Foundations of Artificial Intelligence

42. Board Games: Alpha-Beta Search

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Board Games: Overview

chapter overview:

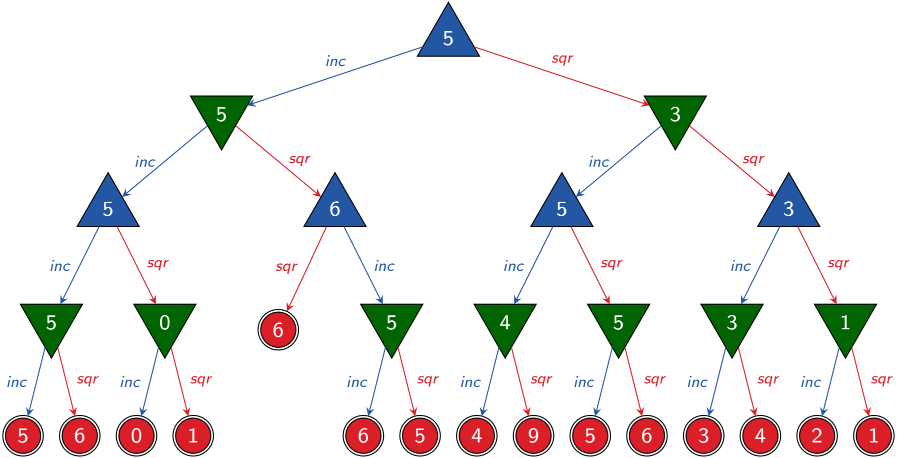
Limitations of Minimax

What if the size of the game tree is too big for minimax?

$\leadsto$ Heuristic Alpha-Beta Search

Alpha-Beta Search

Can We Save Search Effort?



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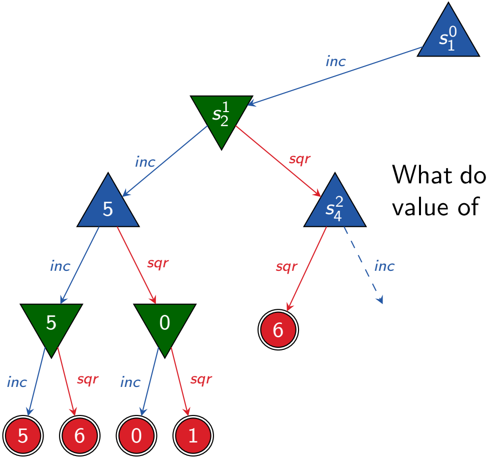

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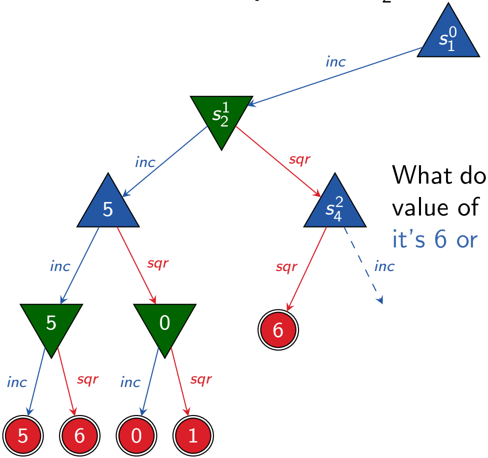
Can We Save Search Effort?

And about the utility value of s_2^1?



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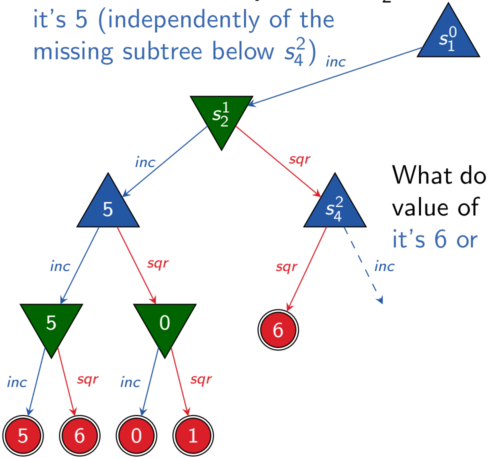
it's 5 (independently of the missing subtree below s_4^2)



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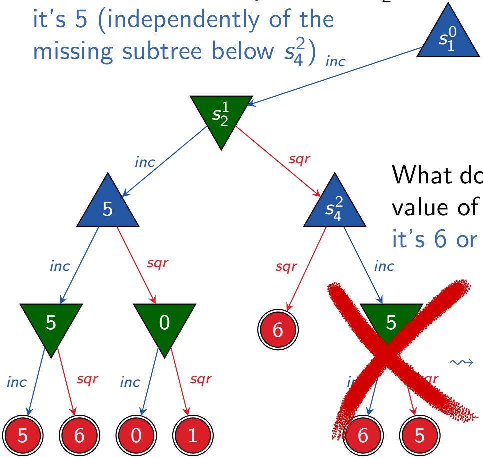

