Foundations of Artificial Intelligence

12. State-Space Search: Depth-first Search & Iterative Deepening

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Foundations of Artificial Intelligence

March 20, 2023 — 12. State-Space Search: Depth-first Search & Iterative Deepening

12.1 Depth-first Search

12.2 Iterative Deepening

12.3 Summary

State-Space Search: Overview

Chapter overview: state-space search

- 5.–7. Foundations
- 8.–12. Basic Algorithms
  - 8. Data Structures for Search Algorithms
  - 9. Tree Search and Graph Search
  - 10. Breadth-first Search
  - 11. Uniform Cost Search
  - 12. Depth-first Search and Iterative Deepening
- 13.–19. Heuristic Algorithms

12.1 Depth-first Search

Idea and Example

depth-first (tree) search:

- expands nodes in opposite order of generation (LIFO)
- open list imlemented as stack
- deepest node expanded first

next

open: [1]

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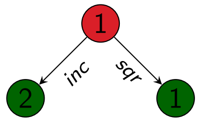

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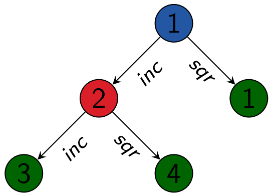

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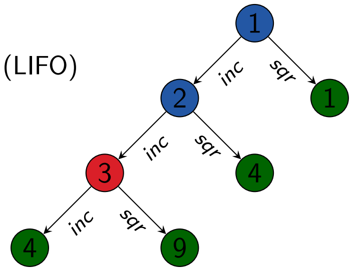

next

open: [1 4 9 4]

Idea and Example

depth-first (tree) search:

- expands nodes in opposite order of generation (LIFO)
- open list imlemented as stack
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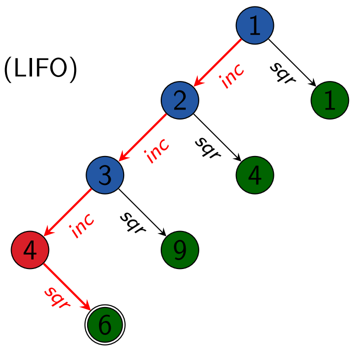
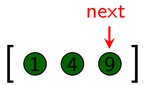
- here: goal test on generation
  (but: depends on implementation)



open:



Depth-first Search: Some Properties

- almost always implemented as a tree search (we will see why)
- not complete, not semi-complete, not optimal (Why?)
- complete for acyclic state spaces,
  e.g., if state space directed tree

Reminder: Generic Tree Search Algorithm

reminder from Chapter 9:

Generic Tree Search

```
$open$ := new OpenList
$open$.insert(make_root_node())
while not $open$.is_empty():
    $n$ := $open$.pop()
    if is_goal($n$.state):
        return extract_path($n$)
    for each $\langle a, s' \rangle \in$ succ($n$.state):
        $n'$ := make_node($n, a, s'$)
        $open$.insert($n'$)
return unsolvable
```

Depth-first Search (Non-recursive Version)

depth-first search (non-recursive version):

Depth-first Search (Non-recursive Version)

```
$open$ := new Stack
$open$.push_back(make_root_node())
while not $open$.is_empty():
    $n$ := $open$.pop_back()
    if is_goal($n$.state):
        return extract_path($n$)
    for each $\langle a, s' \rangle \in$ succ($n$.state):
        $n'$ := make_node($n, a, s'$)
        $open$.push_back($n'$)
return unsolvable
```

Non-recursive Depth-first Search: Discussion

discussion:

- there isn't much wrong with this pseudo-code
  (as long as we ensure to release nodes that are no longer required when using programming languages without garbage collection)
- however, depth-first search as a recursive algorithm is simpler and more efficient

⇝ CPU stack as implicit open list

⇝ no search node data structure needed

Depth-first Search (Recursive Version)

```
function depth_first_search($s$)
if is_goal($s$):
    return $\langle\rangle$
for each $\langle a, s'\rangle \in$ succ($s$):
    $solution$ := depth_first_search($s'$)
    if $solution \neq$ none:
        $solution$.push_front($a$)
        return $solution$
return none
```

main function:

```
Depth-first Search (Recursive Version)
return depth_first_search(init())
```

Depth-first Search: Complexity

time complexity:

- If the state space includes paths of length m, depth-first search can generate $O(b^m)$ nodes, even if much shorter solutions (e.g., of length 1) exist.
- On the other hand: in the best case, solutions of length ℓ can be found with $O(b\ell)$ generated nodes. (Why?)
- improvable to $O(\ell)$ with incremental successor generation

space complexity:

- only need to store nodes along currently explored path
  ("along": nodes on path and their children)

⇝ space complexity $O(bm)$ if m maximal search depth reached

- low memory complexity main reason why depth-first search interesting despite its disadvantages

12.2 Iterative Deepening

Idea and Example

depth-limited search:

- parametrized with depth limit ℓ
- variant of depth-first search, i.e.,
  - expands nodes in opposite order of generation (LIFO)
  - tree search
- prunes (does not expand) search nodes at depth $d \geq \ell$

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consider depth limit $\ell = 2$

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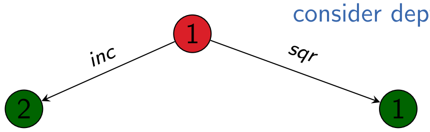

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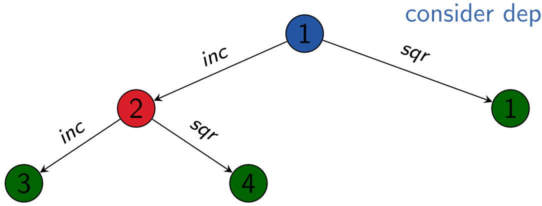

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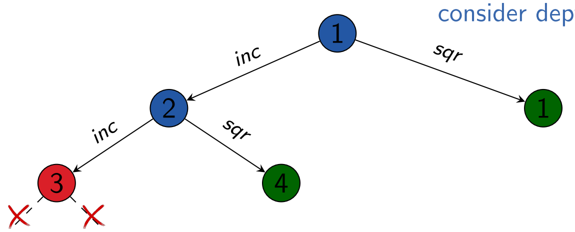

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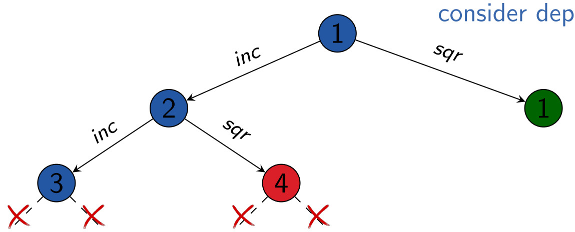

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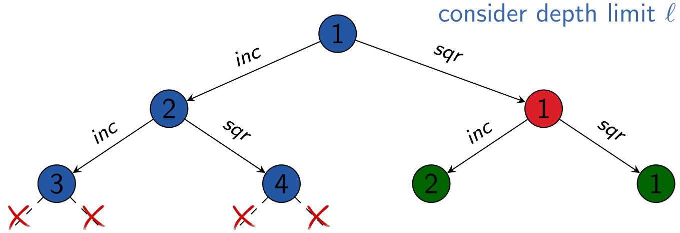

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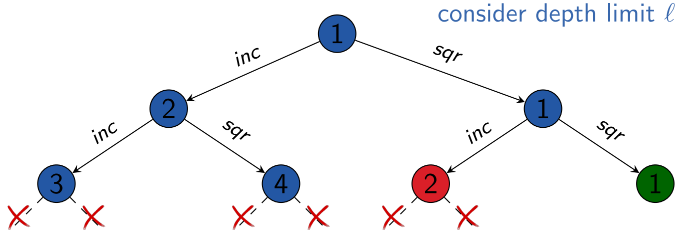

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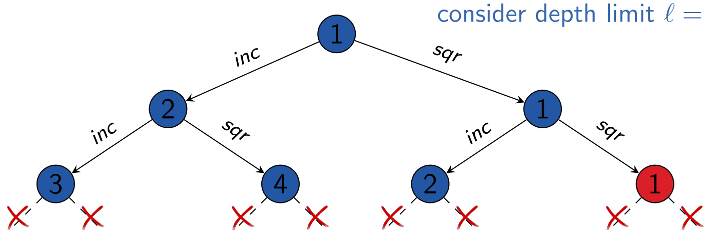
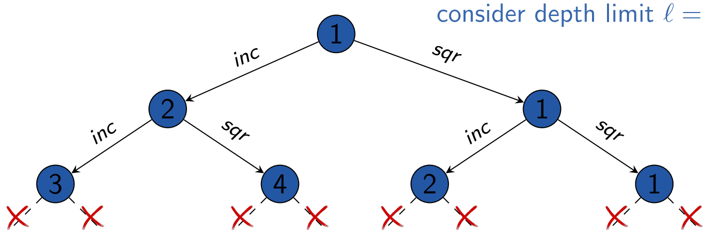


