Foundations of Artificial Intelligence

15. State-Space Search: Best-first Graph Search

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State-Space Search: Overview

Chapter overview: state-space search

- 5.–7. Foundations
- 8.–12. Basic Algorithms
- 13.–19. Heuristic Algorithms
  - 13. Heuristics
  - 14. Analysis of Heuristics
  - 15. Best-first Graph Search
  - 16. Greedy Best-first Search, A*, Weighted A*
  - 17. IDA*
  - 18. Properties of A*, Part I
  - 19. Properties of A*, Part II

Introduction

Heuristic Search Algorithms

Heuristic Search Algorithms

Heuristic search algorithms use heuristic functions
to (partially or fully) determine the order of node expansion.

- this chapter: short introduction
- next chapters: more thorough analysis

Best-first Search

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Best-first Search

A best-first search is a heuristic search algorithm that evaluates search nodes with an evaluation function f and always expands a node n with minimal $f(n)$ value.

- implementation essentially like uniform cost search
- different choices of f $\rightsquigarrow$ different search algorithms

The Most Important Best-first Search Algorithms

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- $f(n) = g(n) + w \cdot h(n.\text{state})$: weighted A^*
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What do we obtain with $f(n) := g(n)$?

Best-first Search: Graph Search or Tree Search?

Best-first search can be graph search or tree search.

- now: graph search (i.e., with duplicate elimination), which is the more common case
- Chapter 17: a tree search variant

Algorithm Details

Reminder: Uniform Cost Search

reminder: uniform cost search

Uniform Cost Search

open := **new** MinHeap ordered by g
open.insert(make_root_node())
closed := **new** HashSet
while not *open*.is_empty():
    n := *open*.pop_min()
    if n.state $\notin$ *closed*:
        closed.insert(n.state)
        if is_goal(n.state):
            return extract_path(n)
        for each $\langle a, s' \rangle \in$ succ(n.state):
            n' := make_node(n, a, s')
            open.insert(n')
return unsolvable

Best-first Search without Reopening (1st Attempt)

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Best-first Search w/o Reopening (1st Attempt): Discussion

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This is already an acceptable implementation of best-first search.

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two useful improvements:

- discard states considered unsolvable by the heuristic
  $\rightsquigarrow$ saves memory in *open*
- if multiple search nodes have identical f values, use h to break ties (preferring low h)
  - not always a good idea, but often
  - obviously unnecessary if $f = h$ (greedy best-first search)

Best-first Search without Reopening (Final Version)

Best-first Search without Reopening

open := **new** MinHeap ordered by $\langle f, h \rangle$
if $h(\text{init}()) < \infty$:
    open.insert(make_root_node())
closed := **new** HashSet
while not *open*.is_empty():
    n := *open*.pop_min()
    if n.state $\notin$ *closed*:
        closed.insert(n.state)
        if is_goal(n.state):
            return extract_path(n)
        for each $\langle a, s' \rangle \in$ succ(n.state):
            if $h(s') < \infty$:
                n' := make_node(n, a, s')
                open.insert(n')
return unsolvable

Best-first Search: Properties

properties:

- complete if h is safe (Why?)
- optimality depends on f $\rightsquigarrow$ next chapters

Reopening

Reopening

- reminder: uniform cost search expands nodes in order of increasing g values

⇝ guarantees that cheapest path to state of a node has been found when the node is expanded

- with arbitrary evaluation functions f in best-first search this does not hold in general

⇝ in order to find solutions of low cost, we may want to expand duplicate nodes when cheaper paths to their states are found (reopening)

German: Reopening

Best-first Search with Reopening

Best-first Search with Reopening

$open := \textbf{new}$ MinHeap ordered by $\langle f, h\rangle$
if $h(\text{init}()) < \infty$:
    $open$.insert(make_root_node())
$distances := \textbf{new}$ HashMap
while not $open$.is_empty():
    $n := open$.pop_min()
    if $distances$.lookup($n.state$) $=$ **none or** $g(n) < distances[n.\text{state}]$:
        $distances[n.\text{state}] := g(n)$
        if is_goal(n.state):
            return extract_path(n)
        for each $\langle a, s'\rangle \in$ succ(n.state):
            if $h(s') < \infty$:
                $n' :=$ make_node(n, a, s')
                $open$.insert(n')
return unsolvable

$\rightsquigarrow$ *distances* controls reopening and replaces *closed*

Summary

Summary

- best-first search: expand node with minimal value of evaluation function f
  - $f = h$: greedy best-first search
  - $f = g + h$: A*
  - $f = g + w \cdot h$ with parameter $w \in \mathbb{R}_0^+$: weighted A*
- here: best-first search as a graph search
- reopening: expand duplicates with lower path costs to find cheaper solutions