Foundations of Artificial Intelligence

37. Automated Planning: Abstraction

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Automated Planning: Overview

Chapter overview: automated planning

- 33. Introduction
- 34. Planning Formalisms
- 35.–36. Planning Heuristics: Delete Relaxation
- 37. Planning Heuristics: Abstraction
- 38.–39. Planning Heuristics: Landmarks

Planning Heuristics

We consider three basic ideas for general heuristics:

- Delete Relaxation
- Abstraction $\rightsquigarrow$ this chapter
- Landmarks

Planning Heuristics

We consider three basic ideas for general heuristics:

- Delete Relaxation
- Abstraction $\rightsquigarrow$ this chapter
- Landmarks

Abstraction: Idea

Estimate solution costs by considering a smaller planning task.

SAS$^+$

SAS$^+$ Encoding

- in this chapter: SAS$^+$ encoding instead of STRIPS (see Chapter 34)
- difference: state variables v not binary, but with finite domain $\mathrm{dom}(v)$
- accordingly, preconditions, effects, goals specified as partial assignments
- everything else equal to STRIPS

(In practice, planning systems convert automatically between STRIPS and SAS$^+$.)

SAS$^+$ Planning Task

Definition (SAS$^+$ planning task)

A SAS$^+$ planning task is a 5-tuple $\Pi = \langle V, \mathrm{dom}, I, G, A \rangle$ with the following components:

- V: finite set of state variables
- dom: domain; $\mathrm{dom}(v)$ finite and non-empty for all $v \in V$
  - states: total assignments for V according to dom
- I: the initial state (state = total assignment)
- G: goals (partial assignment)
- A: finite set of actions a with
  - $pre(a)$: its preconditions (partial assignment)
  - $eff(a)$: its effects (partial assignment)
  - $cost(a) \in \mathbb{N}_0$: its cost

State Space of SAS$^+$ Planning Task

Definition (state space induced by SAS$^+$ planning task)

Let $\Pi = \langle V, \mathrm{dom}, I, G, A\rangle$ be a SAS$^+$ planning task.
Then Π induces the state space $\mathcal{S}(\Pi) = \langle S, A, cost, T, s_0, S_\star\rangle$:

- set of states: total assignments of V according to dom
- actions: actions A defined as in Π
- action costs: *cost* as defined in Π
- transitions: $s \xrightarrow{a} s'$ for states s, s' and action a iff
  - $pre(a)$ complies with s (precondition satisfied)
  - s' complies with $eff(a)$ for all variables mentioned in *eff*; complies with s for all other variables (effects are applied)
- initial state: $s_0 = I$
- goal states: $s \in S_\star$ for state s iff G complies with s

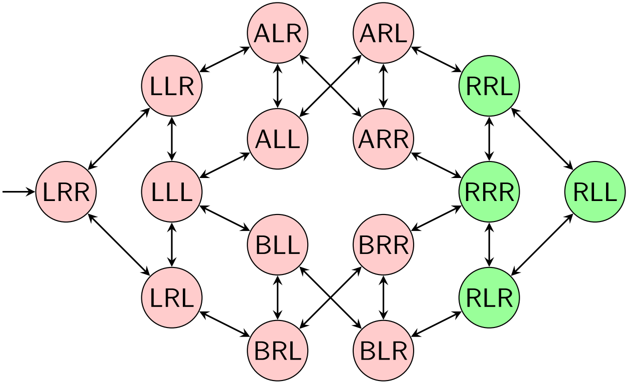
Example: Logistics Task with One Package, Two Trucks

Example (one package, two trucks)

Consider the SAS$^+$ planning task $\langle V, \mathrm{dom}, I, G, A\rangle$ with:

- $V = \{p, t_\mathrm{A}, t_\mathrm{B}\}$
- $\mathrm{dom}(p) = \{\mathrm{L}, \mathrm{R}, \mathrm{A}, \mathrm{B}\}$ and $\mathrm{dom}(t_\mathrm{A}) = \mathrm{dom}(t_\mathrm{B}) = \{\mathrm{L}, \mathrm{R}\}$
- $I = \{p \mapsto \mathrm{L}, t_\mathrm{A} \mapsto \mathrm{R}, t_\mathrm{B} \mapsto \mathrm{R}\}$
- $G = \{p \mapsto \mathrm{R}\}$
- $A = \{\mathit{load}_{i,j} \mid i \in \{\mathrm{A}, \mathrm{B}\}, j \in \{\mathrm{L}, \mathrm{R}\}\}$
  $\cup \{\mathit{unload}_{i,j} \mid i \in \{\mathrm{A}, \mathrm{B}\}, j \in \{\mathrm{L}, \mathrm{R}\}\}$
  $\cup \{\mathit{move}_{i,j,j'} \mid i \in \{\mathrm{A}, \mathrm{B}\}, j, j' \in \{\mathrm{L}, \mathrm{R}\}, j \neq j'\}$ with:
  - $\mathit{load}_{i,j}$ has preconditions $\{t_i \mapsto j, p \mapsto j\}$, effects $\{p \mapsto i\}$
  - $\mathit{unload}_{i,j}$ has preconditions $\{t_i \mapsto j, p \mapsto i\}$, effects $\{p \mapsto j\}$
  - $\mathit{move}_{i,j,j'}$ has preconditions $\{t_i \mapsto j\}$, effects $\{t_i \mapsto j'\}$
  - All actions have cost 1.

State Space for Example Task



- state $\{p \mapsto i, t_A \mapsto j, t_B \mapsto k\}$ denoted as ijk
- annotations of edges not shown for simplicity
- for example, edge from LLL to ALL has annotation $load_{A,L}$

Abstractions

State Space Abstraction

State space abstractions drop distinctions between certain states, but preserve the state space behavior as well as possible.

- An abstraction of a state space $\mathcal{S}$ is defined by an abstraction function α that determines which states can be distinguished in the abstraction.
- Based on $\mathcal{S}$ and α, we compute the abstract state space $\mathcal{S}^\alpha$ which is “similar” to $\mathcal{S}$ but smaller.
- Main idea: Use the cheapest cost in $\mathcal{S}^\alpha$ as a heuristic.

Induced Abstraction

Definition (induced abstraction)

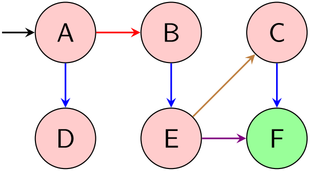
Let $\mathcal{S} = \langle S, A, cost, T, s_0, S_\star \rangle$ be a state space, and let $\alpha : S \to S'$ be a surjective function.

The abstraction of $\mathcal{S}$ induced by α, denoted as $\mathcal{S}^\alpha$, is the state space $\mathcal{S}^\alpha = \langle S', A, cost, T', s'_0, S'_\star \rangle$ with:

- $T' = \{\langle \alpha(s), a, \alpha(t) \rangle \mid \langle s, a, t \rangle \in T\}$
- $s'_0 = \alpha(s_0)$
- $S'_\star = \{\alpha(s) \mid s \in S_\star\}$

Abstraction: Example

concrete state space with
states $S = \{A, B, C, D, E, F\}$



abstract state space with
states $S^\alpha = \{W, X, Y, Z\}$

abstraction function $\alpha : S \to S^\alpha$

$$\alpha(A) = W \quad \alpha(B) = X \quad \alpha(C) = Y$$
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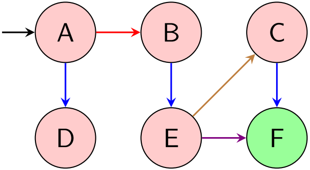

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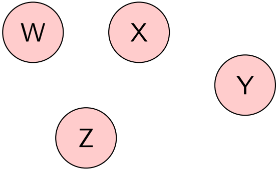

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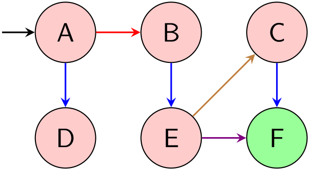

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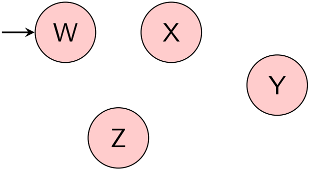