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$\rightsquigarrow$ we don't have to look at this

Idea

idea: use two values α and β during minimax search such that

- α is known lower bound on utility of *max* and
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it holds for every recursive call that a subtree

- is not interesting if its utility value is $\leq \alpha$
  because *max* will never enter it when playing optimally
- is not interesting if its utility value is $\geq \beta$
  because *min* will never enter it when playing optimally
- rooted at a *max* node is pruned if utility $\geq \beta$
- rooted at a *min* node is pruned if utility $\leq \alpha$

Alpha-Beta Search: Pseudo Code

- algorithm skeleton the same as minimax
- function signature extended by two variables α and β

function alpha-beta-main(p)

$\langle v, move \rangle :=$ alpha-beta($p, -\infty, +\infty$)

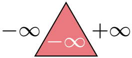
return *move*

Alpha-Beta Search: Pseudo-Code

function alpha-beta(p, α, β)

if p is terminal position:
    return $\langle \mathit{utility}(p), \textbf{none} \rangle$
initialize v and *best_move* [as in minimax]
for each $\langle \mathit{move}, p' \rangle \in \mathrm{succ}(p)$:
    $\langle v', \mathit{best\_move}' \rangle := \mathit{alpha\text{-}beta}(p', \alpha, \beta)$
    update v and *best_move* [as in minimax]
    if $\mathit{player}(p) = \mathit{max}$:
        if $v \geq \beta$:
            return $\langle v, \textbf{none} \rangle$
        $\alpha := \max\{\alpha, v\}$
    if $\mathit{player}(p) = \mathit{min}$:
        if $v \leq \alpha$:
            return $\langle v, \textbf{none} \rangle$
        $\beta := \min\{\beta, v\}$
return $\langle v, \mathit{best\_move} \rangle$

Example



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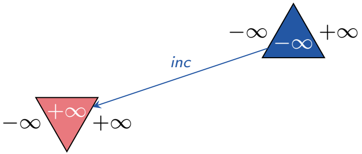

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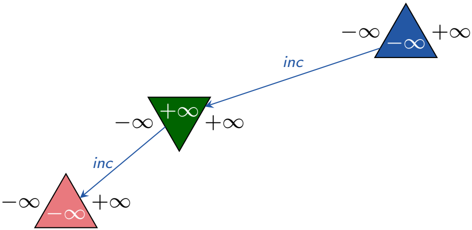

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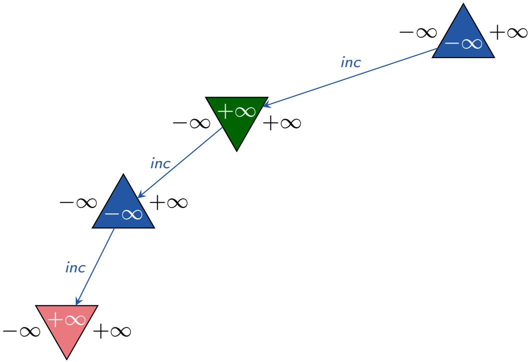

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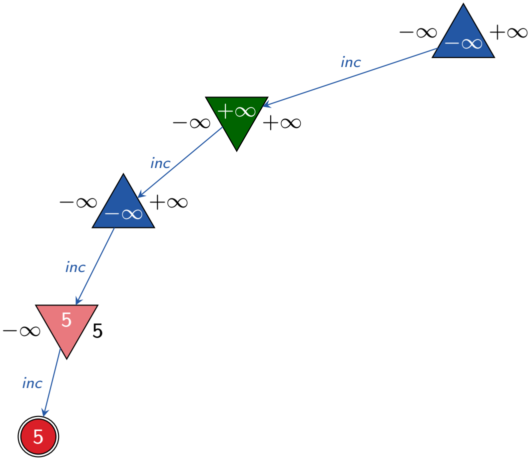

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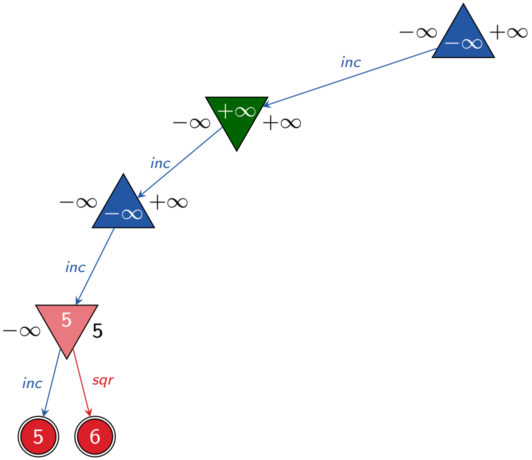