Idea and Example

depth-limited search:

- parametrized with depth limit ℓ
- variant of depth-first search, i.e.,
  - expands nodes in opposite order of generation (LIFO)
  - tree search
- prunes (does not expand) search nodes at depth $d \geq \ell$
- not very useful on its own, but important ingredient of more useful algorithms

consider depth limit $\ell = 2$



Depth-bounded Search: Pseudo-Code

```
function depth_bounded_search($s$, $depth\_bound$):
if is_goal($s$):
    return $\langle\rangle$
if $depth\_bound > 0$:
    for each $\langle a, s'\rangle \in$ succ($s$):
        $solution :=$ depth_bounded_search($s'$, $depth\_bound - 1$)
        if $solution \neq$ none:
            $solution$.push_front($a$)
            return $solution$
return none
```

Iterative Deepening Depth-first Search

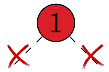
iterative deepening depth-first search (iterative deepening DFS):

- performs a sequence of depth-limited searches
- increases depth limit ℓ in each iteration
- sounds wasteful (each iteration repeats all the useful work of all previous iterations)
- in fact overhead acceptable (analysis follows)

```
Iterative Deepening DFS
for $depth\_bound \in \{0, 1, 2, \dots\}$:
    $solution :=$ depth_bounded_search(init(), $depth\_bound$)
    if $solution \neq$ none:
        return $solution$
```

Example

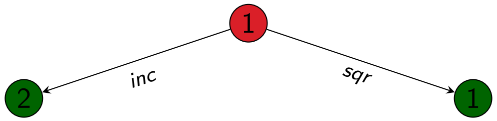
depth limit: 0 generated nodes: 1



(using a non-recursive version of depth-limited search)

Example

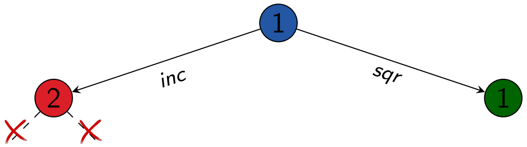
depth limit: 1 generated nodes: 1+3



(using a non-recursive version of depth-limited search)

Example

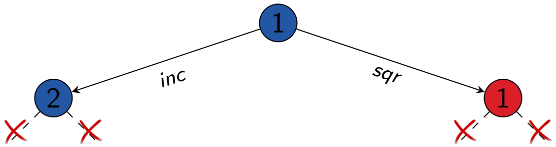
depth limit: 1 generated nodes: 1+3



(using a non-recursive version of depth-limited search)

Example

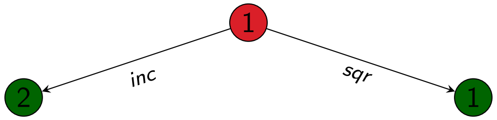
depth limit: 1 generated nodes: 1+3



(using a non-recursive version of depth-limited search)

Example

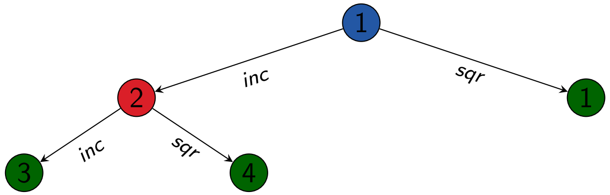
depth limit: 2 generated nodes: 1+3+3



(using a non-recursive version of depth-limited search)

Example

depth limit: 2 generated nodes: 1+3+5



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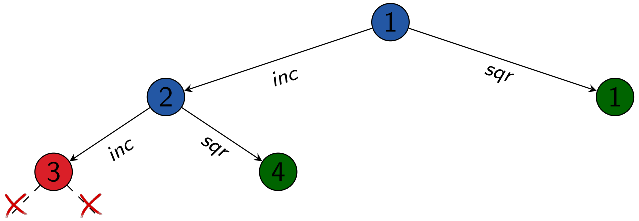

