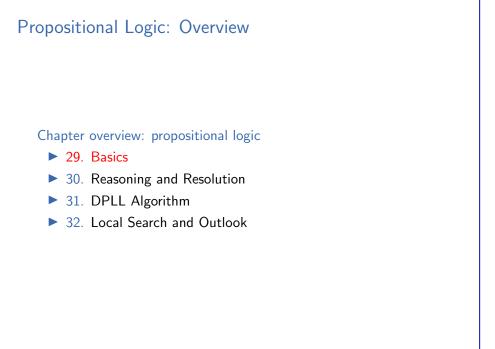
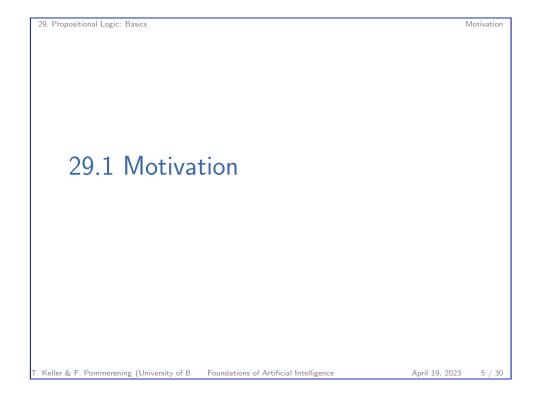




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29.1 Motivation		
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29. Propositional Logic: Basics Relationship to CSPs

- previous topic: constraint satisfaction problems
- satisfiability problem in propositional logic can be viewed as non-binary CSP over {F, T}
- formula encodes constraints
- solution: satisfying assignment of values to variables
- ► SAT algorithms for this problem: ~→ DPLL (next week)

29. Propositional Logic: Basic Motivation Propositional Logic: Motivation propositional logic modeling and representing problems and knowledge basics for general problem descriptions and solving strategies (~→ automated planning ~→ later in this course) allows for automated reasoning

Motivation

Propositional Logic: Description of State Spaces

propositional variables for missionaries and cannibals problem:

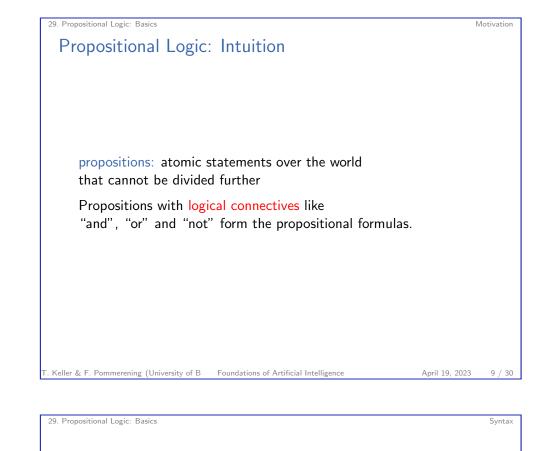
two-missionaries-are-on-left-shore
one-cannibal-is-on-left-shore
boat-is-on-left-shore

• • •

29. Propositional Logic: Basics

- problem description for general problem solvers
- states represented as truth values of atomic propositions

Motivation



29.2 Syntax

29. Propositional Logic: Basics

Syntax and Semantics

Today, we define syntax and semantics of propositional logic.

Syntax

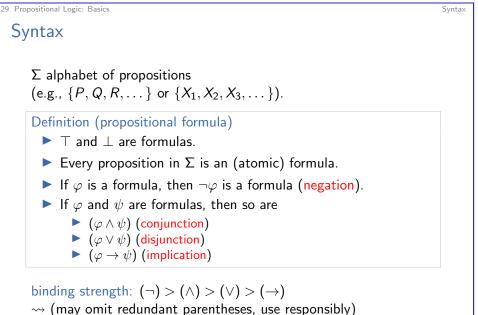
- ▶ defines which symbols are allowed in formulas (,), ℵ, ∧, A, B, C, X, ♡, →, ↗, ...?
- ▶ ... and which sequences of these symbols are correct formuals $(A \land B)$, $((A) \land B)$, $\land)A(B, ...?$

Semantics

- defines the meaning of formulas
- uses interpretations to describe a possible world $I_1 = \{A \mapsto \mathbf{T}, B \mapsto \mathbf{F}\}$
- defines which formulas are true in which worlds

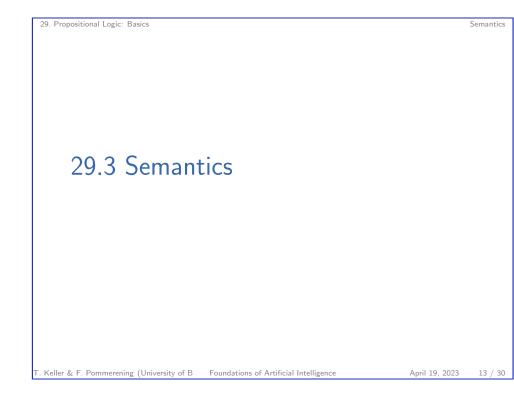
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√→ (may omit redundant parentheses, use responsibly)
 note: minor differences to Discrete Mathematics course

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29. Propositional Logic: Basics Seman Semantics: Formally • defined over interpretation $I : \Sigma \to \{T, F\}$ • interpretation $I : assignment of propositions in <math>\Sigma$ • When is a formula φ true under interpretation I? symbolically: When does $I \models \varphi$ hold? 29. Propositional Logic: Basics

Semantics

A formula is true or false,

depending on the interpretation of the propositions.

