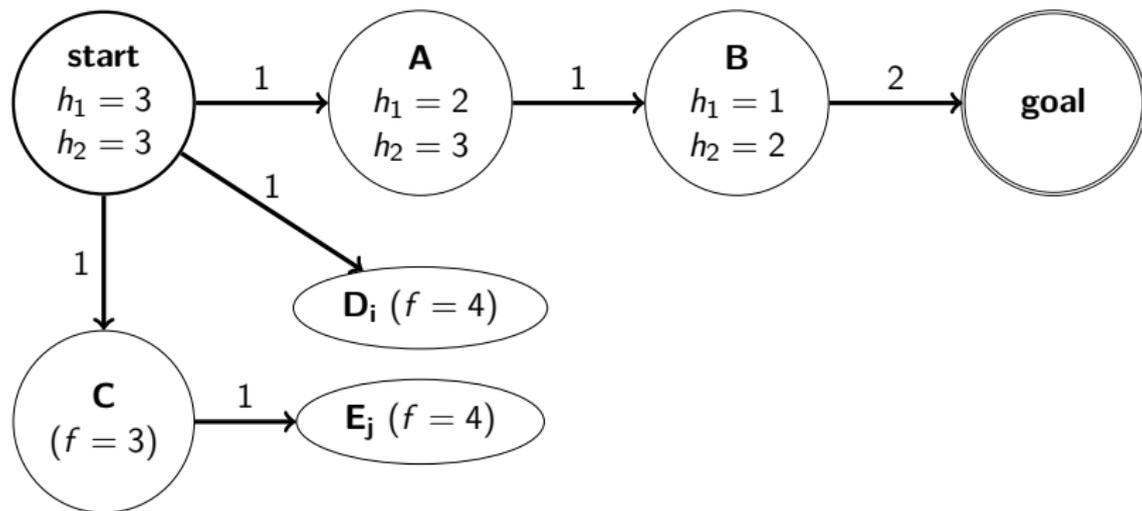
Statement 1: Explanation

More accurate heuristics result in fewer node expansions by A*.

What does more accurate mean?

<u>Definition</u>: Let h_1 and h_2 be two admissible heuristic functions.
h_2 is called **"better than"** h_1 if the following two conditions hold:

- $\forall s \in S \colon h_1(s) \leq h_2(s)$
- $\exists s \in S \colon h_1(s) < h_2(s)$

Statement 1: Comparison

Comparision

Heuristic function	Worst Case	Best Case
h_2 (better)	$4 + i + j$	4
h_1 (dominated)	4	3

What happened?

- All states not on optimal path: $h_1 = h_2$
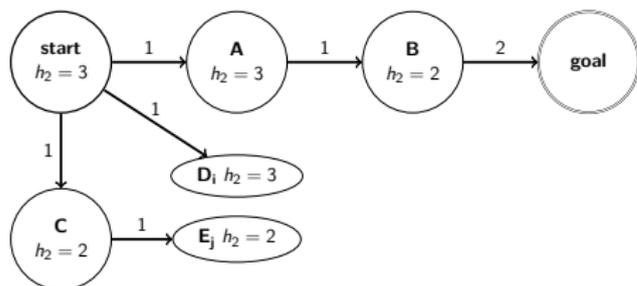- All states on optimal path: $h_2 > h_1$ (by definition)

$\Rightarrow$ Lower (but less accurate) h-values prevent from exploring states which are not on the optimal path.

Can tie-breaking help?

Performing A* with the better heuristic function h_2.

Status after two expansions.

state	$f = g + h$
~~start~~	$3 = 0 + 3$
A	$4 = 1 + 3$
~~C~~	$3 = 1 + 2$
D_i	$4 = 1 + 3$
E_j	$4 = 2 + 2$

$\rightarrow$ **No, tie-breaking cannot help!**

Statement 2: Explanation

> A* does fewer node expansions that any other equally
> informed algorithm that finds optimal solutions.

$\rightarrow$ False if used heuristic is admissible, but not consistent.

Proof sketch:

- State space with admissible, inconsistent heuristic.
- Show: A* does $\mathcal{O}(2^{i-1})$ node expansions.
- Introduce: Variant of A* which does $\mathcal{O}(i^2)$ node expansions.
- $\Rightarrow$ A* is not always optimal w.r.t number of node expansions.