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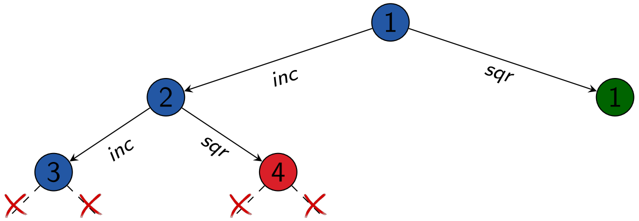
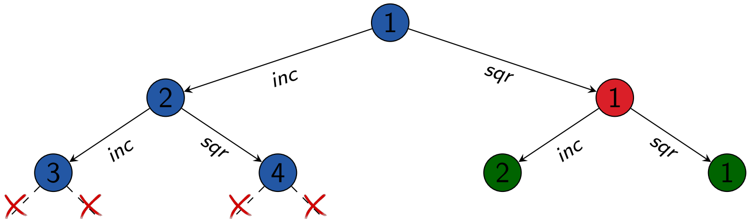


(using a non-recursive version of depth-limited search)

Example

depth limit: 2 generated nodes: 1+3+7



(using a non-recursive version of depth-limited search)

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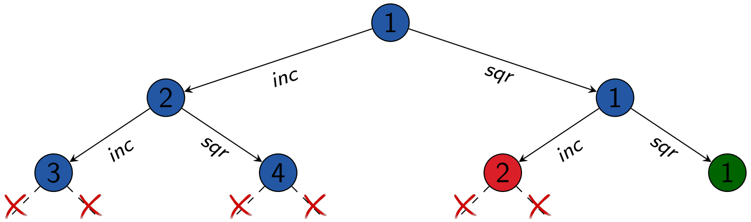

(using a non-recursive version of depth-limited search)

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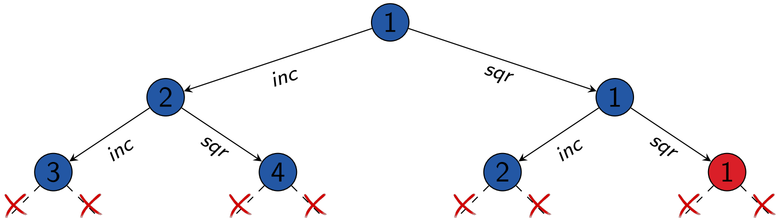
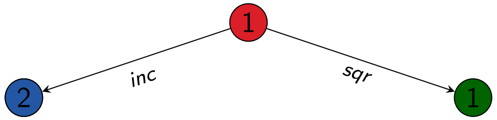
depth limit: 2 generated nodes: 1+3+7



(using a non-recursive version of depth-limited search)

Example

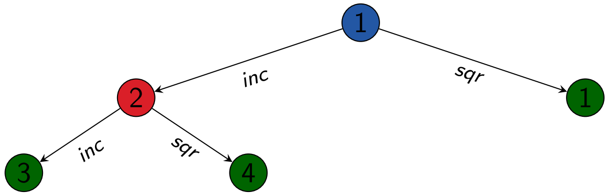
depth limit: 3 generated nodes: 1+3+7+3



(using a non-recursive version of depth-limited search)

Example

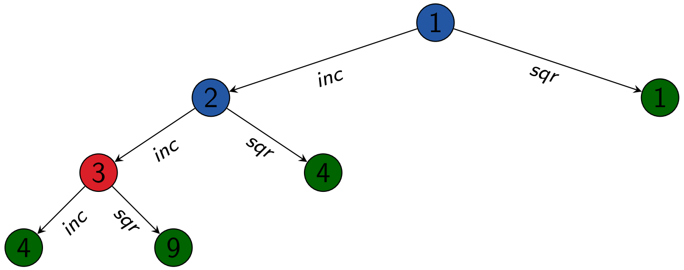
depth limit: 3 generated nodes: 1+3+7+5



(using a non-recursive version of depth-limited search)

Example

depth limit: 3 generated nodes: 1+3+7+7



(using a non-recursive version of depth-limited search)

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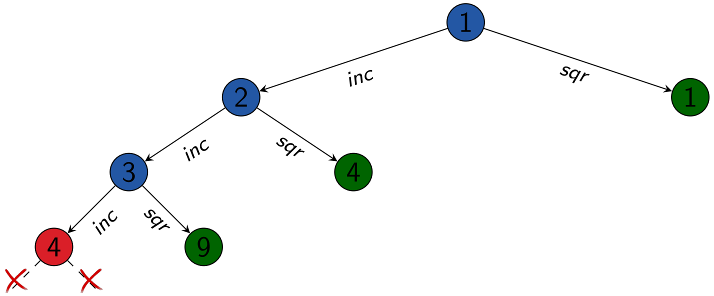
depth limit: 3 generated nodes: 1+3+7+7



(using a non-recursive version of depth-limited search)