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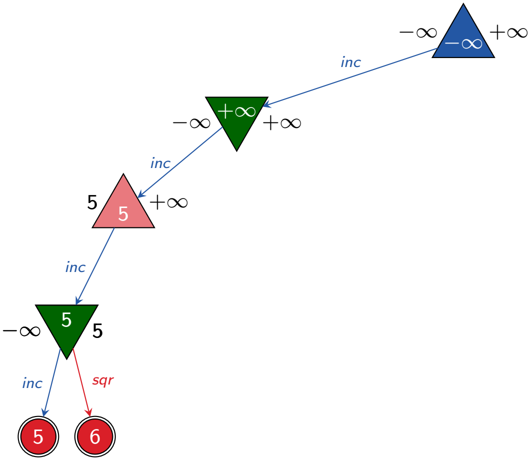

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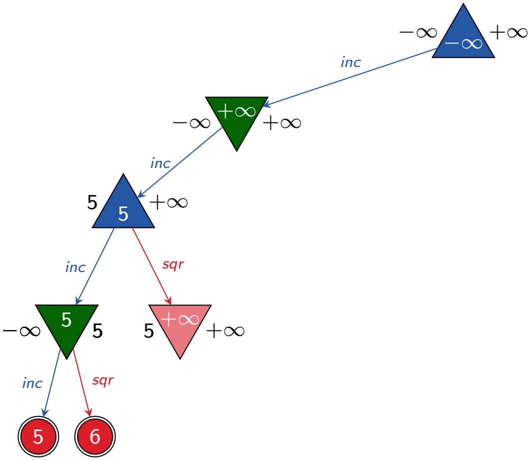

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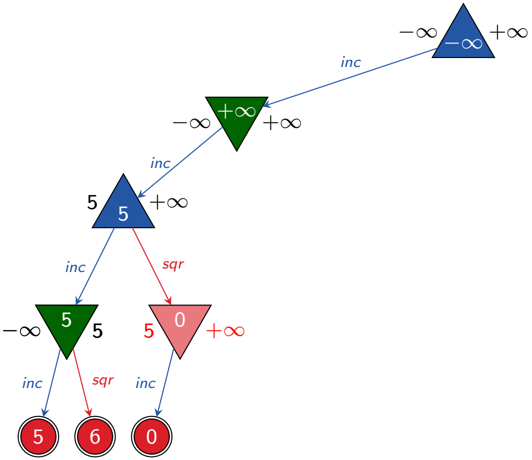

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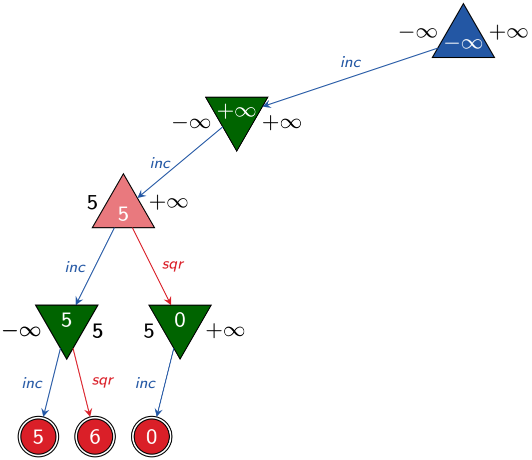

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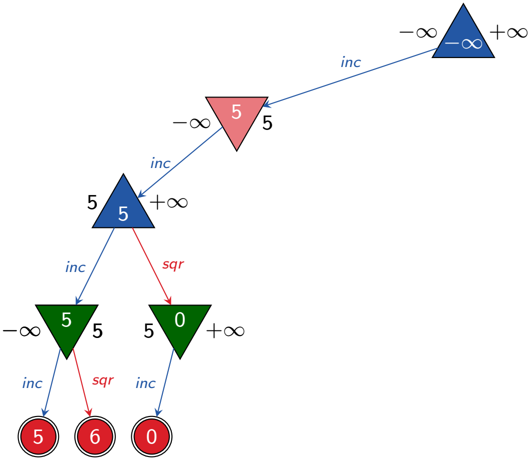

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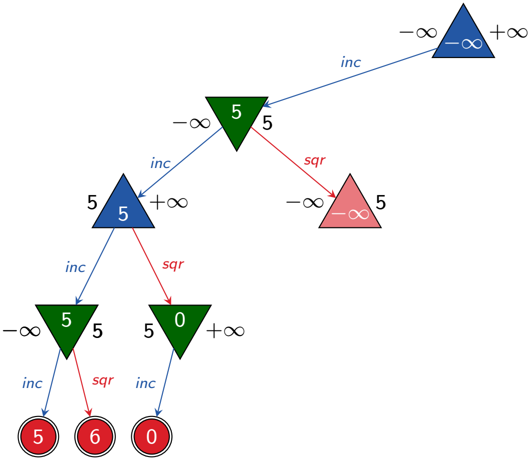

