

Foundations of Artificial Intelligence

36. Automated Planning: Delete Relaxation Heuristics

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Automated Planning: Overview

Chapter overview: automated planning

- 33. Introduction
- 34. Planning Formalisms
- 35.–36. Planning Heuristics: Delete Relaxation
 - 35. Delete Relaxation
 - 36. Delete Relaxation Heuristics
- 37. Planning Heuristics: Abstraction
- 38.–39. Planning Heuristics: Landmarks

Relaxed Planning Graphs

Relaxed Planning Graphs

- **relaxed planning graphs**: represent **which** variables in Π^+ can be reached and **how**
- graphs with **variable layers** V^i and **action layers** A^i
 - variable layer V^0 contains the **variable vertex** v^0 for all $v \in I$
 - action layer A^{i+1} contains the **action vertex** a^{i+1} for action a if V^i contains the vertex v^i for all $v \in pre(a)$
 - variable layer V^{i+1} contains the variable vertex v^{i+1} if previous variable layer contains v^i , or previous action layer contains a^{i+1} with $v \in add(a)$

Relaxed Planning Graphs (Continued)

- **goal vertices** G^i if $v^i \in V^i$ for all $v \in G$
- graph can be constructed for arbitrary many layers but stabilizes after a bounded number of layers
 $\rightsquigarrow V^{i+1} = V^i$ and $A^{i+1} = A^i$ (Why?)
- directed edges:
 - from v^i to a^{i+1} if $v \in pre(a)$ (**precondition edges**)
 - from a^i to v^i if $v \in add(a)$ (**effect edges**)
 - from v^i to G^i if $v \in G$ (**goal edges**)
 - from v^i to v^{i+1} (**no-op edges**)

Illustrative Example

We will write actions a with $pre(a) = \{p_1, \dots, p_k\}$,
 $add(a) = \{q_1, \dots, q_l\}$, $del(a) = \emptyset$ and $cost(a) = c$
as $p_1, \dots, p_k \xrightarrow{c} q_1, \dots, q_l$

$$V = \{m, n, o, p, q, r, s, t\}$$

$$I = \{m\}$$

$$G = \{o, p, q, r, s\}$$

$$A = \{a_1, a_2, a_3, a_4, a_5, a_6\}$$

$$a_1 = m \xrightarrow{3} n, o$$

$$a_2 = m, o \xrightarrow{1} p$$

$$a_3 = n, o \xrightarrow{1} q$$

$$a_4 = n \xrightarrow{1} r$$

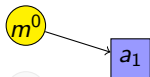
$$a_5 = p \xrightarrow{1} q, r$$

$$a_6 = p \xrightarrow{1} s$$

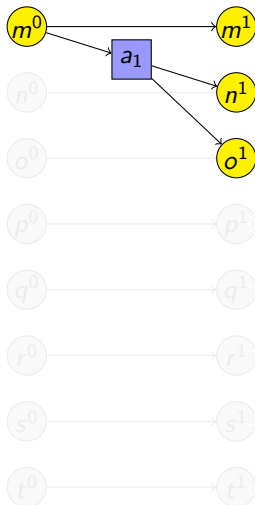
Illustrative Example: Relaxed Planning Graph

 m^0 n^0 o^0 p^0 q^0 r^0 s^0 t^0

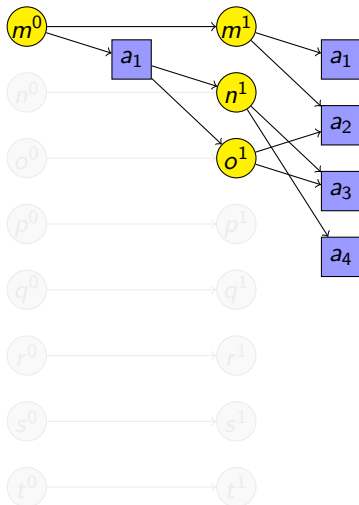
Illustrative Example: Relaxed Planning Graph

 n^0 o^0 p^0 q^0 r^0 s^0 t^0

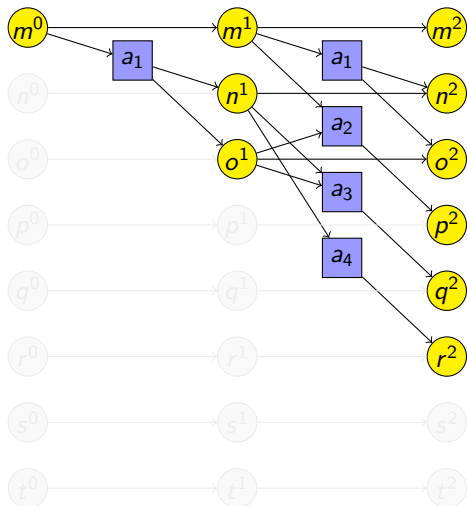
Illustrative Example: Relaxed Planning Graph



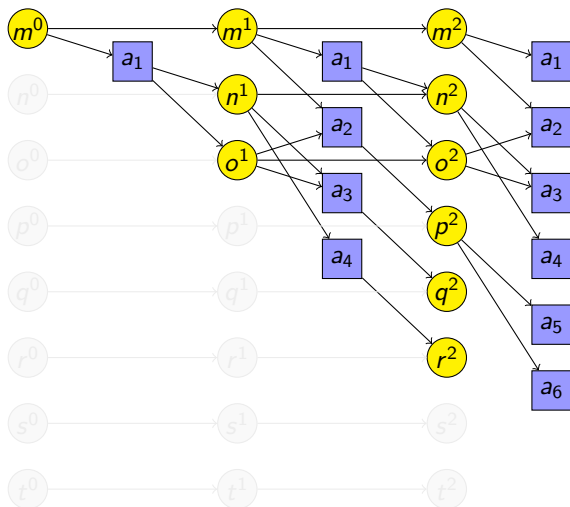
Illustrative Example: Relaxed Planning Graph



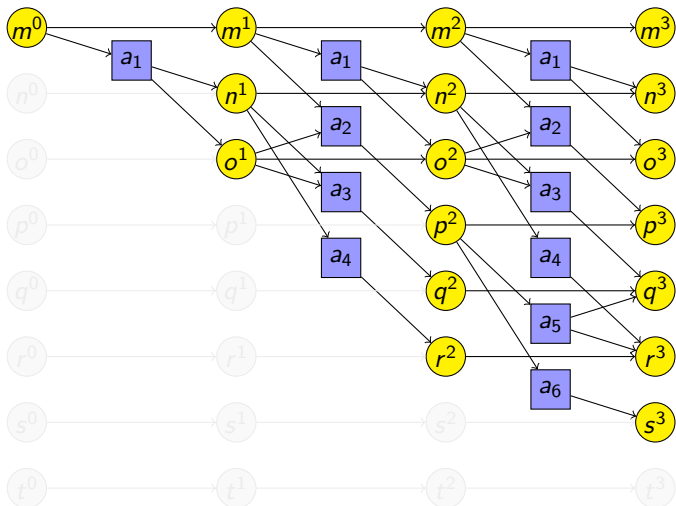
Illustrative Example: Relaxed Planning Graph



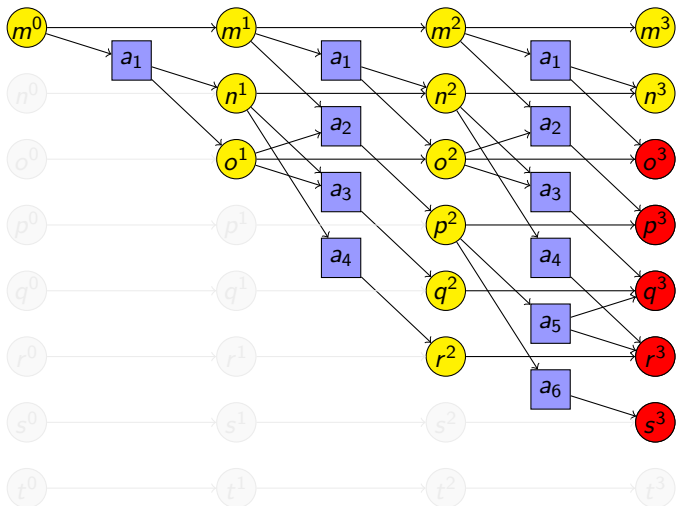
Illustrative Example: Relaxed Planning Graph



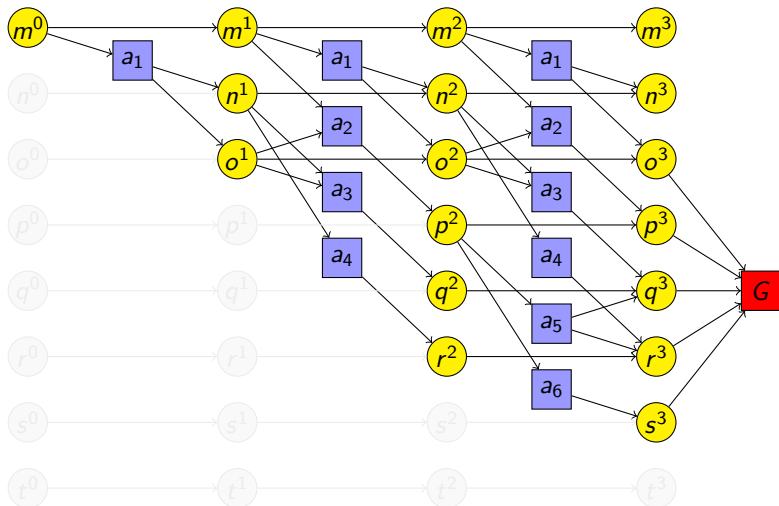
Illustrative Example: Relaxed Planning Graph