Statement 2: Counterexample (Martelli Graph G_5)

Martelli graph family G_i ($\forall i \geq 3$) with $i + 1$ nodes and $\mathcal{O}(2^i)$ expansions (A*).
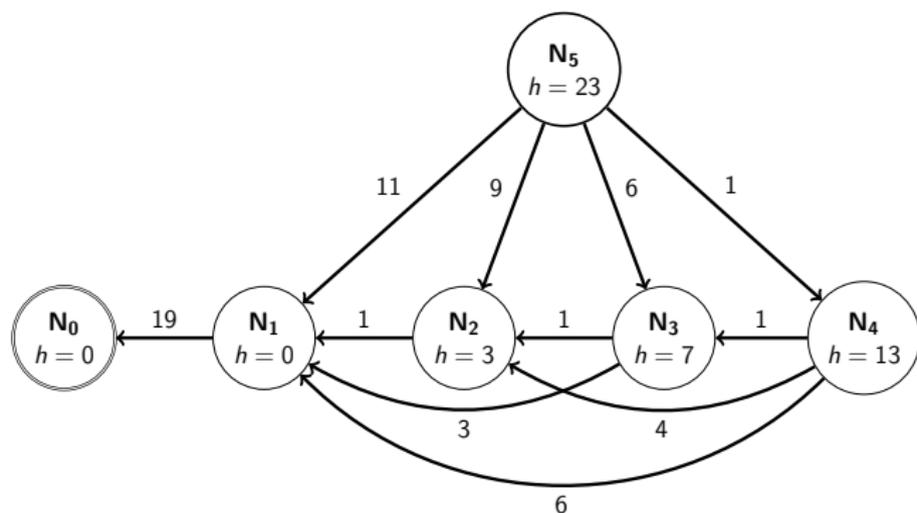
Inconsistency: E.g. $h(N_2) = 3$, $h(N_1) = 0$ and $cost(N_1, N_2) = 1$ but:

$$h(N_2) \leq cost(N_1, N_2) + h(N_1) \nleq$$

Statement 2: Algorithm B

B (variation of A*) finds optimal solution with less node expansions than A* in G_5.

Idea:

- Global variable F: Remembers highest occured f-value.
- Modified node fetching:
  - **If** (minimum f-value in open list $\geq F$) **then** same state as A*
  - **Else** state with minimum g-value among states with $f < F$

Hence:

No state is expanded twice, until F has increased.

$\Rightarrow$ Less expansions of nodes.

> Any admissible heuristic can be turned into a consistent heuristic by a simple technique called pathmax.

Idea behind pathmax:
On a path to a node n the f-value of the nodes should never decrease.
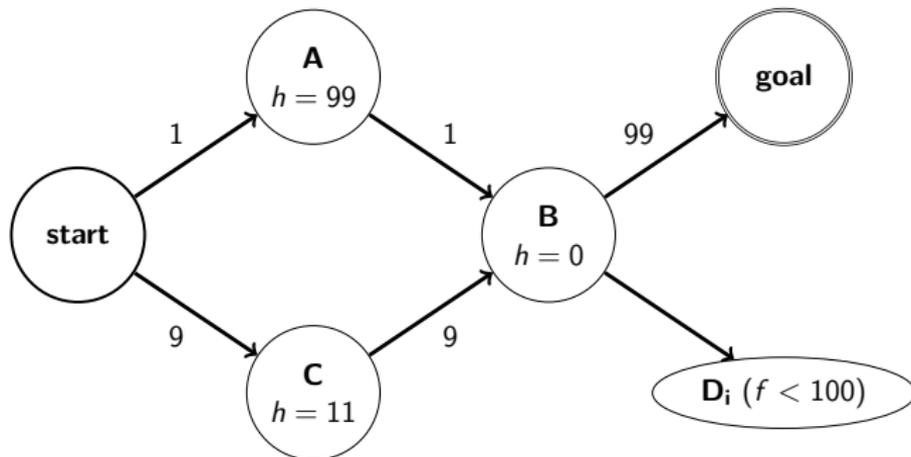$\rightarrow$ Maximum f-value seen on the path is used.