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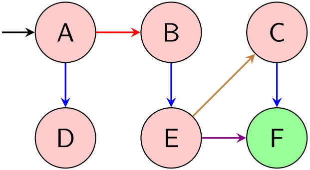

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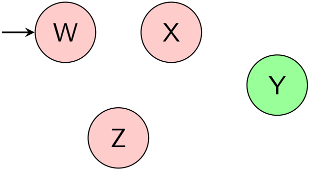

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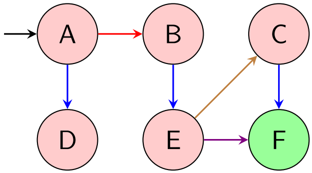

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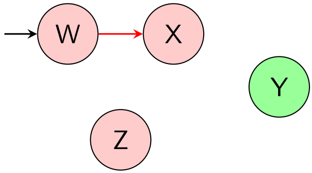

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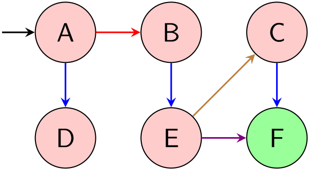

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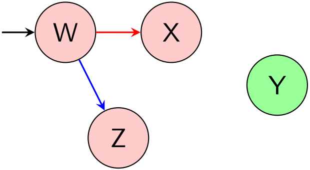

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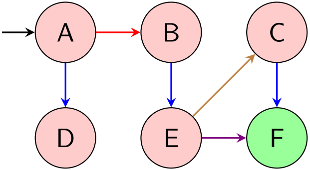

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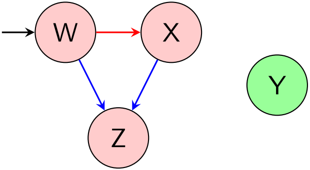

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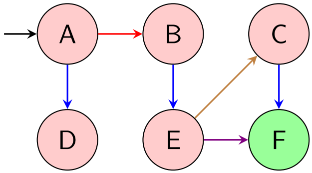

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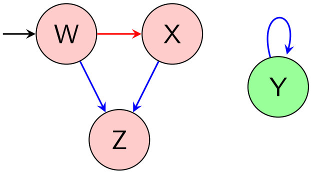

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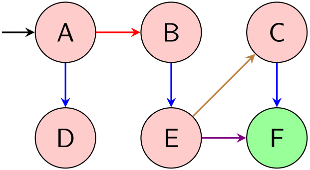

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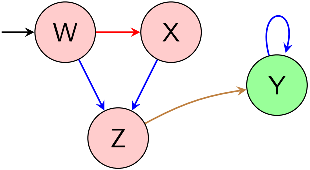

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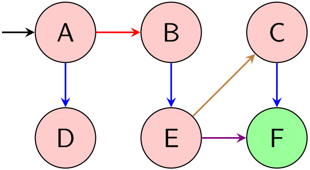

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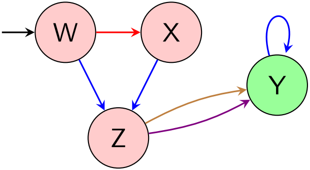

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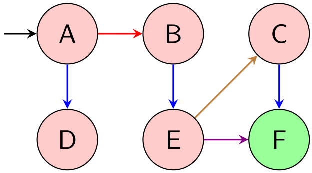

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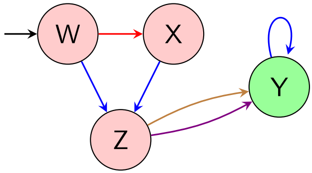

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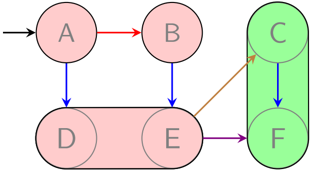
intuition: grouping states



Abstraction Heuristic

Use abstract solution cost (solution cost of $\alpha(s)$ in $\mathcal{S}^\alpha$) as heuristic for concrete solution cost (solution cost of s in $\mathcal{S}$).