Illustrative Example: Relaxed Planning Graph



Illustrative Example: Relaxed Planning Graph



Generic Relaxed Planning Graph Heuristic

Heuristic Values from Relaxed Planning Graph

```
function generic-rpg-heuristic( $\langle V, I, G, A \rangle, s$ ):  
     $\Pi^+ := \langle V, s, G, A^+ \rangle$   
    for  $k \in \{0, 1, 2, \dots\}$ :  
         $rpg := RPG_k(\Pi^+)$     [relaxed planning graph to layer  $k$ ]  
        if  $rpg$  contains a goal node:  
            Annotate nodes of  $rpg$ .  
            if termination criterion is true:  
                return heuristic value from annotations  
        else if graph has stabilized:  
            return  $\infty$ 
```

~> **general template** for RPG heuristics

~> to obtain concrete heuristic: instantiate **highlighted elements**

Concrete Examples for Generic RPG Heuristic

Many planning heuristics fit this general template.

In this course:

- maximum heuristic $h^{\max}$ (Bonet & Geffner, 1999)
- additive heuristic h^{add} (Bonet, Loerincs & Geffner, 1997)
- Keyder & Geffner's (2008) variant of the FF heuristic h^{FF} (Hoffmann & Nebel, 2001)

remark:

- The most efficient implementations of these heuristics do not use explicit planning graphs, but rather alternative (equivalent) definitions.

Maximum and Additive Heuristics

Maximum and Additive Heuristics

- $h^{\max}$ and h^{add} are the simplest RPG heuristics.
- Vertex annotations are **numerical values**.
- The vertex values estimate the costs
 - to make a given variable true
 - to reach and apply a given action
 - to reach the goal

Maximum and Additive Heuristics: Filled-in Template

$h^{\max}$ and h^{add}

computation of annotations:

- costs of variable vertices:
0 in layer 0;
otherwise **minimum** of the costs of predecessor vertices
- costs of action and goal vertices:
maximum ($h^{\max}$) or **sum** (h^{add}) of predecessor vertex costs;
for action vertices a^i , also add $\text{cost}(a)$

termination criterion:

- **stability**: terminate if $V^i = V^{i-1}$ and costs of all vertices in V^i equal corresponding vertex costs in V^{i-1}

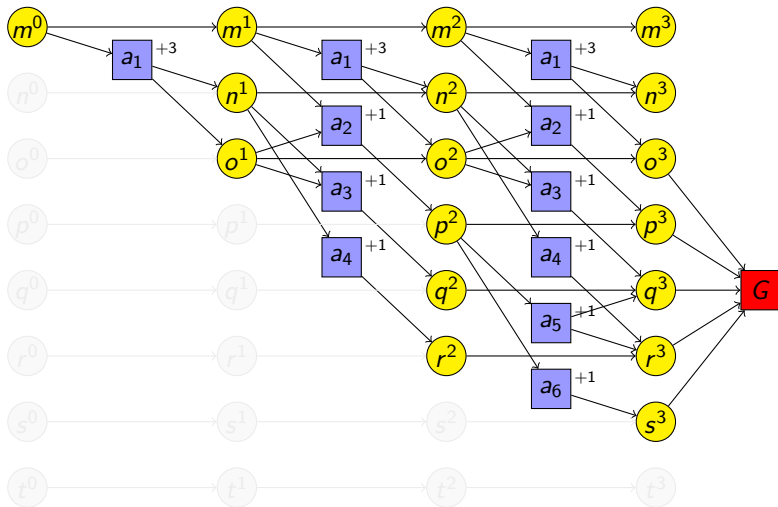
heuristic value:

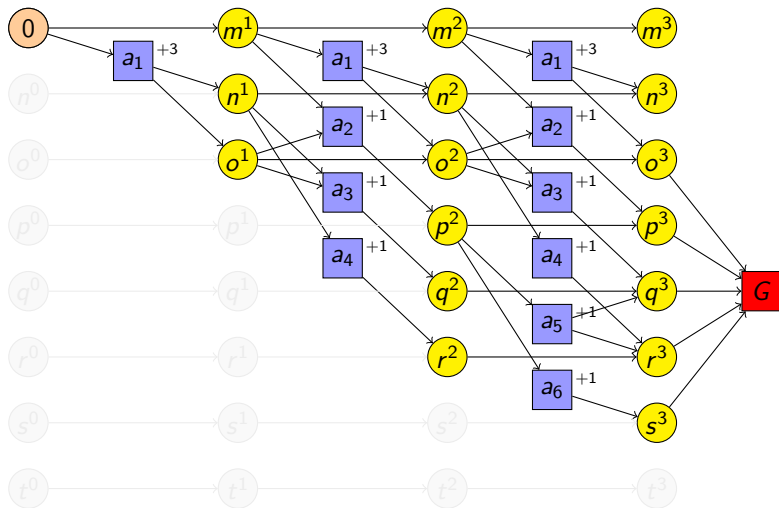
- value of goal vertex in the last layer

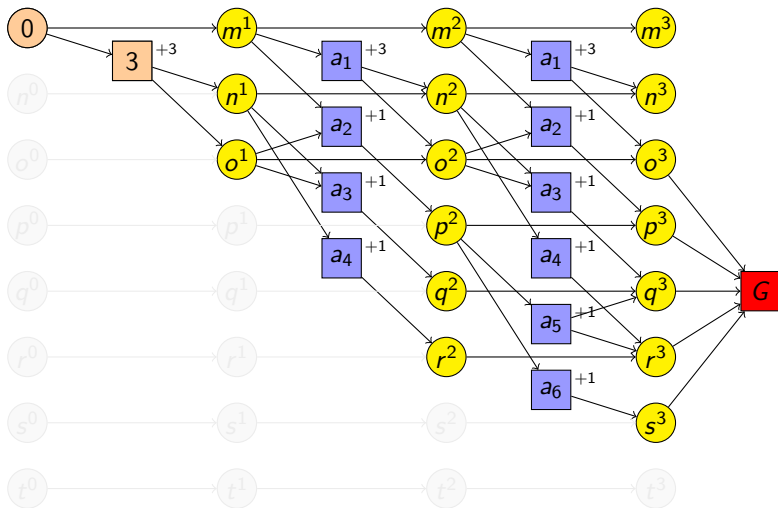
Maximum and Additive Heuristics: Intuition

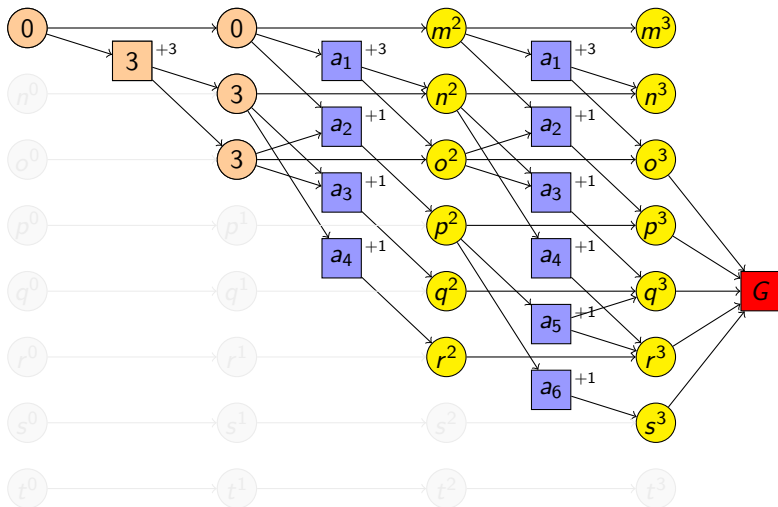
intuition:

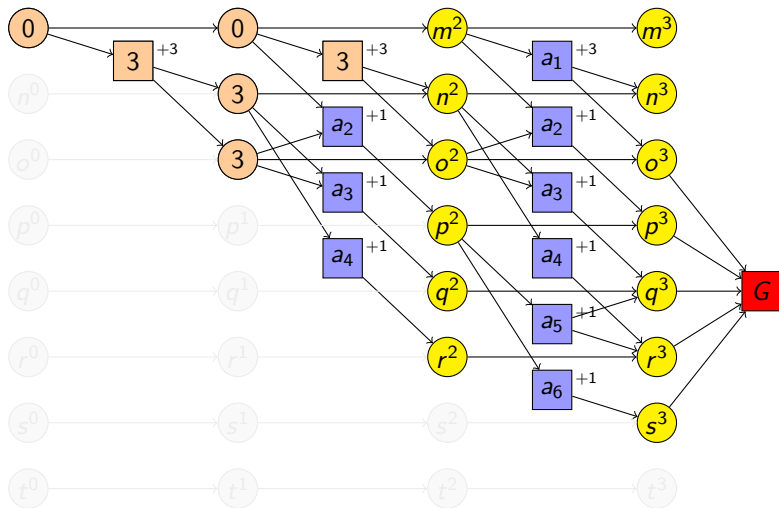
- variable vertices:
 - choose **cheapest** way of reaching the variable
- action/goal vertices:
 - $h^{\max}$ is **optimistic**: assumption:
when reaching the **most expensive** precondition variable,
we can reach the other precondition variables in parallel
(hence maximization of costs)
 - h^{add} is **pessimistic**: assumption:
all precondition variables must be reached completely
independently of each other (hence summation of costs)

