Foundations of Artificial Intelligence

9. State-Space Search: Tree Search and Graph Search

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State-Space Search: Overview

Chapter overview: state-space search

- 5.–7. Foundations
- 8.–12. Basic Algorithms
  - 8. Data Structures for Search Algorithms
  - 9. Tree Search and Graph Search
  - 10. Breadth-first Search
  - 11. Uniform Cost Search
  - 12. Depth-first Search and Iterative Deepening
- 13.–19. Heuristic Algorithms

Introduction

Search Algorithms

General Search Algorithm

iteratively create a search tree:

- starting with the initial state,
- repeatedly expand a state by generating its successors (which state depends on the used search algorithm)
- stop when a goal state is expanded (sometimes: generated)
- or all reachable states have been considered

Search Algorithms

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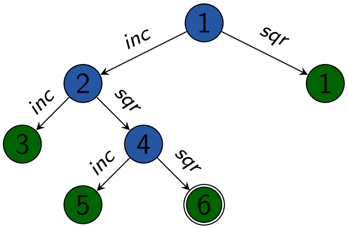
In this chapter, we study two essential classes of search algorithms:

- tree search and
- graph search

(Each class consists of a large number of concrete algorithms.)

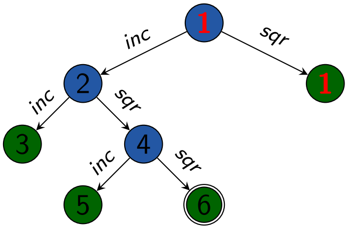
Tree Search

Tree Search: General Idea



- possible paths to be explored organized in a tree (search tree)
- search nodes correspond 1:1 to paths from initial state

Tree Search: General Idea



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- duplicates or transpositions (i.e., multiple nodes with identical state) possible

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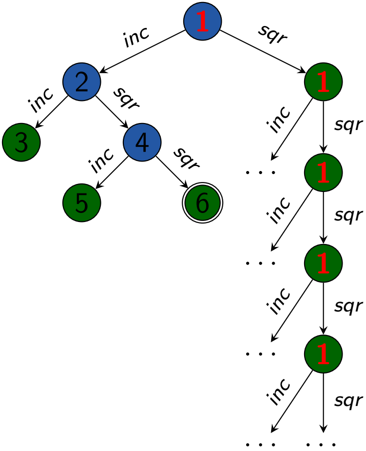


- possible paths to be explored organized in a tree (search tree)
- search nodes correspond 1:1 to paths from initial state
- duplicates or transpositions (i.e., multiple nodes with identical state) possible
- search tree can have unbounded depth

Generic Tree Search Algorithm

Generic Tree Search Algorithm

```
open := new OpenList
open.insert(make_root_node())
while not open.is_empty():
    n := open.pop()
    if is_goal(n.state):
        return extract_path(n)
    for each ⟨a, s'⟩ ∈ succ(n.state):
        n' := make_node(n, a, s')
        open.insert(n')
return unsolvable
```

Generic Tree Search Algorithm: Discussion

discussion:

- generic template for tree search algorithms

$\rightsquigarrow$ for concrete algorithm, we must (at least) decide how to implement the open list

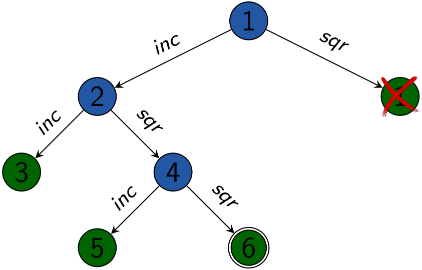
- concrete algorithms often conceptually follow template, (= generate the same search tree), but deviate from details for efficiency reasons

Graph Search

Graph Search

differences to tree search:

- recognize duplicates: when a state is reached on multiple paths, only keep one search node
- search nodes correspond 1:1 to reachable states
- depth of search tree bounded



remarks:

- some graph search algorithms do not immediately eliminate all duplicates (⇝ later)
- one possible reason: find optimal solutions when a path to state s found later is cheaper than one found earlier

Generic Graph Search Algorithm

Generic Graph Search Algorithm

```
open := new OpenList
open.insert(make_root_node())
closed := new ClosedList
while not open.is_empty():
    n := open.pop()
    if closed.lookup(n.state) = none:
        closed.insert(n)
        if is_goal(n.state):
            return extract_path(n)
        for each ⟨a, s′⟩ ∈ succ(n.state):
            n′ := make_node(n, a, s′)
            open.insert(n′)
return unsolvable
```

Generic Graph Search Algorithm: Discussion

discussion:

- same comments as for generic tree search apply
- in "pure" algorithm, closed list does not actually need to store the search nodes
  - sufficient to implement *closed* as set of states
  - advanced algorithms often need access to the nodes, hence we show this more general version here
- some variants perform goal and duplicate tests elsewhere (earlier) ⇝ following chapters

Evaluating Search Algorithms

Criteria: Completeness

four criteria for evaluating search algorithms:

Completeness

Is the algorithm guaranteed to find a solution if one exists?

Does it terminate if no solution exists?

first property: semi-complete
both properties: complete

Criteria: Optimality

four criteria for evaluating search algorithms:

Optimality

Are the solutions returned by the algorithm always optimal?

Criteria: Time Complexity

four criteria for evaluating search algorithms:

Time Complexity

How much time does the algorithm need until termination?

- usually worst case analysis
- usually measured in generated nodes

often a function of the following quantities:

- b: (branching factor) of state space
  (max. number of successors of a state)
- d: search depth
  (length of longest path in generated search tree)

Criteria: Space Complexity

four criteria for evaluating search algorithms:

Space Complexity

How much memory does the algorithm use?

- usually worst case analysis
- usually measured in (concurrently) stored nodes

often a function of the following quantities:

- b: (branching factor) of state space
  (max. number of successors of a state)
- d: search depth
  (length of longest path in generated search tree)

Analyzing the Generic Search Algorithms

Generic Tree Search Algorithm

- Is it complete? Is it semi-complete?
- Is it optimal?
- What is its worst-case time complexity?
- What is its worst-case space complexity?

Generic Graph Search Algorithm

- Is it complete? Is it semi-complete?
- Is it optimal?
- What is its worst-case time complexity?
- What is its worst-case space complexity?

Summary

Summary (1)

tree search:

- search nodes correspond 1:1 to paths from initial state

graph search:

- search nodes correspond 1:1 to reachable states

$\rightsquigarrow$ duplicate elimination

generic methods with many possible variants

Summary (2)

evaluating search algorithms:

- completeness and semi-completeness
- optimality
- time complexity and space complexity