

Foundations of Artificial Intelligence

26. Constraint Satisfaction Problems: Path Consistency

Malte Helmert

University of Basel

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Constraint Satisfaction Problems: Overview

Chapter overview: constraint satisfaction problems:

- 22.–23. Introduction
- 24.–26. Basic Algorithms
 - 24. Backtracking
 - 25. Arc Consistency
 - 26. Path Consistency
- 27.–28. Problem Structure

Beyond Arc Consistency

Beyond Arc Consistency: Path Consistency

idea of arc consistency:

- For every assignment to a variable u there must be a suitable assignment to every other variable v .
- If not: remove values of u for which no suitable “partner” assignment to v exists.

⇒ tighter **unary constraint** on u

This idea can be extended to three variables (**path consistency**):

- For every joint assignment to variables u, v there must be a suitable assignment to every third variable w .
- If not: remove pairs of values of u and v for which no suitable “partner” assignment to w exists.

⇒ tighter **binary constraint** on u and v

German: Pfadkonsistenz

Beyond Arc Consistency: i -Consistency

general concept of i -consistency for $i \geq 2$:

- For every joint assignment to variables $v_1, \dots, v_{i-1}$ there must be a suitable assignment to every i -th variable v_i .
 - If not: remove value tuples of $v_1, \dots, v_{i-1}$ for which no suitable “partner” assignment for v_i exists.
- ↪ tighter $(i-1)$ -ary constraint on $v_1, \dots, v_{i-1}$
- 2-consistency = arc consistency
 - 3-consistency = path consistency (*)

We do not consider general i -consistency further as larger values than $i = 3$ are rarely used and we restrict ourselves to binary constraints in this course.

- (*) usual definitions of 3-consistency vs. path consistency differ when ternary constraints are allowed

Path Consistency

Path Consistency: Definition

Definition (path consistent)

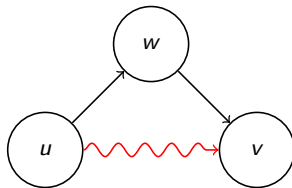
Let $\mathcal{C} = \langle V, \text{dom}, (R_{uv}) \rangle$ be a constraint network.

- a Two different variables $u, v \in V$ are **path consistent** with respect to a third variable $w \in V$ if for all values $d_u \in \text{dom}(u)$, $d_v \in \text{dom}(v)$ with $\langle d_u, d_v \rangle \in R_{uv}$ there is a value $d_w \in \text{dom}(w)$ with $\langle d_u, d_w \rangle \in R_{uw}$ and $\langle d_v, d_w \rangle \in R_{vw}$.
- b The constraint network $\mathcal{C}$ is **path consistent** if for any three variables u, v, w , the variables u and v are path consistent with respect to w .

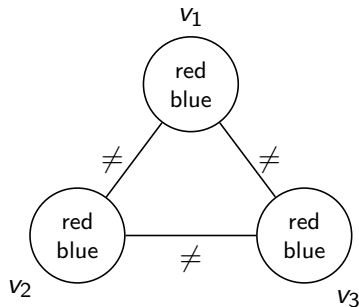
Path Consistency: Remarks

remarks:

- Even if the constraint R_{uv} is trivial, path consistency can infer nontrivial constraints between u and v .
- name “path consistency”:
path $u \rightarrow w \rightarrow v$ leads to new information on $u \rightarrow v$



Path Consistency: Example



arc consistent, but not path consistent

Processing Variable Triples: revise-3

analogous to [revise](#) for arc consistency:

function revise-3($\mathcal{C}, u, v, w$):

$\langle V, \text{dom}, (R_{uv}) \rangle := \mathcal{C}$

for each $\langle d_u, d_v \rangle \in R_{uv}$:

if there is no $d_w \in \text{dom}(w)$ with

$\langle d_u, d_w \rangle \in R_{uw}$ **and** $\langle d_v, d_w \rangle \in R_{vw}$:

remove $\langle d_u, d_v \rangle$ from R_{uv}

input: constraint network $\mathcal{C}$ and three variables u, v, w of $\mathcal{C}$

effect: u, v path consistent with respect to w .

All violating pairs are removed from R_{uv} .

time complexity: $O(k^3)$ where k is maximal domain size

Enforcing Path Consistency: PC-2

analogous to [AC-3](#) for arc consistency:

function PC-2($\mathcal{C}$):

$\langle V, \text{dom}, (R_{uv}) \rangle := \mathcal{C}$

$queue := \emptyset$

for each set of two variables $\{u, v\}$:

for each $w \in V \setminus \{u, v\}$:

 insert $\langle u, v, w \rangle$ into $queue$

while $queue \neq \emptyset$:

 remove any element $\langle u, v, w \rangle$ from $queue$

 revise-3($\mathcal{C}, u, v, w$)

if R_{uv} changed in the call to revise-3:

for each $w' \in V \setminus \{u, v\}$:

 insert $\langle w', u, v \rangle$ into $queue$

 insert $\langle w', v, u \rangle$ into $queue$

PC-2: Discussion

The comments for AC-3 hold analogously.

- PC-2 enforces path consistency
- **proof idea:** invariant of the **while** loop:
if $\langle u, v, w \rangle \notin \text{queue}$, then u, v path consistent
with respect to w
- time complexity $O(n^3 k^5)$ for n variables and maximal domain
size k (**Why?**)

Summary

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- generalization of
 arc consistency (considers pairs of variables)
 to path consistency (considers triples of variables)
 and i -consistency (considers i -tuples of variables)
- arc consistency tightens unary constraints
- path consistency tightens binary constraints
- i -consistency tightens $(i - 1)$ -ary constraints
- higher levels of consistency more powerful
 but more expensive than arc consistency