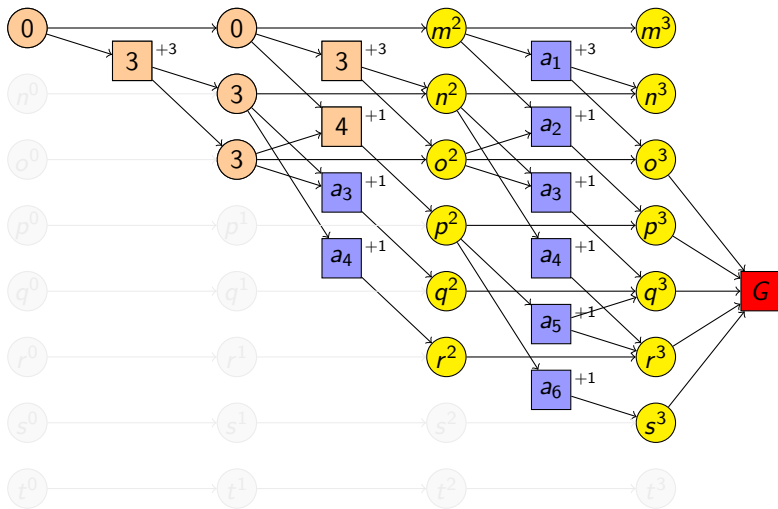
Illustrative Example: $h^{\max}$ 

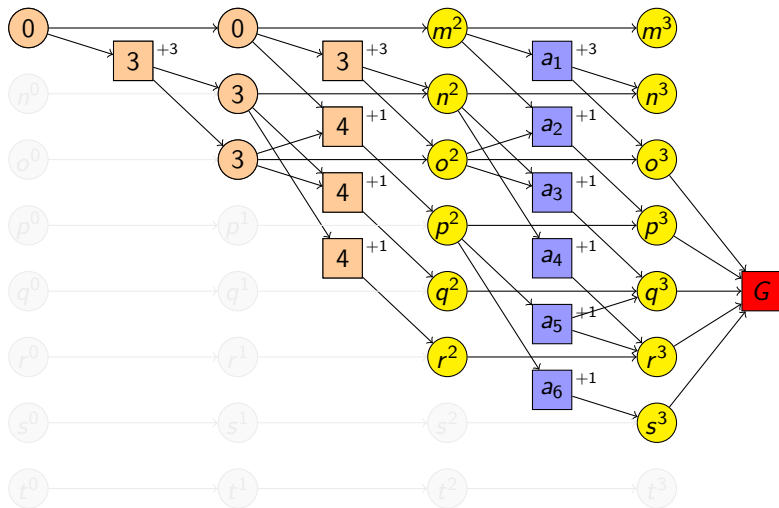
Illustrative Example: $h^{\max}$ 

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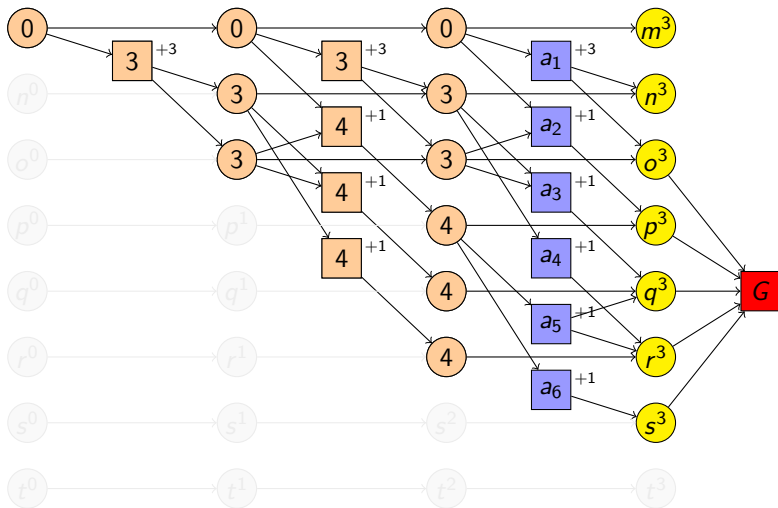
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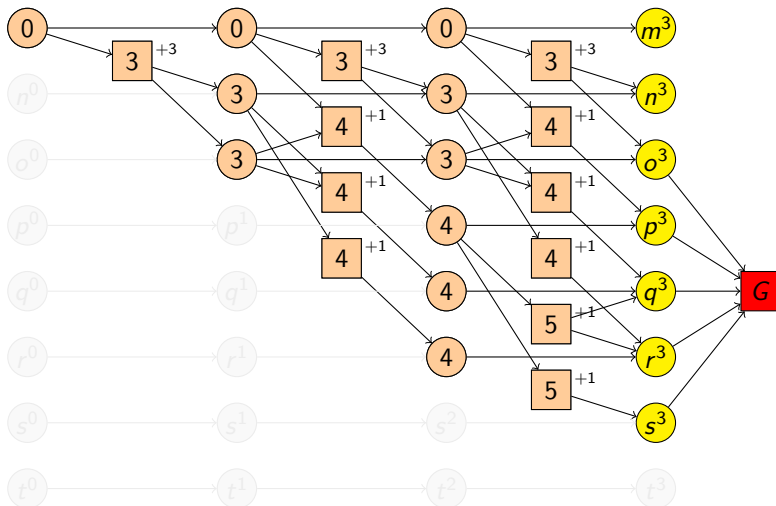
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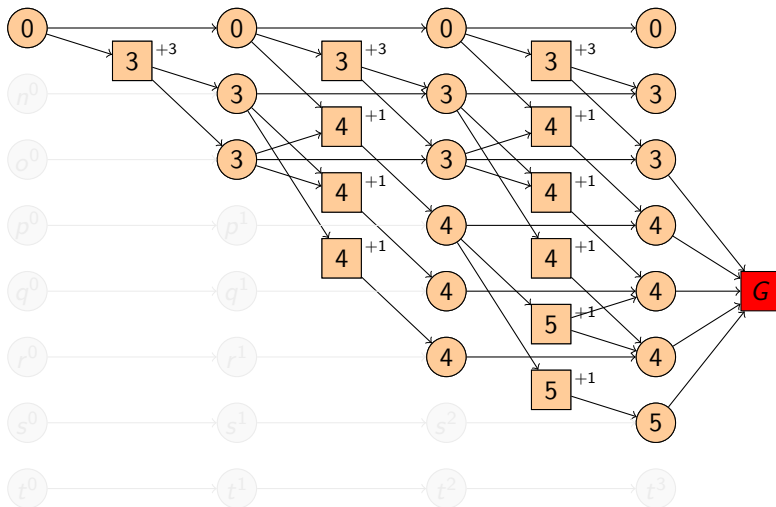
Illustrative Example: $h^{\max}$ 

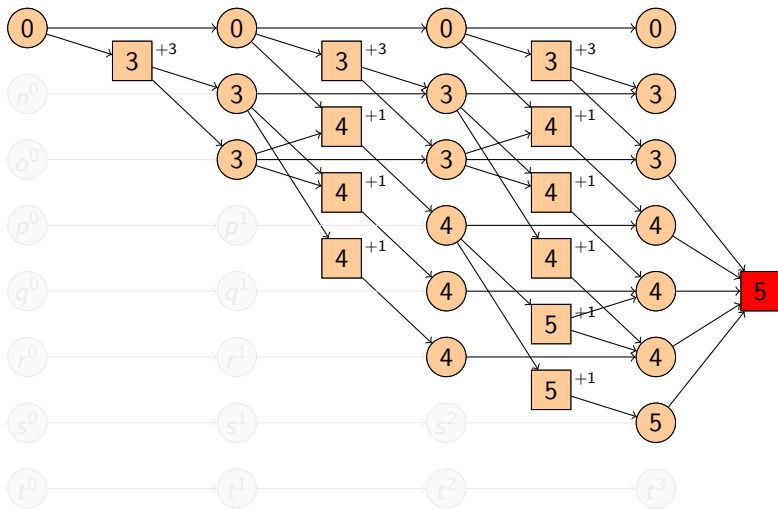
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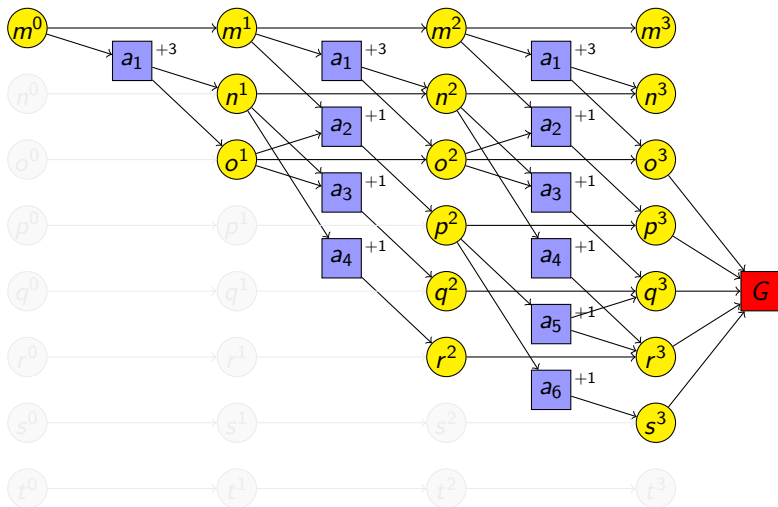