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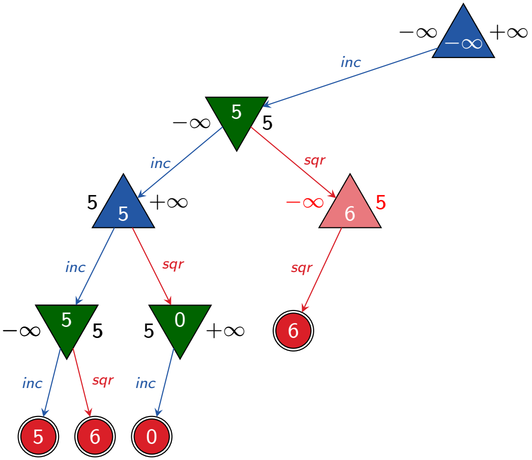

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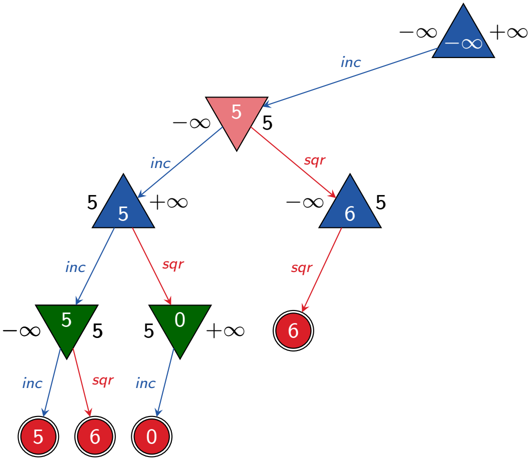

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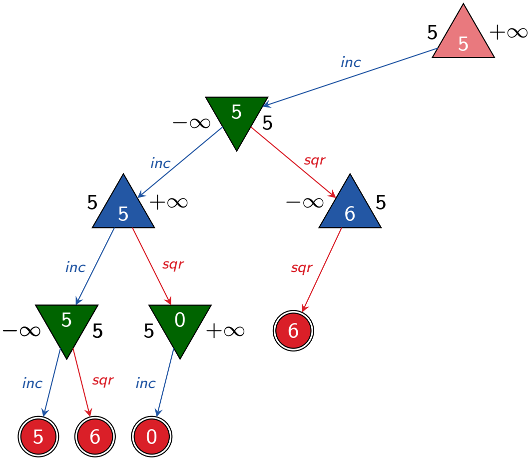

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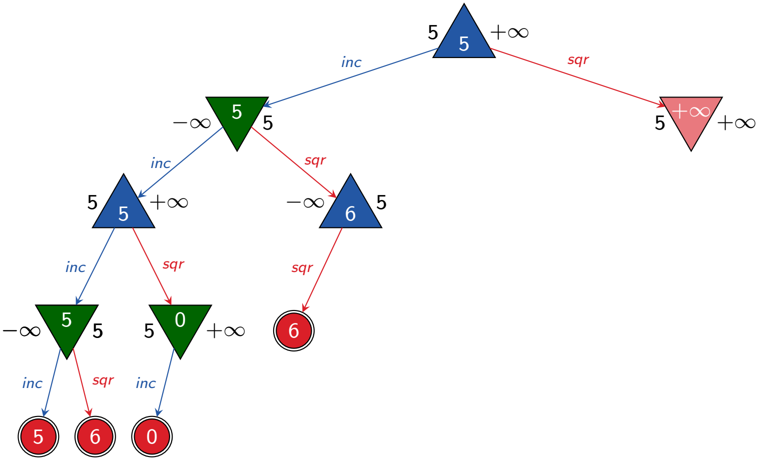

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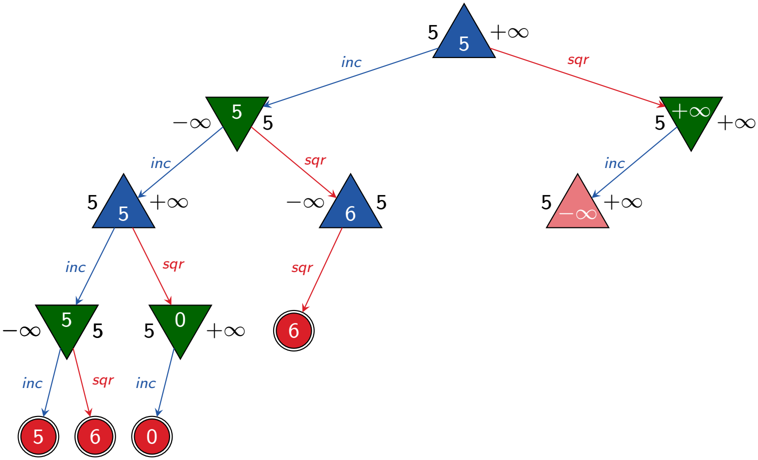