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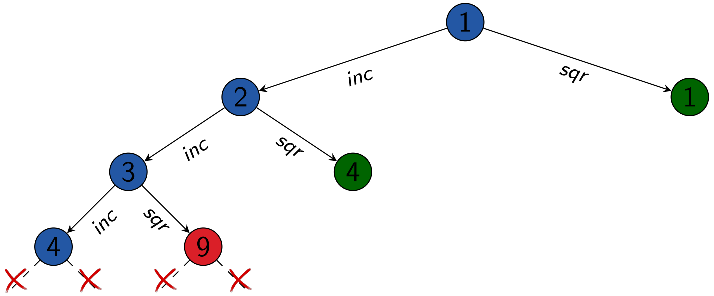
depth limit: 3 generated nodes: 1+3+7+7

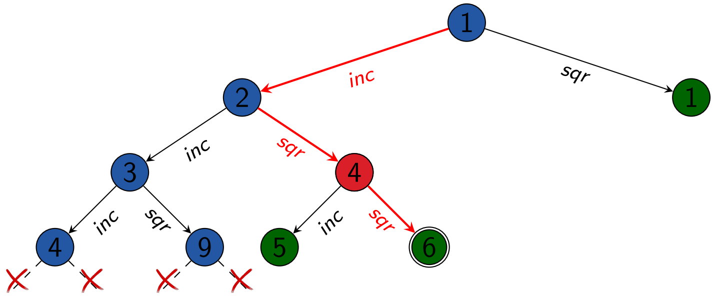


(using a non-recursive version of depth-limited search)

Example

depth limit: 3

generated nodes: 1+3+7+9=20



(using a non-recursive version of depth-limited search)

Iterative Deepening DFS: Properties

combines advantages of breadth-first and depth-first search:

- (almost) like BFS: semi-complete (however, not complete)
- like BFS: optimal if all actions have same cost
- like DFS: only need to store nodes along one path
  $\rightsquigarrow$ space complexity $O(bd)$, where d minimal solution length
- time complexity only slightly higher than BFS
  ($\rightsquigarrow$ next slide)

Iterative Deepening DFS: Complexity Example

time complexity (generated nodes):

breadth-first search	$1 + b + b^2 + \dots + b^{d-1} + b^d$
iterative deepening DFS	$(d+1) + db + (d-1)b^2 + \dots + 2b^{d-1} + 1b^d$

example: $b = 10$, $d = 5$

breadth-first search	$1 + 10 + 100 + 1000 + 10000 + 100000$ $= 111111$
iterative deepening DFS	$6 + 50 + 400 + 3000 + 20000 + 100000$ $= 123456$

for $b = 10$, only 11% more nodes than breadth-first search

Iterative Deepening DFS: Time Complexity

Theorem (time complextive of iterative deepening DFS)

Let b be the branching factor and d be the minimal solution length of the given state space. Let $b \geq 2$.

Then the time complexity of iterative deepening DFS is

$$(d+1) + db + (d-1)b^2 + (d-2)b^3 + \dots + 1b^d = O(b^d)$$

and the memory complexity is

$$O(bd).$$

Iterative Deepening DFS: Evaluation

Iterative Deepening DFS: Evaluation

Iterative Deepening DFS is often the method of choice if

- tree search is adequate (no duplicate elimination necessary),
- all action costs are identical, and
- the solution depth is unknown.

12.3 Summary

Summary

depth-first search: expand nodes in LIFO order

- usually as a tree search
- easy to implement recursively
- very memory-efficient
- can be combined with iterative deepening to combine many of the good aspects of breadth-first and depth-first search

Comparison of Blind Search Algorithms

completeness, optimality, time and space complexity

	search algorithm				
criterion	breadth-first	uniform cost	depth-first	depth-bounded	iterative deepening
complete?	yes*	yes	no	no	semi
optimal?	yes**	yes	no	no	yes**
time	$O(b^d)$	$O(b^{\lfloor c^*/\varepsilon \rfloor + 1})$	$O(b^m)$	$O(b^\ell)$	$O(b^d)$
space	$O(b^d)$	$O(b^{\lfloor c^*/\varepsilon \rfloor + 1})$	$O(bm)$	$O(b\ell)$	$O(bd)$

$b \geq 2$	branching factor
d	minimal solution depth
m	maximal search depth
ℓ	depth bound
c^*	optimal solution cost
$\varepsilon > 0$	minimal action cost

remarks:
* for BFS-Tree: semi-complete
** only with uniform action costs