## Foundations of Artificial Intelligence

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# Exercise Sheet 8

**Due: April 30, 2023**

**Important: for submission, consult the rules at the end of the exercise. Non-adherence to the rules will lead to your submission not being corrected.**

**Exercise 8.1** (3 marks)

In the lecture, we have shown that the resolution method can be used for reasoning by a reduction to testing unsatisfiability. In this way, use the resolution method to show that $\psi = \neg A \wedge (B \vee C)$ follows logically from $\varphi = \{\{\neg A, \neg B, E\}, \{B, \neg C\}, \{C, D, E\}, \{B, \neg D\}, \{\neg E\}\}$, i.e., $\varphi \models \psi$. Provide each application of the resolution rule and explain the result obtained from resulution. Compare the number of required resolution steps with the number of entries that would have been necessary to show the same result with a truth table.

**Exercise 8.2** (2+1 marks)

(a) Perfom DPLL on

$$\{\{\neg A, C\}, \{A, B, E\}, \{\neg B, \neg D\}, \{\neg B, \neg E\}, \{\neg C, D, E\}, \{B, \neg C, E\}, \{B, \neg E\}\}$$

choosing variables alphabetically (also for unit propagation if there are several choices) and always assigning **T** first for splitting. Provide the application of all rules and the resulting tree as done in the lecture (chapter 31). What is the result of the algorithm?

(b) Would always assigning **F** first result in a smaller tree?

**Exercise 8.3** (1+1 mark)

(a) Consider the following idea to solve SAT efficiently: "Given a CNF formula $\varphi$, transform it into an equivalent horn formula $\varphi_h$. Then use DPLL to decide $\varphi_h$ in polynomial time." Where is the flaw in the idea?

(b) If the above idea worked, which important problem would we have solved?

**Exercise 8.4** (1+1 mark)

(a) In the lecture (chapter 32), we saw that 3-SAT (clauses of fixed size 3) has a phase transition at a ratio of $\approx 4.3$ clauses per variable. We want to investigate the phase transition for SAT with a fixed clause size of 10 variables. Would the phase transition be more to the left (i.e., $< 4.3$ clauses per variable) or more to the right (i.e., $> 4.3$ clauses per variable) compared to the phase transition for 3-SAT? Justify your answer.

(b) Considering the phase transition for 3-SAT and your answer to (a), why do you think the formulas by Goldberg (1979) with $n$ variables and $k$ clauses are in general easy to solve? You may assume $n \geq 10$ and $k < n$.

**Submission rules:**

- Exercise sheets must be submitted in groups of two students. Please submit a single copy of the exercises per group (only one member of the group does the submission).

- Create a single PDF file (ending .pdf) for all non-programming exercises. Use a file name that does not contain any spaces or special characters other than the underscore "_". If you want to submit handwritten solutions, include their scans in the single PDF. Make sure it is in a reasonable resolution so that it is readable, but ensure at the same time that the PDF size is not astronomically large. Put the names of all group members on top of the first page. Either use page numbers on all pages or put your names on each page. Make sure your PDF has size A4 (fits the page size if printed on A4).
- For programming exercises, only create those code textfiles required by the exercise. Put your names in a comment on top of each file. Make sure your code compiles and test it. Code that does not compile or which we cannot successfully execute will not be graded.
- For the submission: if the exercise sheet does not include programming exercises, simply upload the single PDF. If the exercise sheet includes programming exercises, upload a ZIP file (ending .zip, .tar.gz or .tgz; *not* .rar or anything else) containing the single PDF and the code textfile(s) and nothing else. Do not use directories within the ZIP, i.e., zip the files directly.
- Do not upload several versions to ADAM, i.e., if you need to resubmit, use the same file name again so that the previous submission is overwritten.