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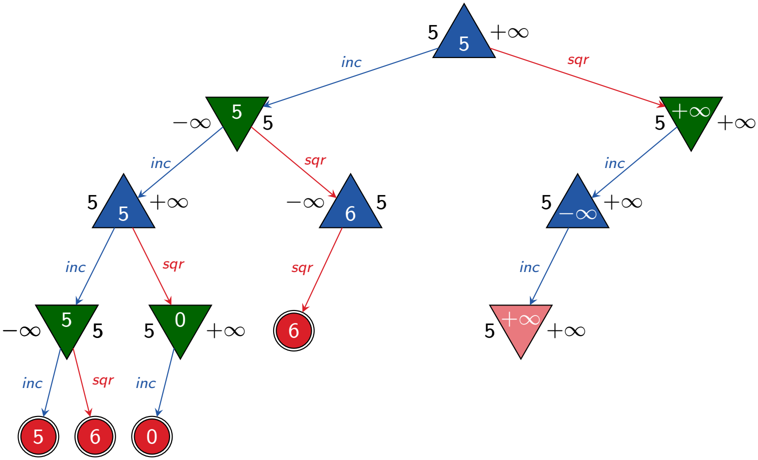

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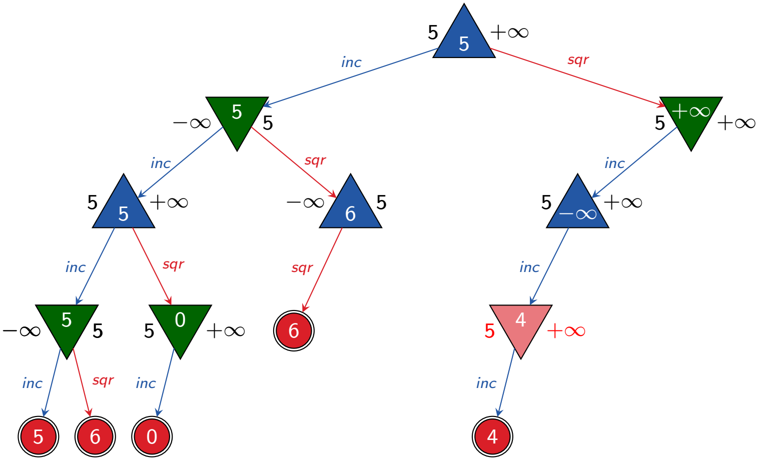

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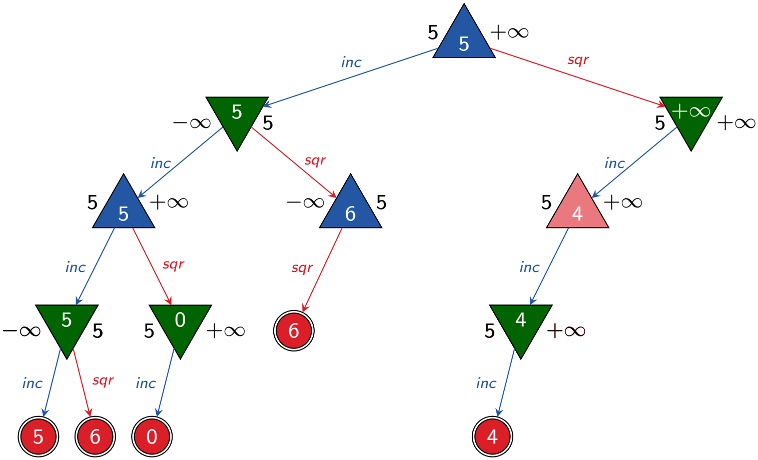

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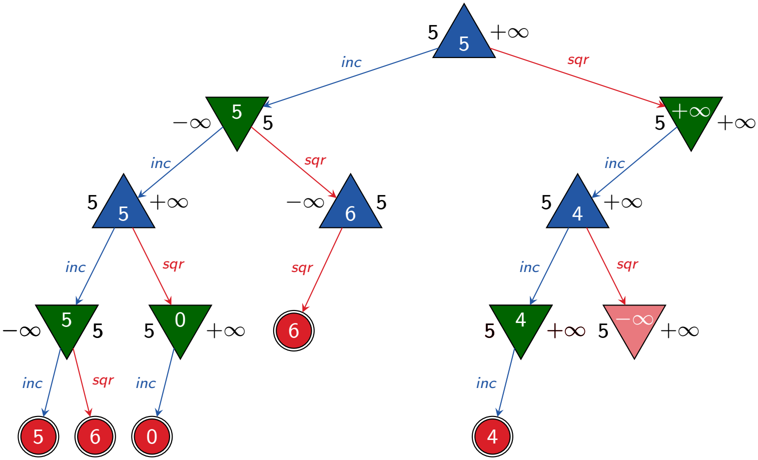