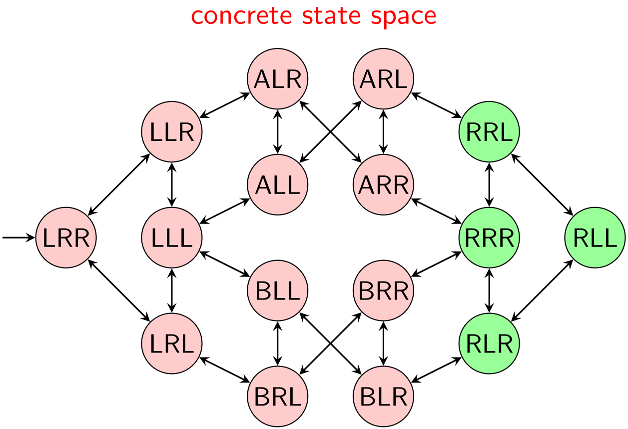
Definition (abstraction heuristic)

The abstraction heuristic for abstraction α maps each state s to its abstract solution costs

$$h^\alpha(s) := h^*_{\mathcal{S}^\alpha}(\alpha(s))$$

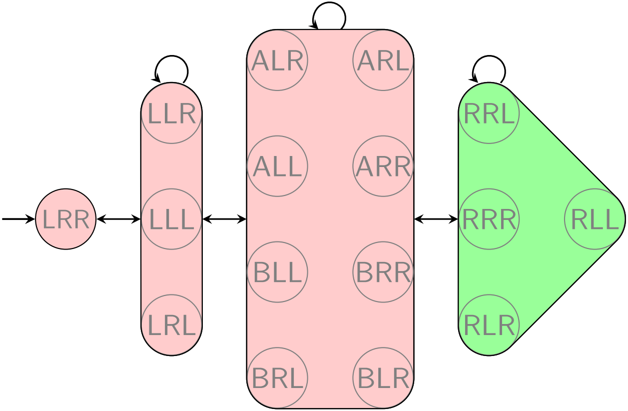
where $h^*_{\mathcal{S}^\alpha}$ is the perfect heuristic in $\mathcal{S}^\alpha$.

Abstraction: Example



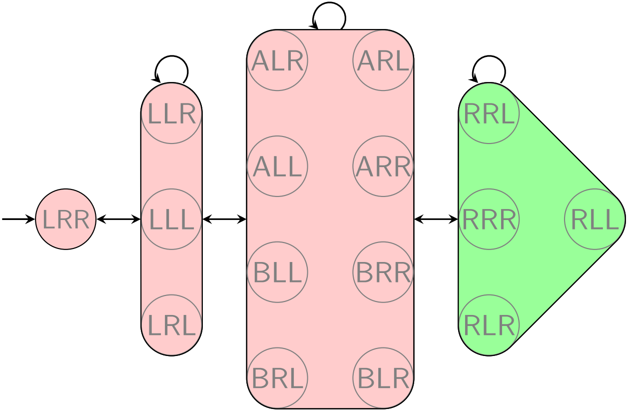
Abstraction: Example

(an) abstract state space



remark: Most edges correspond to several (parallel) transitions with different annotations.

Abstraction Heuristic: Example

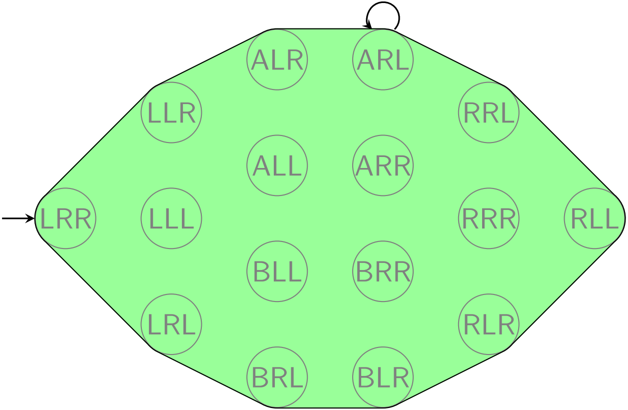


$$h^\alpha(\{p \mapsto \mathsf{L}, t_{\mathsf{A}} \mapsto \mathsf{R}, t_{\mathsf{B}} \mapsto \mathsf{R}\}) = 3$$

Abstraction Heuristics: Discussion

- Every abstraction heuristic is admissible and consistent. (proof idea?)
- The choice of the abstraction function α is very important.
  - Every α yields an admissible and consistent heuristic.
  - But most α lead to poor heuristics.
- An effective α must yield an informative heuristic ...
- ... as well as being efficiently computable.
- How to find a suitable α?