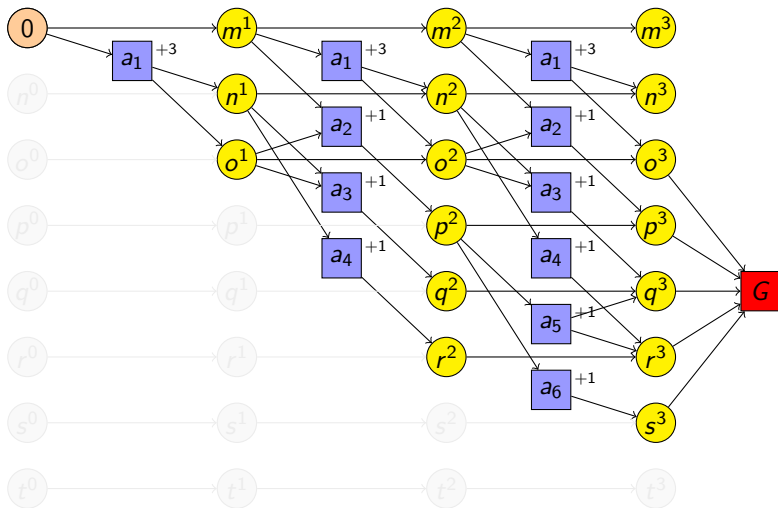
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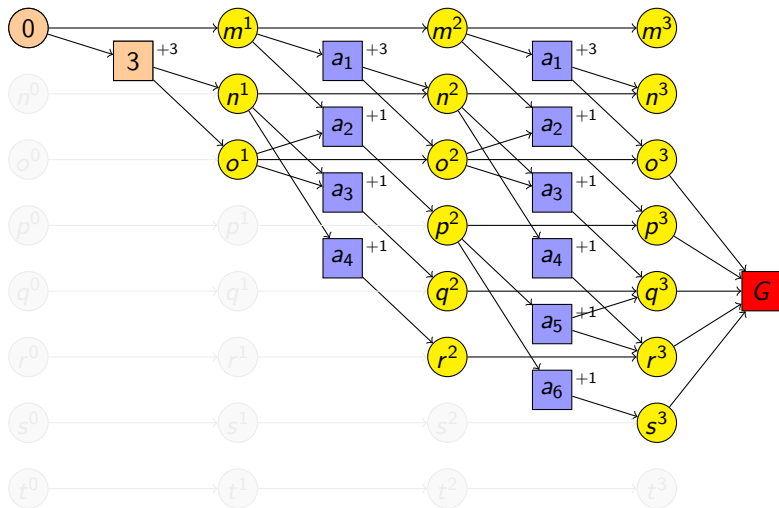
Illustrative Example: $h^{\max}$ 

$$h^{\max}(\{m\}) = 5$$

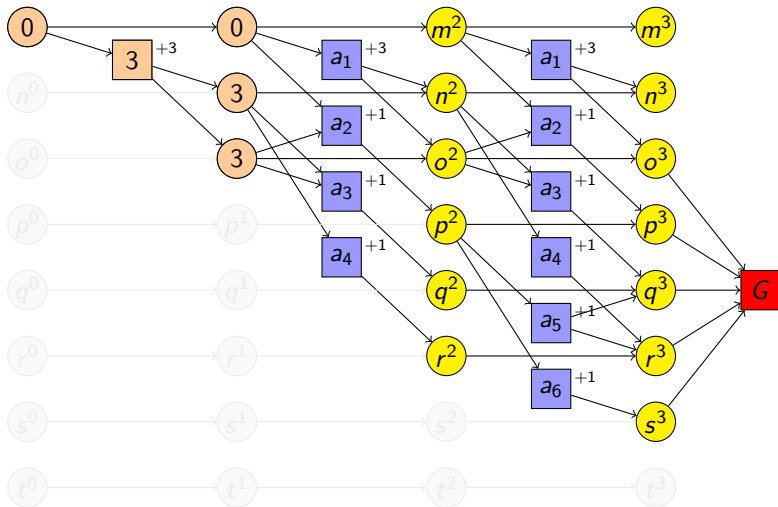
Illustrative Example: h^{add} 

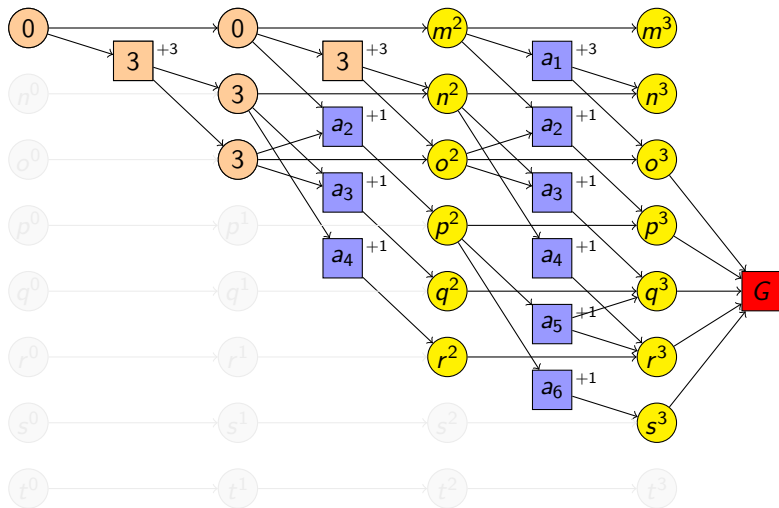
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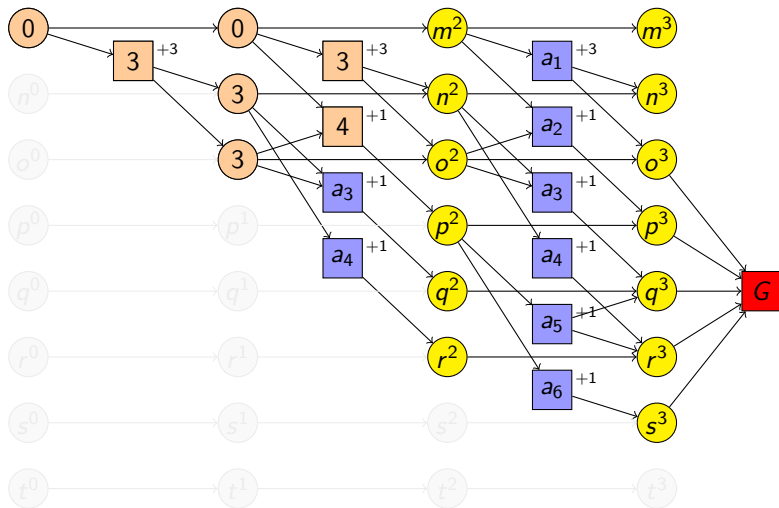
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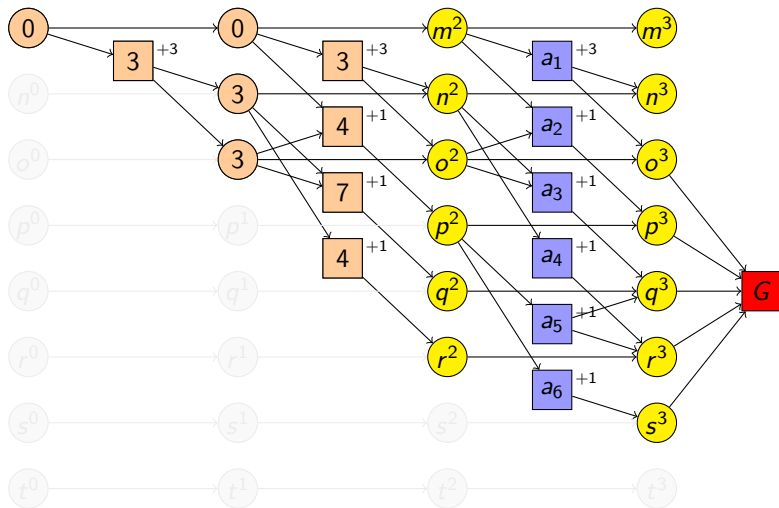