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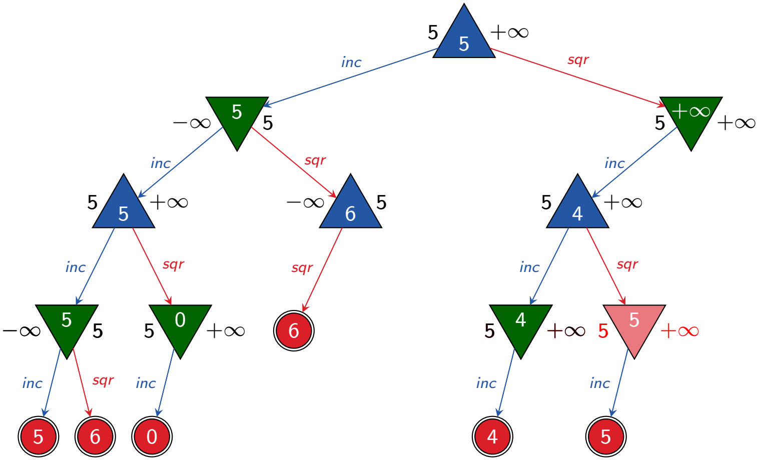

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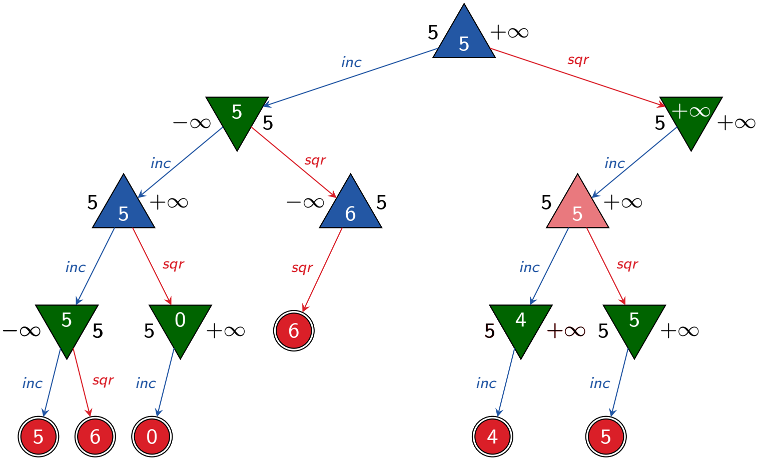

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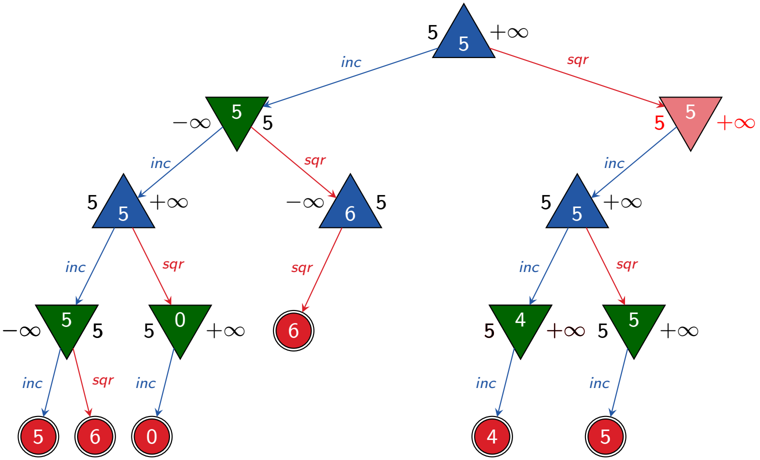

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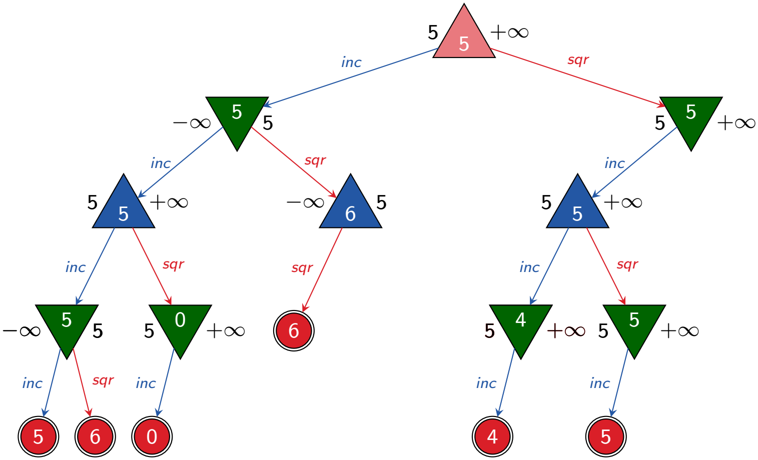