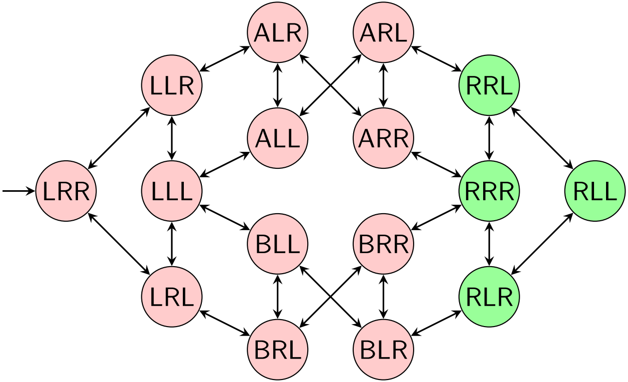
Usually a Bad Idea: Single-State Abstraction



one state abstraction: $\alpha(s) :=$ const

- $+$ compactly representable and α easy to compute
- $-$ very uninformed heuristic

Usually a Bad Idea: Identity Abstraction



identity abstraction: $\alpha(s) := s$

- $+$ perfect heuristic and α easy to compute
- $-$ too many abstract states $\rightsquigarrow$ computation of h^α too hard

Automatic Computation of Suitable Abstractions

Main Problem with Abstraction Heuristics

How to find a good abstraction?

Several successful methods:

- pattern databases (PDBs) $\rightsquigarrow$ this course
  (Culberson & Schaeffer, 1996)
- merge-and-shrink abstractions
  (Dräger, Finkbeiner & Podelski, 2006)
- Cartesian abstractions
  (Seipp & Helmert, 2013)

Pattern Databases

Pattern Databases: Background

- The most common abstraction heuristics are pattern database heuristics.
- originally introduced for the 15-puzzle (Culberson & Schaeffer, 1996) and for Rubik's Cube (Korf, 1997)
- introduced for automated planning by Edelkamp (2001)
- for many search problems the best known heuristics
- many many research papers studying
  - theoretical properties
  - efficient implementation and application
  - pattern selection
  - ...

Pattern Databases: Projections

A PDB heuristic for a planning task is an abstraction heuristic where

- some aspects (= state variables) of the task are preserved with perfect precision while
- all other aspects are not preserved at all.

formalized as projections to a pattern $P \subseteq V$:

$$\pi_P(s) := \{v \mapsto s(v) \mid v \in P\}$$

example:

- $s = \{p \mapsto L, t_A \mapsto R, t_B \mapsto R\}$
- projection on $P = \{p\}$ (= ignore trucks):
  $\pi_P(s) = \{p \mapsto L\}$
- projection on $P = \{p, t_A\}$ (= ignore truck B):
  $\pi_P(s) = \{p \mapsto L, t_A \mapsto R\}$

Pattern Databases: Definition

Definition (pattern database heuristic)

Let P be a subset of the variables of a planning task.

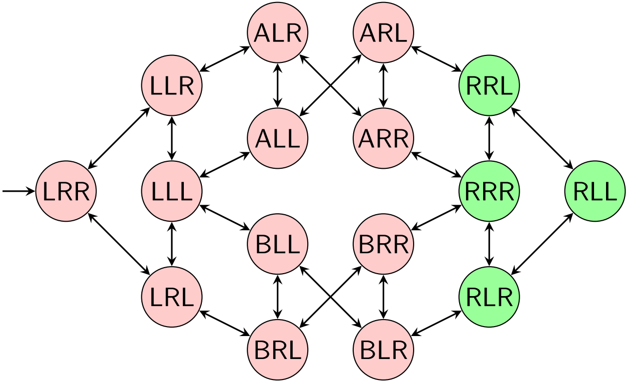
The abstraction heuristic induced by the projection π_P on P is called pattern database heuristic (PDB heuristic) with pattern P.