Connection to consistency?
Path through s. t lies later on that path, with decreasing $f(t)$:

$$h(s) \leq cost(s, t) + h(t) \cancel{} \quad \text{(without pathmax)}$$

But: Pathmax can help by prevent from decreasing $f(t)$.

Reminder: A* with consistent heuristic never reopens a state. ⚡

$\rightarrow$ B is expanded after reaching through a suboptimal path.
$\rightarrow$ B is reopened later reaching through an optimal path.
$\Rightarrow$ Contradiction to reminder.

Statement 4: Explanation

> In search spaces whose operators all have the same cost
> A* with the constant heuristic function $h = 0$
> is the same as breadth-first search.

Where does this assumption come from?

Algorithm General framework for search algorithms

1: OpenList = {*root*}
2: ClosedList = {}
3: **while** OpenList is not empty **do**
4:     n := fetch node from OpenList; /* to be specified*/
5:     **if** $n \notin \mathrm{ClosedList}$ **then**
6:         **if** n is a goal **then**
7:             **return** plan to n from the root;
8:         save n in the ClosedList;
9:         expand n;
10:        save successors of n in OpenList;
11: **return** no solution;
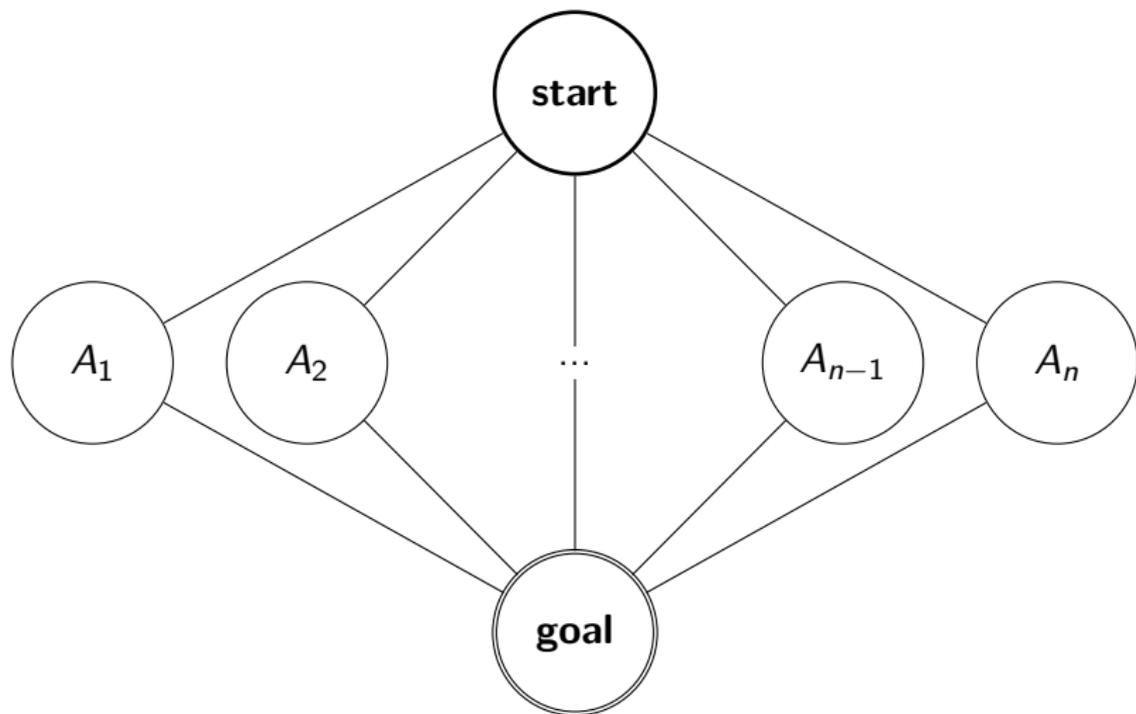
State space with uniform cost and constant heuristic function $h = 0$

Reformulated version of the discussed statements

1. Better heuristics can result in more search.
2. A* is not always optimal with respect to the number of node expansions.
3. Pathmax does not make heuristics consistent.
4. A* with $h = 0$ and uniform costs is not breadth-first search.

Questions?