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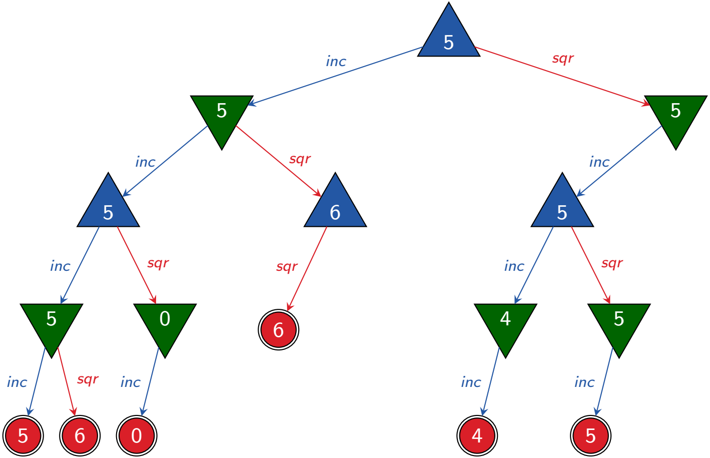
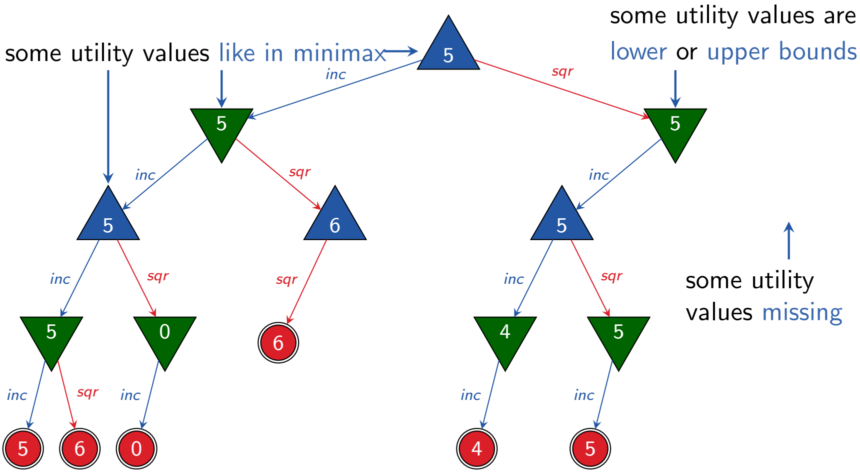


What do the utility values express?

Discussion



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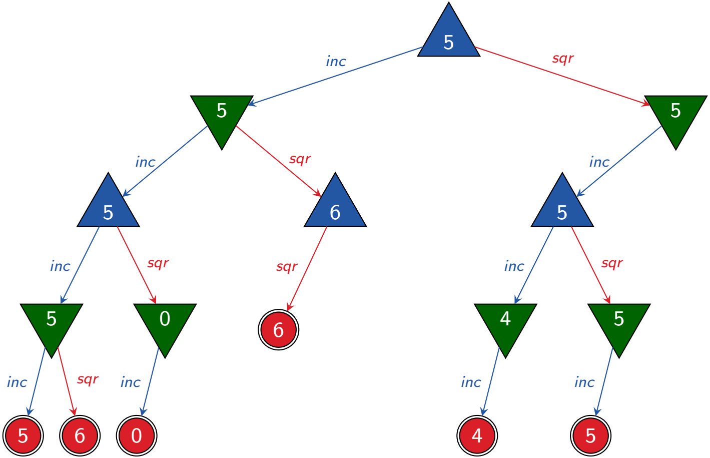

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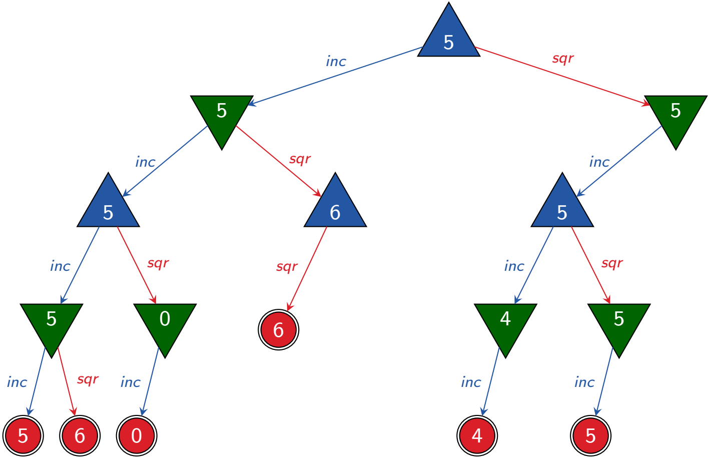
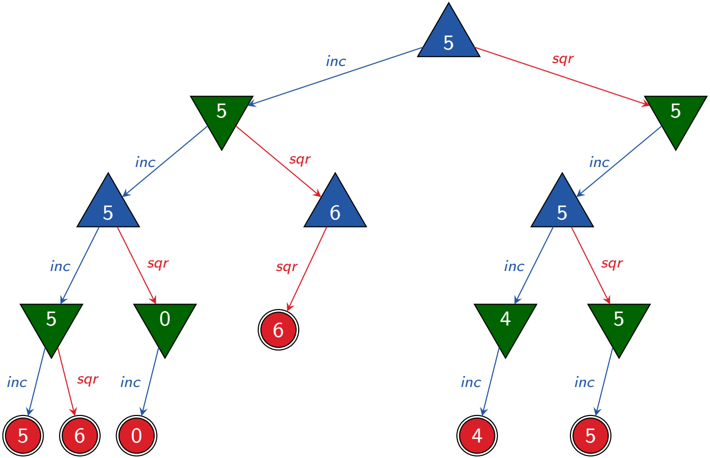


- only partial
- optimal in positions reachable under optimal play
- need to take earliest move in case of ties

Move Ordering

How Much Effort Did We Save?