abbreviated notation: h^P for h^{π_P}

remark:

- "pattern databases" in analogy to endgame databases (which have been successfully applied in 2-person-games)

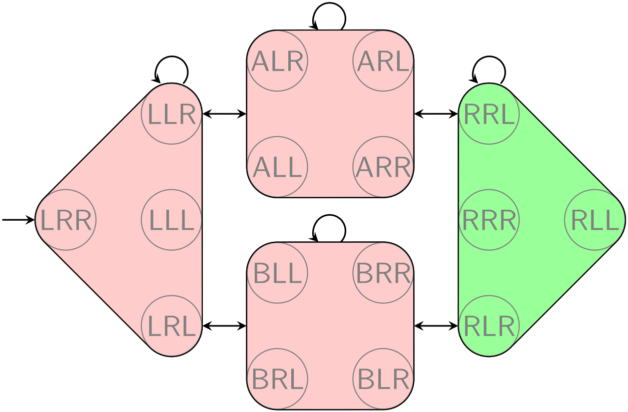
Example: Concrete State Space



- state variable *package*: $\{L, R, A, B\}$
- state variable *truck A*: $\{L, R\}$
- state variable *truck B*: $\{L, R\}$

Example: Projection (1)

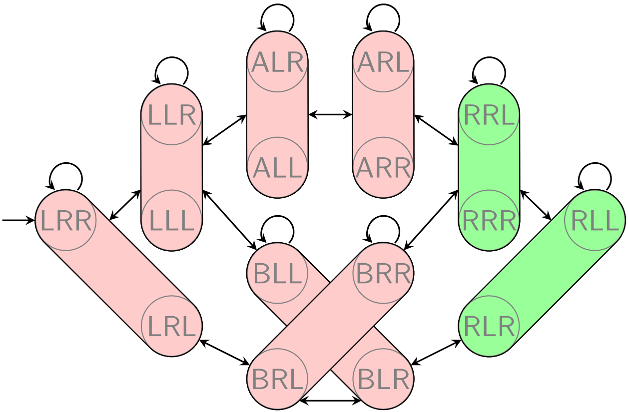
abstraction induced by $\pi_{\{package\}}$:



$$h^{\{package\}}(\text{LRR}) = 2$$

Example: Projection (2)

abstraction induced by $\pi_{\{package, truck\ A\}}$:



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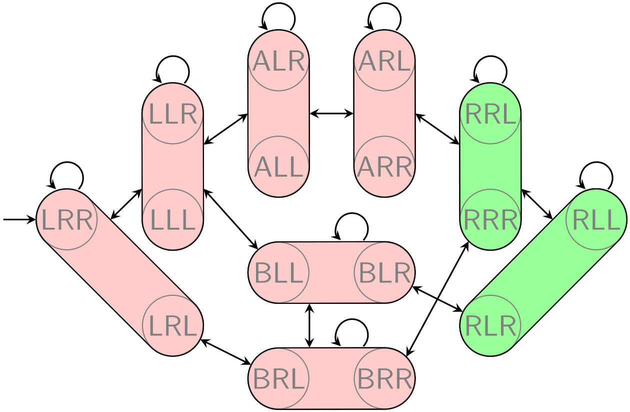

$$h^{\{package, truck\ A\}}(\mathsf{LRR}) = 2$$

Pattern Databases in Practice

practical aspects which we do not discuss in detail:

- How to automatically find good patterns?
- How to combine multiple PDB heuristics?
- How to implement PDB heuristics efficiently?
  - good implementations efficiently handle abstract state spaces with 10^7, 10^8 or more abstract states
  - effort independent of the size of the concrete state space
  - usually all heuristic values are precomputed $\rightsquigarrow$ space complexity $=$ number of abstract states

Summary

Summary

- basic idea of abstraction heuristics: estimate solution cost by considering a smaller planning task.
- formally: abstraction function α maps states to abstract states and thus defines which states can be distinguished by the resulting heuristic.
- induces abstract state space whose solution costs are used as heuristic
- Pattern database heuristics are abstraction heuristics based on projections onto state variable subsets (patterns): states are distinguishable iff they differ on the pattern.