Semantics: Intuition

- A proposition p is either true or false.
 The truth value of p is determined by an interpretation.
- The truth value of a formula follows from the truth values of the propositions.

Example

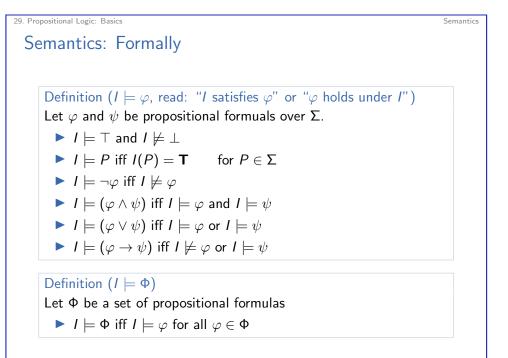
 $\varphi = (P \lor Q) \land R$

- If P and Q are false, then φ is false (independent of the truth value of R).
- If P and R are true, then φ is true (independent of the truth value of Q).

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Semantics



29. Propositional Logic: Basics

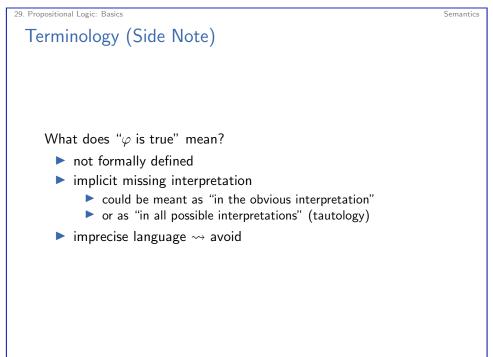
Examples

Example (Interpretation I) $I = \{P \mapsto \mathbf{T}, Q \mapsto \mathbf{T}, R \mapsto \mathbf{F}, S \mapsto \mathbf{F}\}$

Which formulas are true under *I*? • $\varphi_1 = \neg (P \land Q) \land (R \land \neg S)$. Does $I \models \varphi_1$ hold?

- $\varphi_2 = (P \land Q) \land \neg (R \land \neg S)$. Does $I \models \varphi_2$ hold?
- $\varphi_3 = (R \rightarrow P)$. Does $I \models \varphi_3$ hold?

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29. Propositional Logic: Basics
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Terminology

Definition (model)

An interpretation I is called a model of φ if $I \models \varphi$.

Definition (satisfiable etc.)

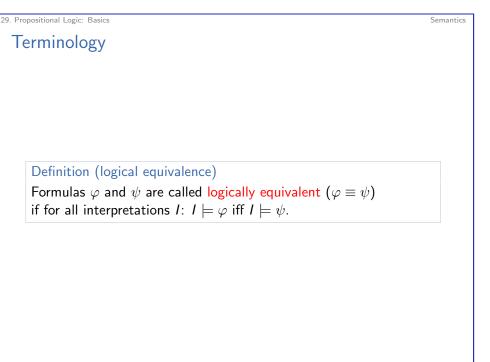
A formula φ is called

- **satisfiable** if there is an interpretation I such that $I \models \varphi$.
- unsatisfiable if φ is not satisfiable.
- **b** falsifiable if there is an interpretation I such that $I \not\models \varphi$.
- ▶ valid (= a tautology) if $I \models \varphi$ for all interpretations I.

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Semantics



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Semantic



Semantics

Equivalences

Logical Equivalences

Let φ , ψ , and η be formulas.

- $(\varphi \land \psi) \equiv (\psi \land \varphi)$ and $(\varphi \lor \psi) \equiv (\psi \lor \varphi)$ (commutativity)
- ((φ ∧ ψ) ∧ η) ≡ (φ ∧ (ψ ∧ η)) and ((φ ∨ ψ) ∨ η) ≡ (φ ∨ (ψ ∨ η))
 ((φ ∧ ψ) ∨ η) ≡ ((φ ∨ η) ∧ (ψ ∨ η))
 (φ → ψ) ≡ (¬φ ∨ ψ)
 ¬(φ ∧ ψ) ≡ (¬φ ∨ ¬ψ)
 (De Morgan)
 ¬(φ ∨ ψ) ≡ (¬φ ∧ ¬ψ)
- $\neg \neg \varphi \equiv \varphi$ (double negation)

Commutativity and associativity are often used implicitly \rightsquigarrow We write $(X_1 \land X_2 \land X_3 \land X_4)$ instead of $(X_1 \land (X_2 \land (X_3 \land X_4)))$

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29. Propositional Logic: Basics

29. Propositional Logic: Basics

Truth Tables

Truth Tables

How to determine automatically if a given formula is (un)satisfiable, falsifiable, valid?