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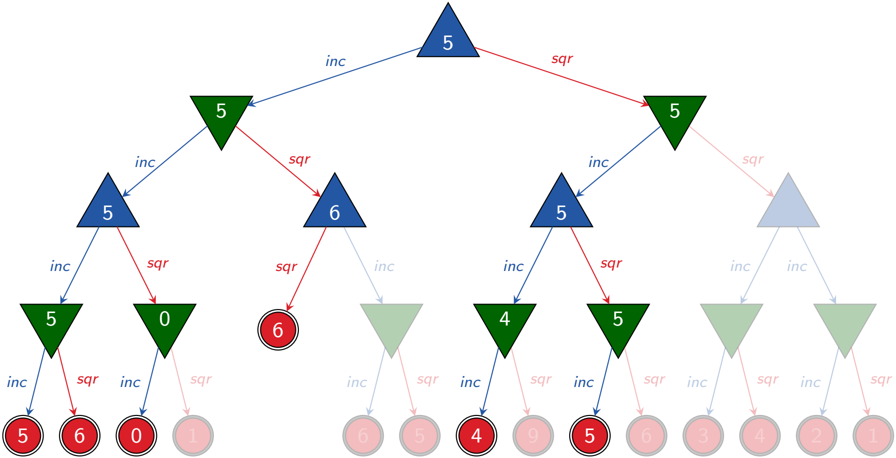
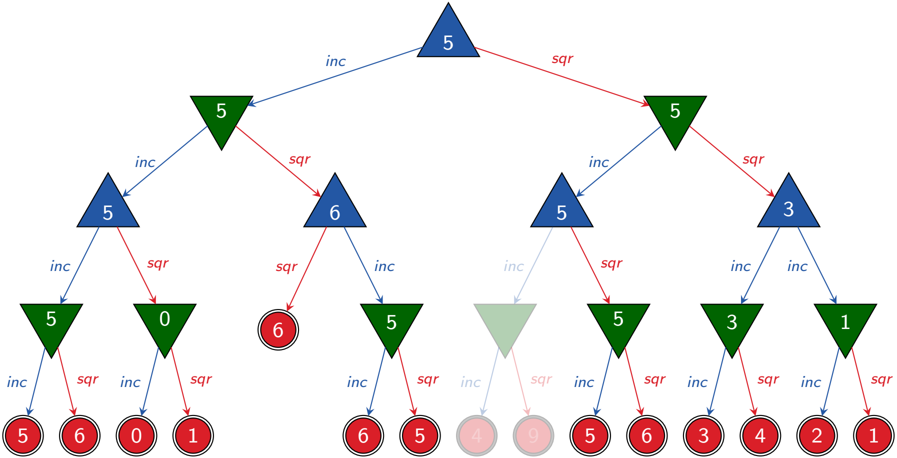


Have We Been Lucky?



if successors are considered in reverse order, we prune only a few positions

Move Ordering

idea: first consider the successors that are likely to be best

- domain-specific ordering function
  e.g. chess: captures $<$ threats $<$ forward moves $<$ backward moves
- dynamic move-ordering
  - first try moves that have been good in the past
  - e.g., in iterative deepening search:
    best moves from previous iteration

How Much Do We Gain with Alpha-Beta Pruning?

assumption: uniform game tree, depth d, branching factor $b \geq 2$; *max* and *min* positions alternating

- perfect move ordering
  - best move in every position is considered first (this cannot be done in practice)
  - effort reduced from $O(b^d)$ (minimax) to $O(b^{d/2})$
  - doubles the search depth that can be achieved in same time
- random move ordering
  - effort still reduced to $O(b^{3d/4})$ (for moderate b)

in practice, it is often possible to get close to the optimum

Heuristic Alpha-Beta Search

- combines evaluation function and alpha-beta search
- often uses additional techniques, e.g.
  - quiescence search
  - transposition tables
  - forward pruning
  - specialised sub-procedure for critical parts of the game (e.g., endgame database in chess)
  - ...

$\rightsquigarrow$ reaches expert level of play in chess

Summary

Summary

alpha-beta search

- stores which utility both players can force somewhere else in the game tree
- exploits this information to avoid unnecessary computations
- can have significantly lower search effort than minimax
- best case: search twice as deep in the same time