

Foundations of Artificial Intelligence

13. State-Space Search: Heuristics

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State-Space Search: Overview

Chapter overview: state-space search

- 5.–7. Foundations
- 8.–12. Basic Algorithms
- 13.–19. Heuristic Algorithms
 - 13. Heuristics
 - 14. Analysis of Heuristics
 - 15. Best-first Graph Search
 - 16. Greedy Best-first Search, A^* , Weighted A^*
 - 17. IDA *
 - 18. Properties of A^* , Part I
 - 19. Properties of A^* , Part II

Introduction

Informed Search Algorithms

search algorithms considered so far:

- **uninformed** (“blind”): use **no information** besides **formal definition** to solve a problem
- **scale poorly**: prohibitive time (and space) requirements for seemingly **simple** problems (**time complexity** usually $O(b^d)$)

Informed Search Algorithms

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example: $b = 13$; 10^5 nodes/second

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d	nodes	time
4	30 940	0.3 s
6	$5.2 \cdot 10^6$	52 s
8	$8.8 \cdot 10^8$	147 min
10	10^{11}	17 days
12	10^{13}	8 years
14	10^{15}	1 352 years
16	10^{17}	$2.2 \cdot 10^5$ years
18	10^{20}	$38 \cdot 10^6$ years

Informed Search Algorithms

Rubik's cube:



- branching factor: ≈ 13
- typical solution length: 18

example: $b = 13$; 10^5 nodes/second

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Informed Search Algorithms

Rubik's cube:



search algorithms considered now:

- **idea:** try to find (problem-specific) criteria to distinguish **good** and **bad states**
 - **heuristic** (“informed”) search algorithms prefer **good states**
-
- branching factor: ≈ 13
 - typical solution length: 18

Heuristics

Heuristics

Definition (heuristic)

Let \mathcal{S} be a state space with states S .

A **heuristic function** or **heuristic** for \mathcal{S} is a function

$$h : S \rightarrow \mathbb{R}_0^+ \cup \{\infty\},$$

mapping each state to a non-negative number (or ∞).

Heuristics: Intuition

idea: $h(s)$ estimates distance (= cost of cheapest path) from s to closest goal state

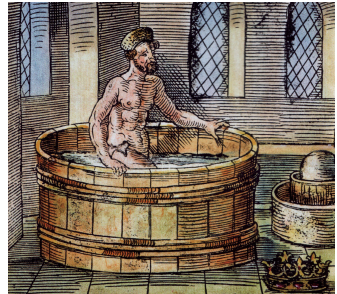
- heuristics can be **arbitrary** functions
- **intuition:**
 - 1 the closer h is to true goal distance, the more efficient the search using h
 - 2 the better h separates states that are **close** to the goal from states that are **far**, the more efficient the search using h

Heuristics are sometimes defined for **search nodes** instead of states, but this increased generality is rarely useful. (**Why?**)

Why “Heuristic”?

What does “heuristic” mean?

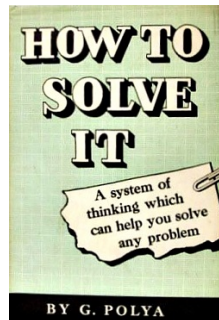
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- same origin as ἐυρηκᾶ!



Why “Heuristic”?

What does “heuristic” mean?

- from ancient Greek εὕρισκω (= I find)
- same origin as εὕρηκα!
- popularized by George Pólya:
How to Solve It (1945)
- in computer science often used for:
rule of thumb, inexact algorithm
- in state-space search technical term
for **goal distance estimator**



Representation of Heuristics

In our black box model, heuristics are an additional element of the state space interface:

State Spaces as Black Boxes (Extended)

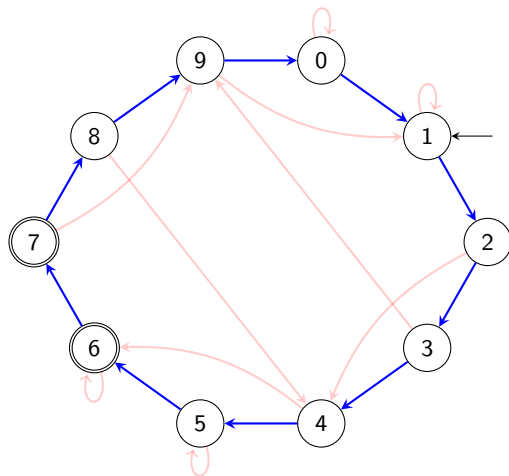
- `init()`
- `is_goal(s)`
- `succ(s)`
- `cost(a)`
- `h(s)`: heuristic value for state s
 result: non-negative integer or ∞

