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Propositional Logic: Overview

Chapter overview: propositional logic

- ▶ 29. Basics
- ▶ 30. Reasoning and Resolution
- ▶ 31. DPLL Algorithm
- ► 32. Local Search and Outlook

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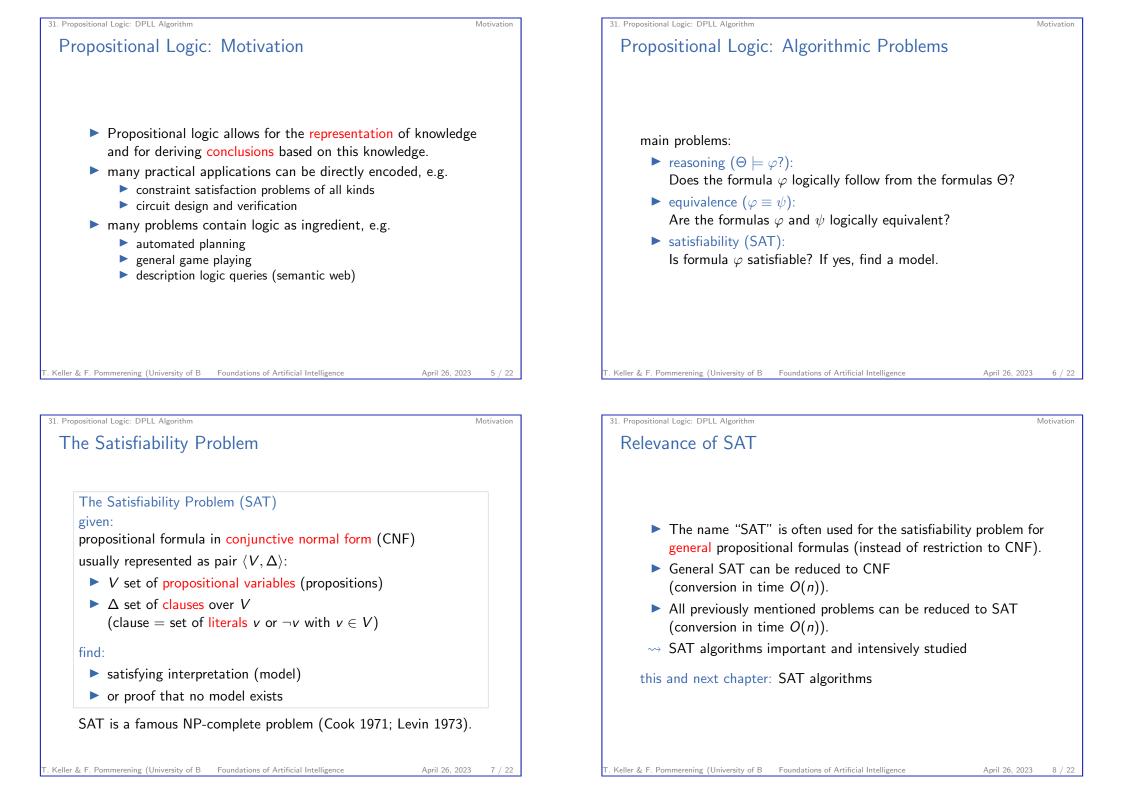
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31.1 Motivation

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31.2 Systematic Search: DPLL

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Systematic Search: DPLL

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The DPLL Algorithm

The DPLL algorithm (Davis/Putnam/Logemann/Loveland) corresponds to backtracking with inference for CSPs.

- \blacktriangleright recursive call DPLL(Δ , I) for clause set Δ and partial interpretation I
- result is consistent extension of *I*: **unsatisfiable** if no such extension exists
- First call DPLL(Δ, \emptyset)

inference in DPLL:

- **implify**: after assigning value d to variable v, simplify all clauses that contain v
 - → forward checking (for constraints of potentially higher arity)
- unit propagation: variables that occur in clauses without other variables (unit clauses) are assigned immediately
 - → minimum remaining values variable order

31. Propositional Logic: DPLL Algorithm

SAT vs CSP

SAT can be considered as constraint satisfaction problem:

- CSP variables = propositions
- domains = { \mathbf{F}, \mathbf{T} }
- constraints = clauses

However, we often have constraints that affect > 2 variables.