Illustrative Example: h^{add}

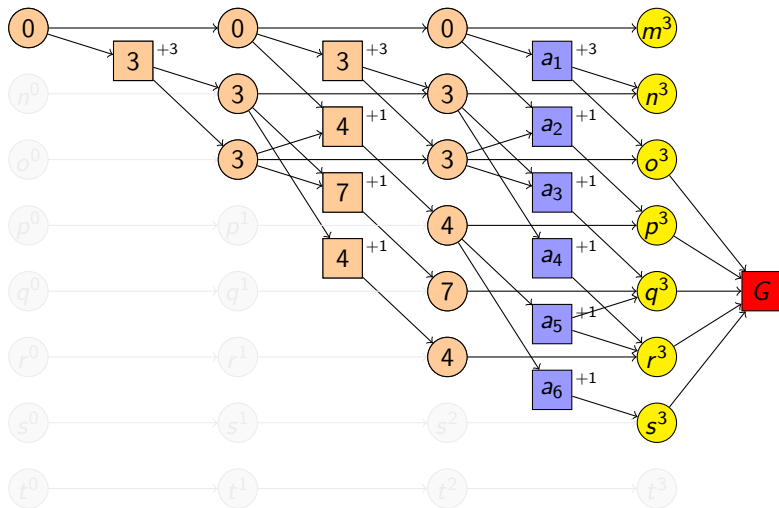


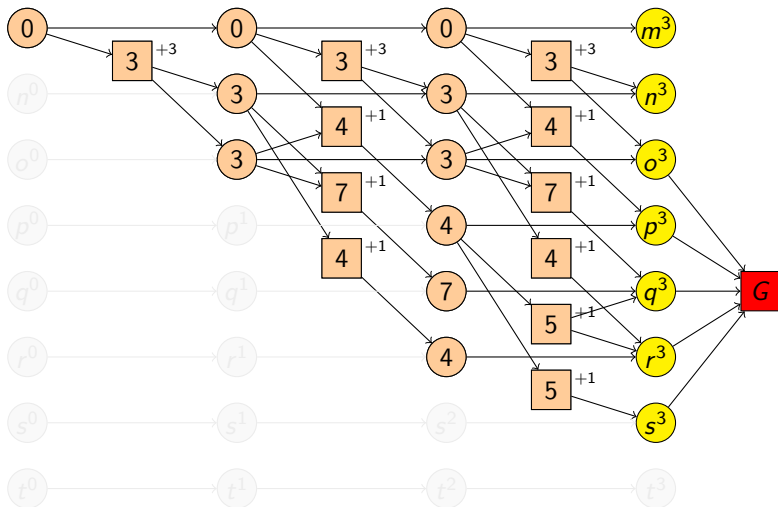
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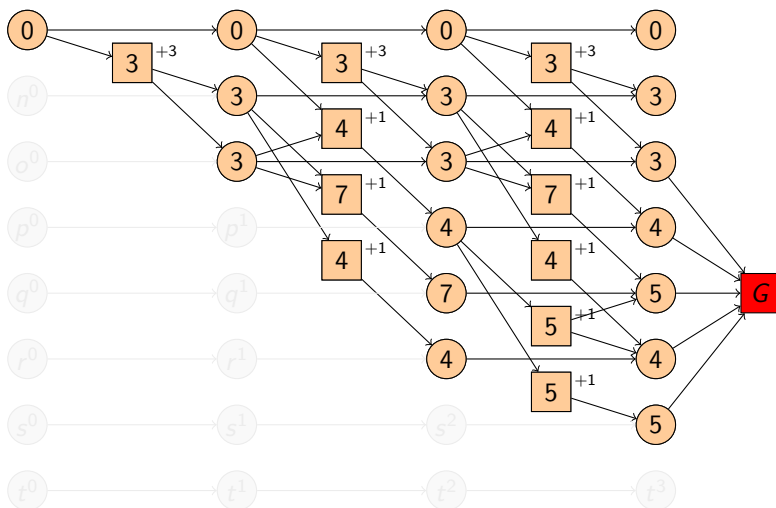
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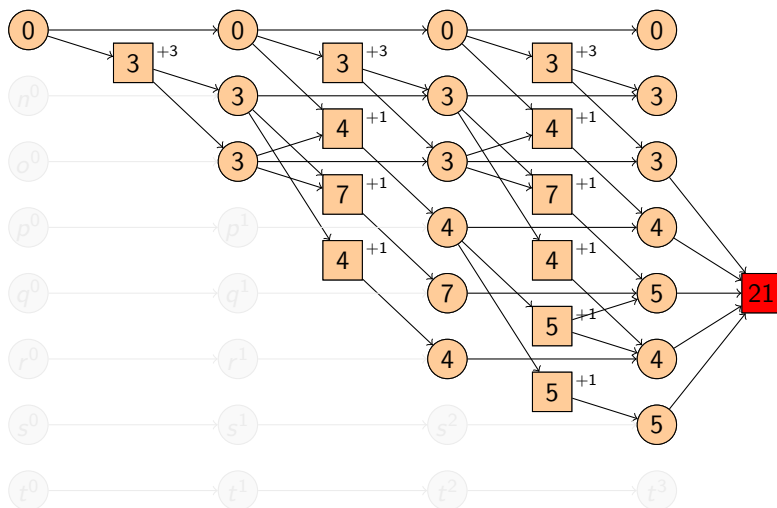
Illustrative Example: h^{add} 

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$$h^{\text{add}}(\{m\}) = 21$$

$h^{\max}$ and h^{add} : Remarks

comparison of $h^{\max}$ and h^{add} :

- both are safe and goal-aware
- $h^{\max}$ is admissible and consistent; h^{add} is neither.

⇒ h^{add} not suited for **optimal** planning

$h^{\max}$ and h^{add} : Remarks

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- $h^{\max}$ is admissible and consistent; h^{add} is neither.
- ⇒ h^{add} not suited for **optimal** planning
- However, h^{add} is usually **much more informative** than $h^{\max}$.
Greedy best-first search with h^{add} is a decent algorithm.

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- However, h^{add} is usually **much more informative** than $h^{\max}$. Greedy best-first search with h^{add} is a decent algorithm.
- Apart from not being admissible, h^{add} often **vastly** overestimates the actual costs because **positive synergies** between subgoals are not recognized.

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~> FF heuristic

FF Heuristic

FF Heuristic

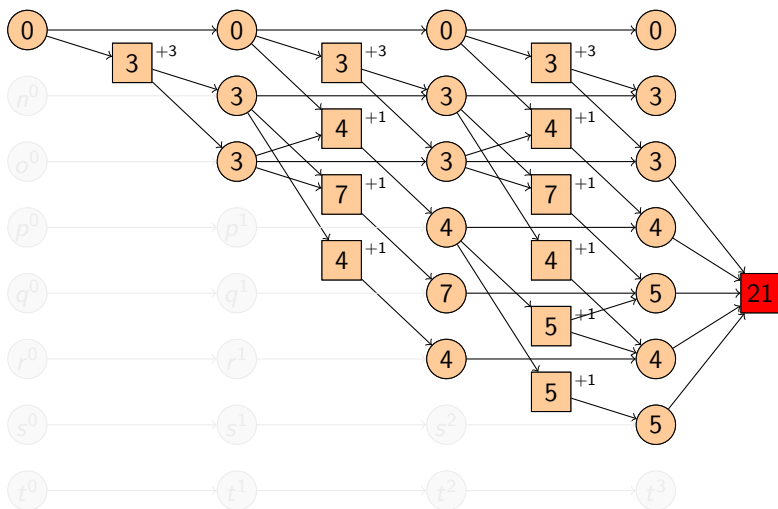
The FF Heuristic

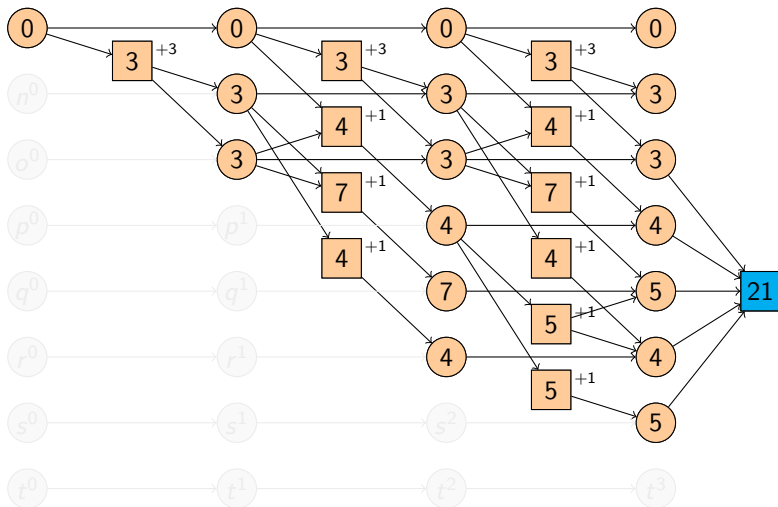
identical to h^{add} , but **additional steps** at the end:

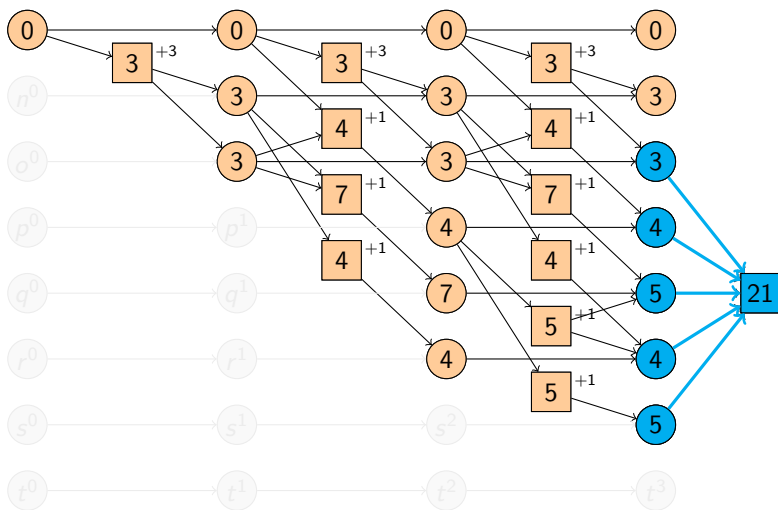
- **Mark** goal vertex in the last graph layer.
- Apply the following **marking rules** until nothing more to do:
 - marked action or goal vertex?
 $\rightsquigarrow$ mark **all** predecessors
 - marked variable vertex v^i in layer $i \geq 1$?
 $\rightsquigarrow$ mark **one** predecessor with **minimal** h^{add} value
 (tie-breaking: prefer variable vertices; otherwise arbitrary)

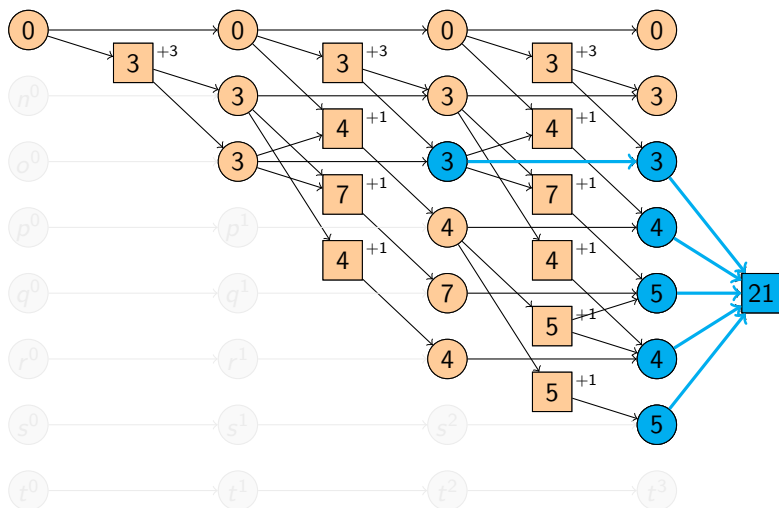
heuristic value:

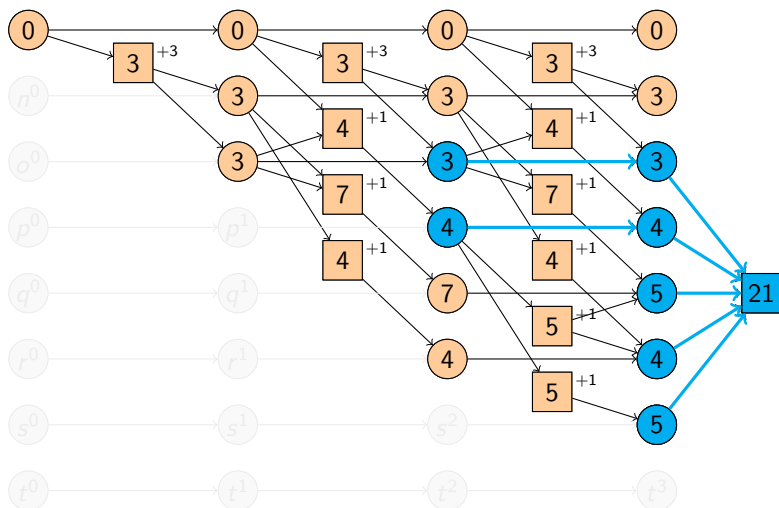
- The actions corresponding to the marked action vertices build a relaxed plan.
- The **cost of this plan** is the heuristic value.