Examples

Bounded Inc-and-square

bounded inc-and-square:

possible heuristics:



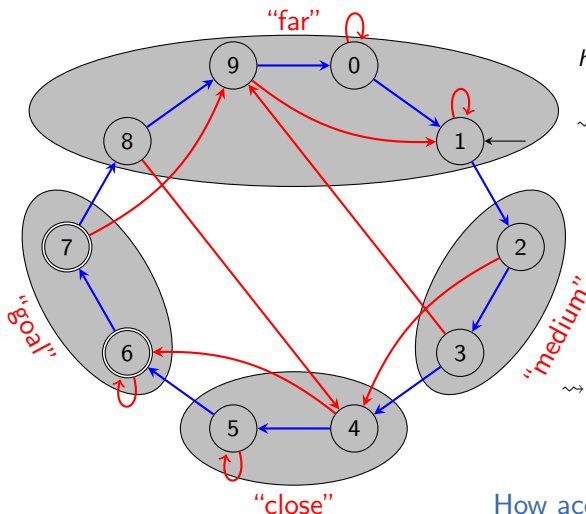
$$h_1(s) = \begin{cases} 0 & \text{if } s = 7 \\ (16 - s) \% 10 & \text{otherwise} \end{cases}$$

\rightsquigarrow number of *inc* actions to goal

How accurate is this heuristic?

Bounded Inc-and-square

bounded inc-and-square:



possible heuristics:

$$h_1(s) = \begin{cases} 0 & \text{if } s = 7 \\ (16 - s) \% 10 & \text{otherwise} \end{cases}$$

\leadsto number of *inc* actions to goal

$$h_2(s) = \begin{cases} 0 & \text{if } s \text{ is a "goal"} \\ 1 & s \text{ is "close"} \\ 2 & s \text{ is "medium"} \\ 3 & s \text{ is "far"} \end{cases}$$

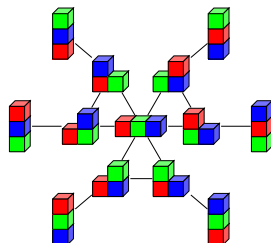
\leadsto categorize states

How accurate is this heuristic?

Example: Blocks World

possible heuristic:

count blocks x that currently lie on y
and must lie on $z \neq y$ in the goal
(including case where y or z is the table)

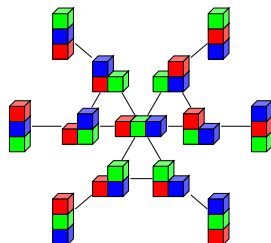


Example: Blocks World

possible heuristic:

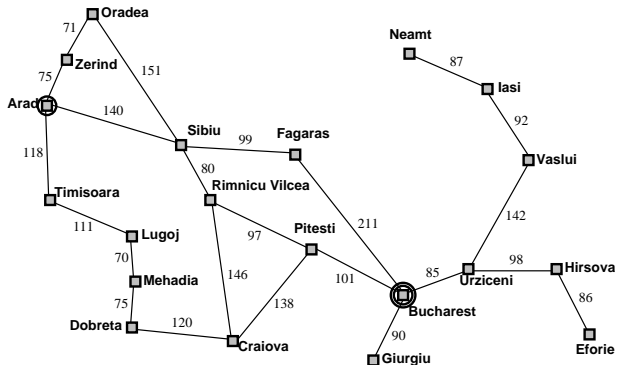
count blocks x that currently lie on y
and must lie on $z \neq y$ in the goal
(including case where y or z is the table)

How accurate is this heuristic?



Example: Route Planning in Romania

possible heuristic: straight-line distance to Bucharest



Arad	366
Bucharest	0
Craiova	160
Drobeta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Example: Missionaries and Cannibals

Setting: Missionaries and Cannibals

- Six people must cross a river.
- Their rowing boat can carry one or two people across the river at a time (it is too small for three).
- Three people are missionaries, three are cannibals.
- Missionaries may never stay with a majority of cannibals.

possible heuristic: number of people on the wrong river bank

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~> with our formulation of states as triples $\langle m, c, b \rangle$:

$$h(\langle m, c, b \rangle) = m + c$$

Summary

Summary

- **heuristics** estimate distance of a state to the goal
 - can be used to **focus** search on **promising** states
- ~> **soon:** search algorithms that use heuristics