Due to this relationship, all ideas for CSPs are applicable to SAT:

- search
- ▶ inference
- variable and value orders

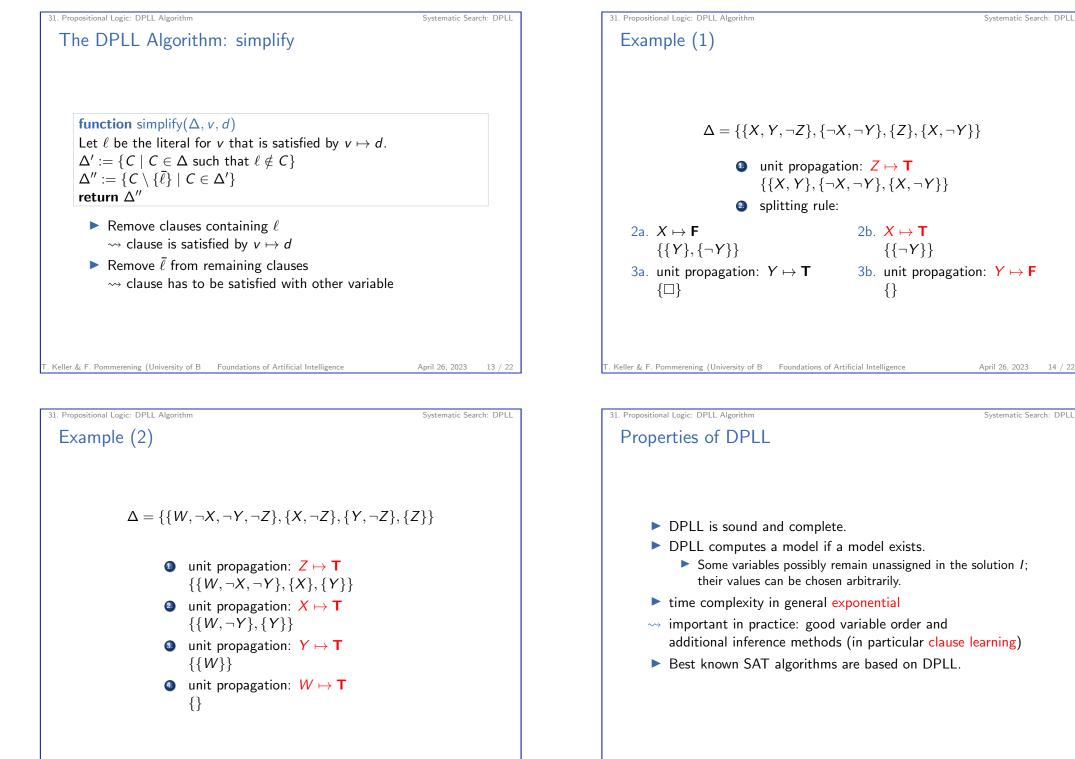
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Systematic Search: DPLL

31. Propositional Logic: DPLL Algorithm Systematic Search: DPLL The DPLL Algorithm: Pseudo-Code function DPLL(Δ , *I*): if $\Box \in \Delta$: [empty clause exists ~> unsatisfiable] return unsatisfiable else if $\Delta = \emptyset$: [no clauses left \rightsquigarrow interpretation *I* satisfies formula] return / else if there exists a unit clause $\{v\}$ or $\{\neg v\}$ in Δ : [unit propagation] Let v be such a variable, d the truth value that satisfies the clause. $\Delta' := \operatorname{simplify}(\Delta, v, d)$ return DPLL($\Delta', I \cup \{v \mapsto d\}$) splitting rule else: Select some variable v which occurs in Δ . for each $d \in \{F, T\}$ in some order: $\Delta' := \operatorname{simplify}(\Delta, v, d)$ $I' := \mathsf{DPLL}(\Delta', I \cup \{v \mapsto d\})$ if $I' \neq$ unsatisfiable return // return unsatisfiable Keller & F. Pommerening (University of B Foundations of Artificial Intelligence

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31.3 DPLL on Horn Formulas

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DPLL on Horn Formulas

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DPLL on Horn Formulas

Proposition (DPLL on Horn formulas) If the input formula φ is a Horn formula, then the time complexity of DPLL is polynomial in the length of φ .

Proof.

properties:

- If Δ is a Horn formula, then so is simplify (Δ, v, d) . (Why?)
 - → all formulas encountered during DPLL search are Horn formulas if input is Horn formula
- Every Horn formula without empty or unit clauses is satisfiable:
 - all such clauses consist of at least two literals
 - ► Horn property: at least one of them is negative
 - assigning F to all variables satisfies formula

. . .

Horn Formulas

important special case: Horn formulas

Definition (Horn formula) A Horn clause is a clause with at most one positive literal, i.e., of the form

 $\neg x_1 \lor \cdots \lor \neg x_n \lor y \text{ or } \neg x_1 \lor \cdots \lor \neg x_n$

(n = 0 is allowed.)

A Horn formula is a propositional formula in conjunctive normal form that only consists of Horn clauses.

- ► foundation of logic programming (e.g., PROLOG)
- critical in many kinds of practical reasoning problems

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