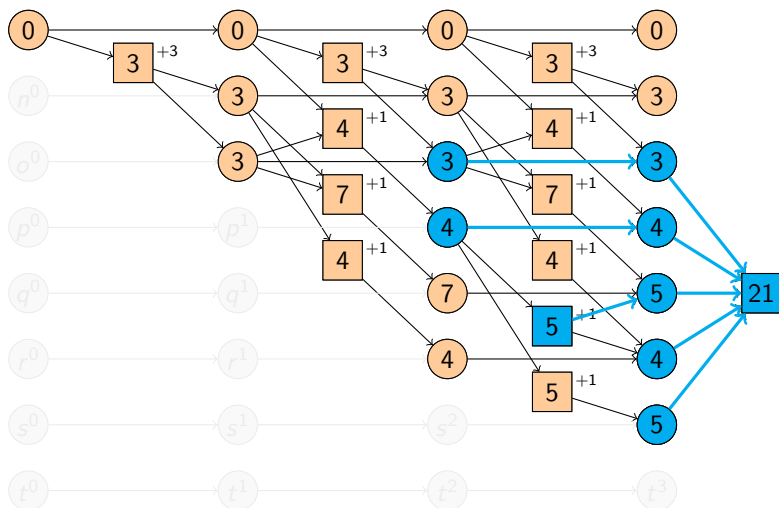
Illustrative Example: h^{FF} 

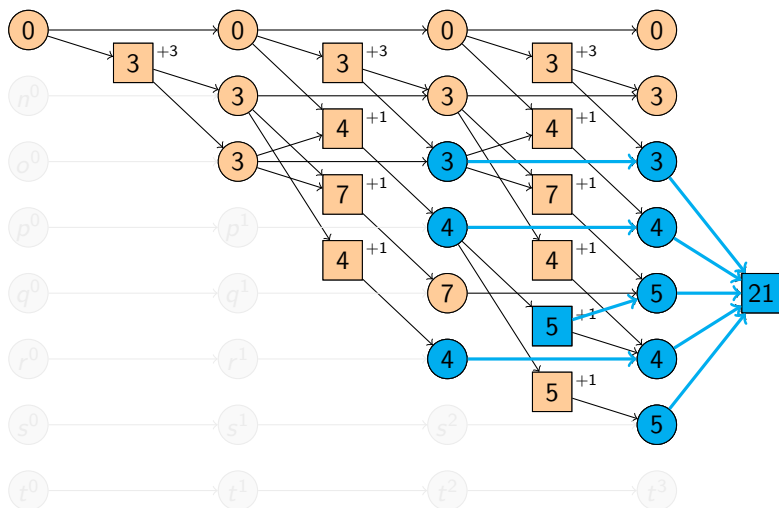
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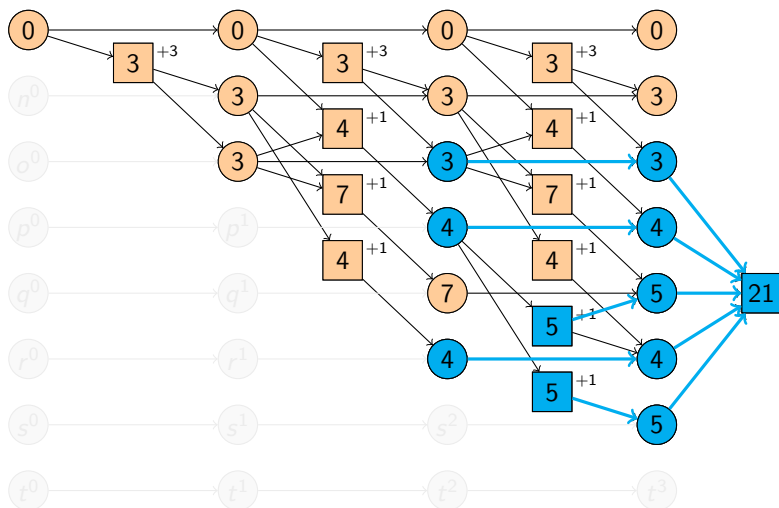
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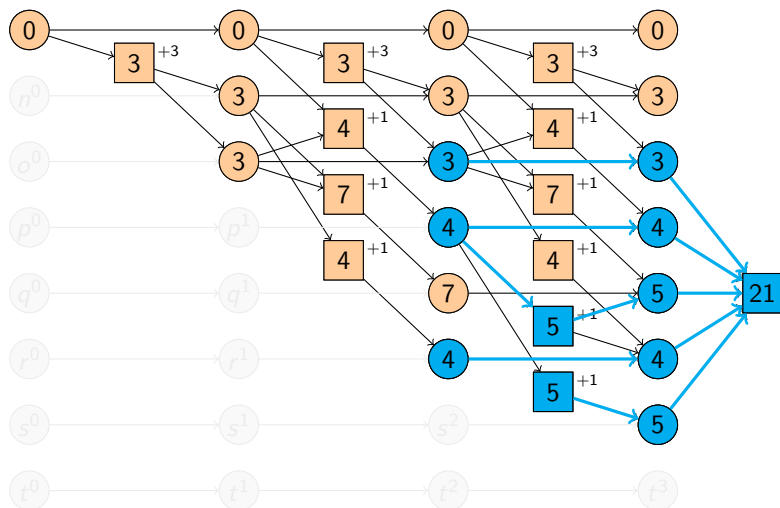
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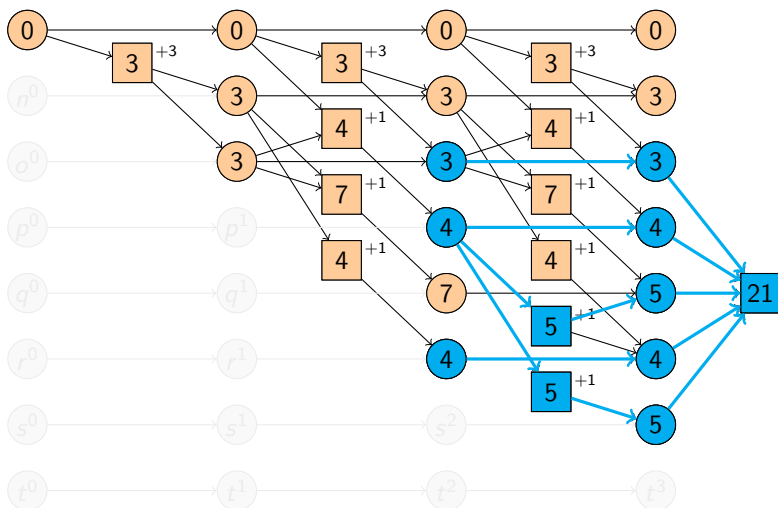
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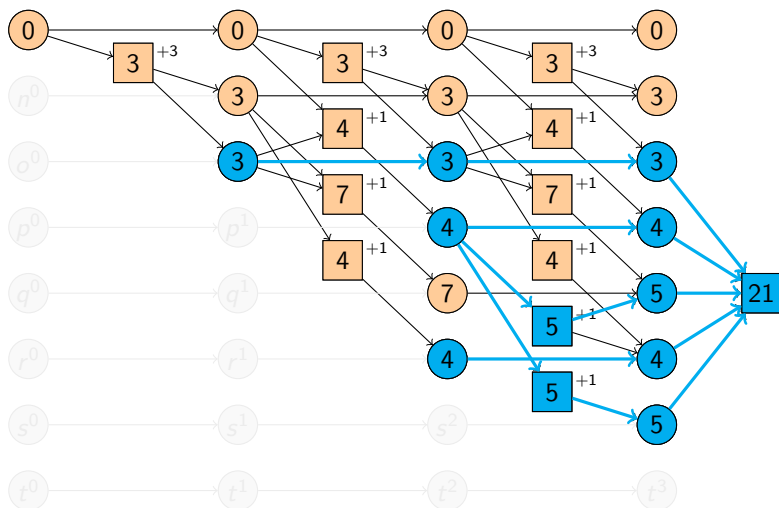
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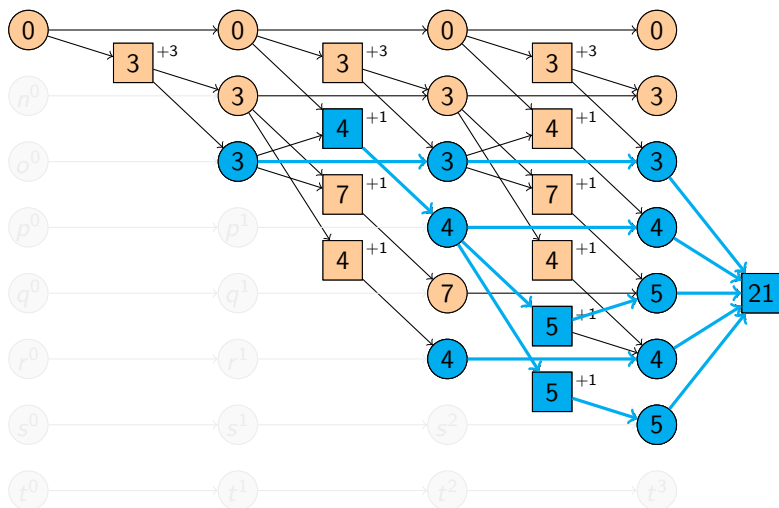
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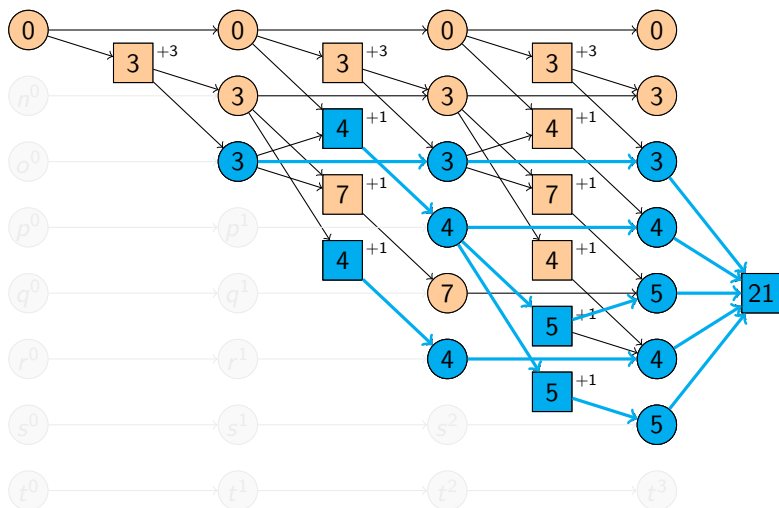


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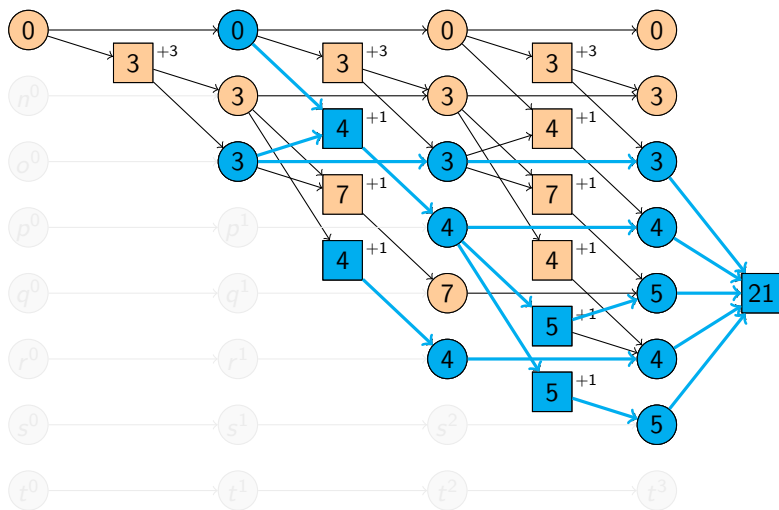
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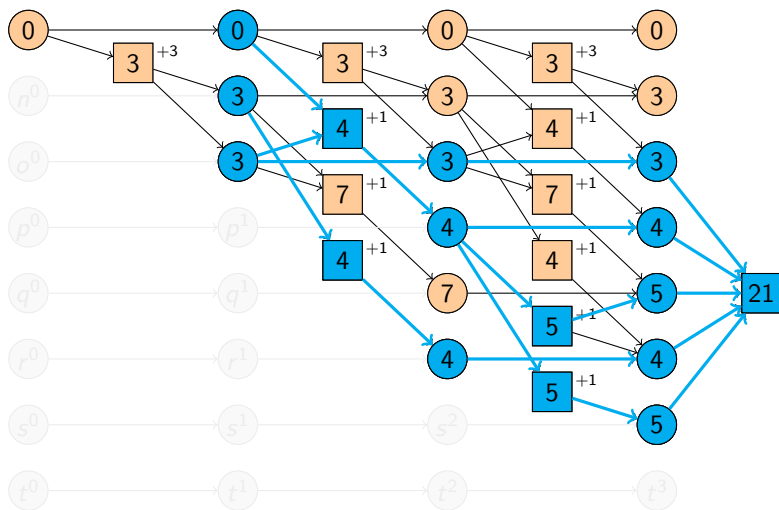
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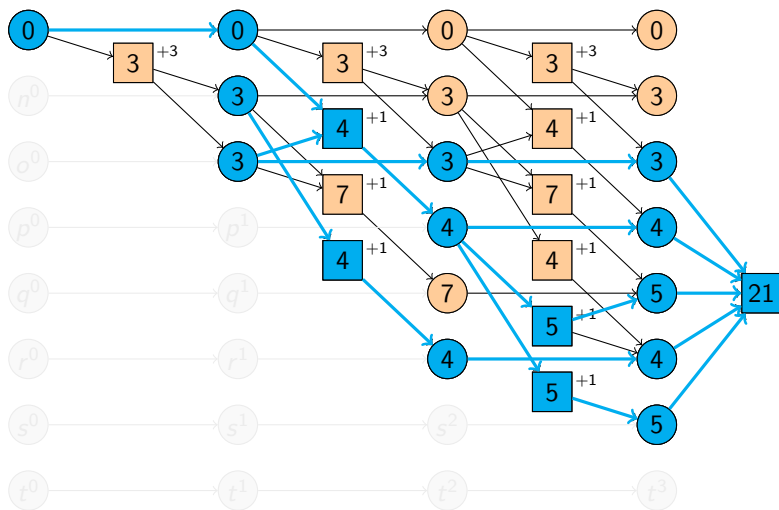
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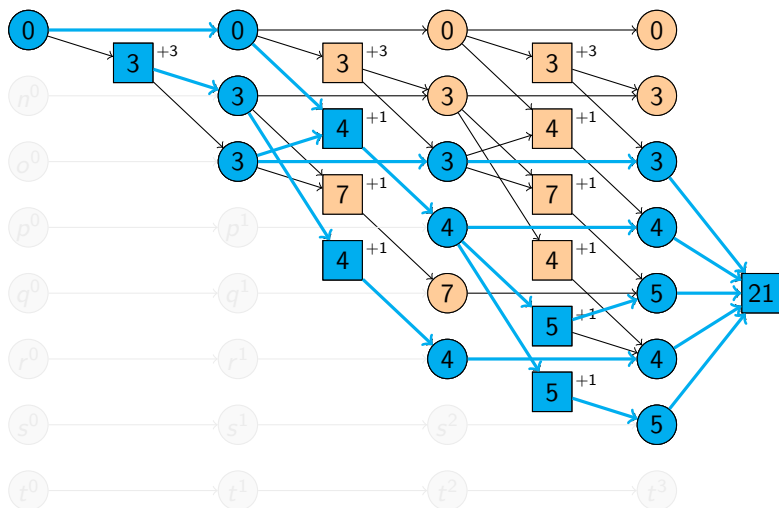
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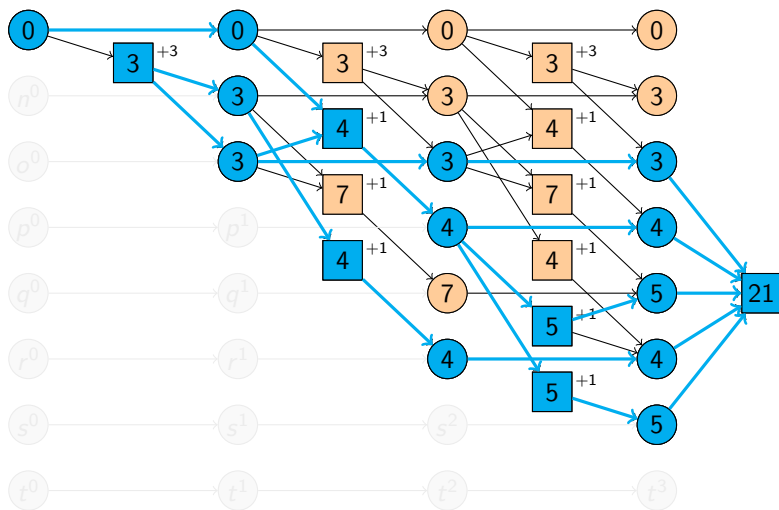