 \rightsquigarrow simple method: truth tables

example: Is $\varphi = ((P \lor H) \land \neg H) \to P$ valid?

Ρ	H	$P \lor H$	$((P \lor H) \land \neg H)$	$((P \lor H) \land \neg H) \to P$
F	F	F	F	Т
F	Т	Т	F	Т
Т	F	Т	Т	Т
Т	Т	Т	F	Т
Т	Т	Т	F	Т

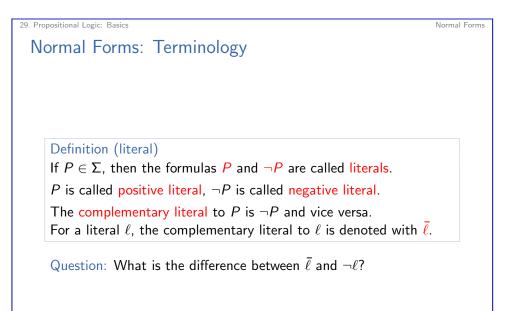
 $I\models\varphi \text{ for all interpretations }I\rightsquigarrow\varphi \text{ is valid.}$

satisfiability, falsifiability, unsatisfiability?

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Semantics



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Normal Forms

Normal Forms: Terminology

Definition (clause)

A disjunction of 0 or more literals is called a clause. The empty clause \perp is also written as \square . Clauses consisting of only one literal are called unit clauses.

Definition (monomial)

A conjunction of 0 or more literals is called a monomial.

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Normal Forms

Normal Forms

29. Propositional Logic: Basics

For every propositional formula, there exists a logically equivalent propositional formula in CNF and in DNF.

Conversion to CNF with equivalences				
eliminate implications				
$(arphi ightarrow \psi) \equiv (eg arphi ee \psi)$	((ightarrow)-elimination)			
2 move negations inside				
$ eg(arphi\wedge\psi)\equiv(egarphi\vee eg\psi)$	(De Morgan)			
$ eg(arphi \lor \psi) \equiv (\neg arphi \land \neg \psi)$	(De Morgan)			
$\neg\neg\varphi\equiv\varphi$	(double negation)			
3 distribute \lor over \land				
$((arphi \wedge \psi) ee \eta) \equiv ((arphi ee \eta) \wedge (\psi ee \eta))$	(distributivity)			
• simplify trivial subformulas (op, ot)				
There are formulas φ for which every logically equivalent formula				

There are formulas φ for which every logically equivalent formula in CNF and DNF is exponentially longer than φ .

29. Propositional Logic: Basics

Normal Forms

Definition (normal forms)

A formula φ is in conjunctive normal form (CNF, clause form) if φ is a conjunction of 0 or more clauses:

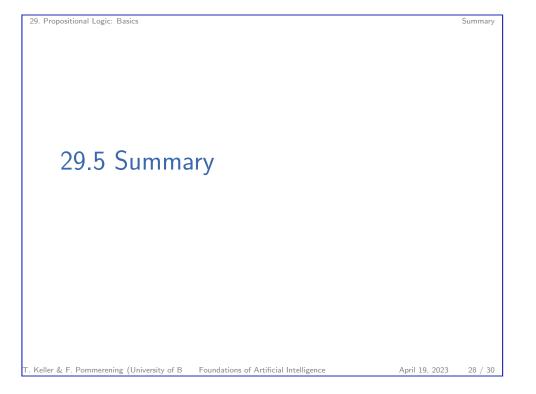
$$\varphi = \bigwedge_{i=1}^{n} \left(\bigvee_{j=1}^{m_i} \ell_{i,j}\right)$$

A formula φ is in disjunctive normal form (DNF) if φ is a disjunction of 0 or more monomials:

$$\varphi = \bigvee_{i=1}^{n} \left(\bigwedge_{j=1}^{m_i} \ell_{i,j} \right)$$

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29. Propositional Logic: Basics Summary 29. Propositional Logic: Basics Summary Summary (1) Summary (2) Propositional logic forms the basis for a general important terminology: representation of problems and knowledge. model Propositions (atomic formulas) are statements over the world satisfiable, unsatisfiable, falsifiable, valid logically equivalent which cannot be divided further. different kinds of formulas: Propositional formulas combine atomic formulas atomic formulas and literals with \neg , \land , \lor , \rightarrow to more complex statements. clauses and monomials Interpretations determine which atomic formulas are true conjunctive normal form and disjunctive normal form and which ones are false. . Keller & F. Pommerening (University of B Foundations of Artificial Intelligence April 19, 2023 29 / 30 T. Keller & F. Pommerening (University of B Foundations of Artificial Intelligence April 19, 2023 30 / 30