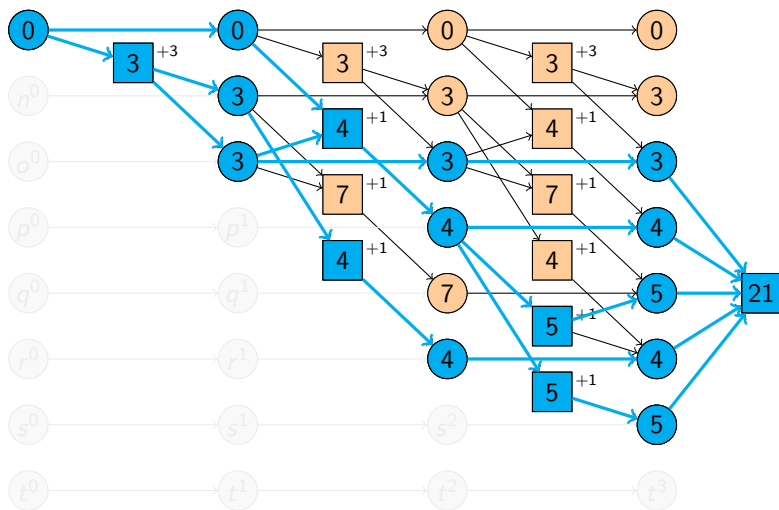
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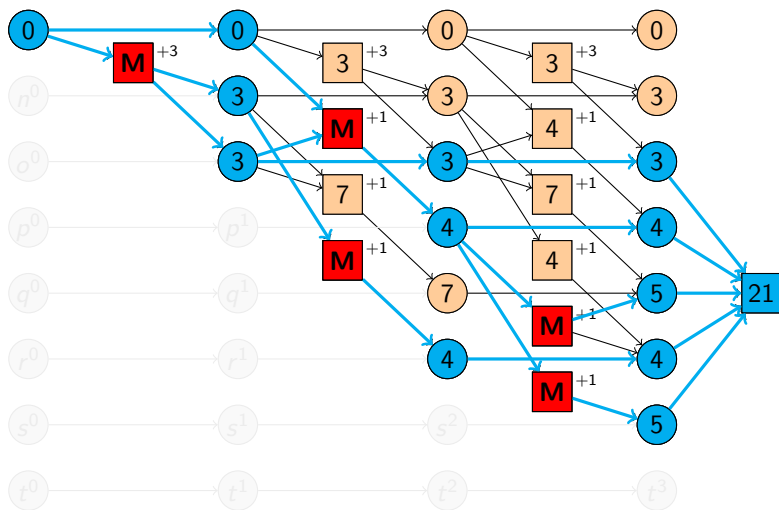


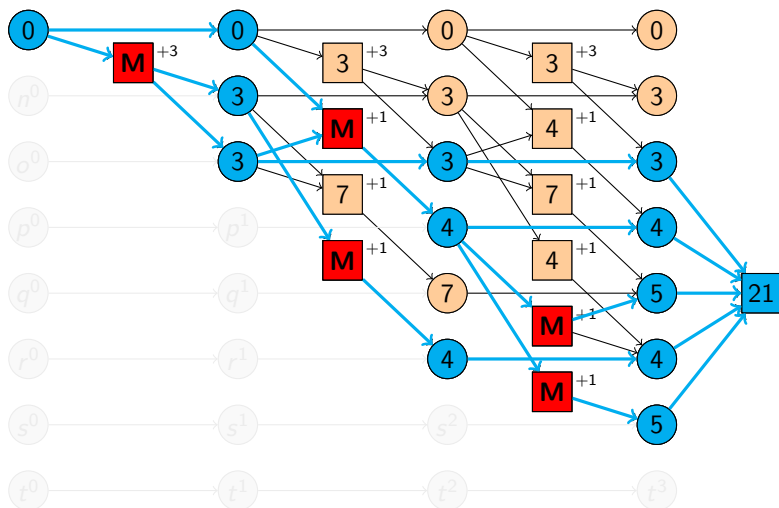
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$$h^{FF}(\{m\}) = 3 + 1 + 1 + 1 + 1 = 7$$

FF Heuristic: Remarks

- Like h^{add} , h^{FF} is safe and goal-aware, but neither admissible nor consistent.
- approximation of h^+ which is **always** at least as good as h^{add}
- **usually** significantly better
- can be computed in **almost linear time** ($O(n \log n)$) in the size of the description of the planning task

FF Heuristic: Remarks

- Like h^{add} , h^{FF} is safe and goal-aware, but neither admissible nor consistent.
- approximation of h^+ which is **always** at least as good as h^{add}
- **usually** significantly better
- can be computed in **almost linear time** ($O(n \log n)$) in the size of the description of the planning task
- computation of heuristic value depends on **tie-breaking** of marking rules (h^{FF} not well-defined)
- one of the **most successful** planning heuristics

Comparison of Relaxation Heuristics

Relationships of Relaxation Heuristics

Let s be a state in the STRIPS planning task $\langle V, I, G, A \rangle$.

Then

- $h^{\max}(s) \leq h^+(s) \leq h^*(s)$
- $h^{\max}(s) \leq h^+(s) \leq h^{\text{FF}}(s) \leq h^{\text{add}}(s)$
- h^* and h^{FF} are incomparable
- h^* and h^{add} are incomparable

further remarks:

- For **non-admissible** heuristics, it is generally neither good nor bad to compute higher values than another heuristic.
- For relaxation heuristics, the objective is to approximate h^+ as closely as possible.

Summary

Summary

- Many delete relaxation heuristics can be viewed as computations on **relaxed planning graphs** (RPGs).
- examples: $h^{\max}$, h^{add} , h^{FF}
- $h^{\max}$ and h^{add} propagate **numeric values** in the RPGs
 - difference: $h^{\max}$ computes the **maximum** of predecessor costs for action and goal vertices; h^{add} computes the **sum**
- h^{FF} **marks** vertices and sums the costs of marked action vertices.
- generally: $h^{\max}(s) \leq h^+(s) \leq h^{\text{FF}}(s) \leq h^{\text{add}}(s)$