

Foundations of Artificial Intelligence

16. State-Space Search: Greedy BFS, A*, Weighted A*

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16.1 Introduction

16.2 Greedy Best-first Search

16.3 A*

16.4 Weighted A*

16.5 Summary

State-Space Search: Overview

Chapter overview: state-space search

- ▶ 5.–7. Foundations
- ▶ 8.–12. Basic Algorithms
- ▶ 13.–19. Heuristic Algorithms
 - ▶ 13. Heuristics
 - ▶ 14. Analysis of Heuristics
 - ▶ 15. Best-first Graph Search
 - ▶ 16. Greedy Best-first Search, A*, Weighted A*
 - ▶ 17. IDA*
 - ▶ 18. Properties of A*, Part I
 - ▶ 19. Properties of A*, Part II

16.1 Introduction

What Is It About?

In this chapter we study last chapter's algorithms in more detail:

- ▶ greedy best-first search
- ▶ A*
- ▶ weighted A*

16.2 Greedy Best-first Search

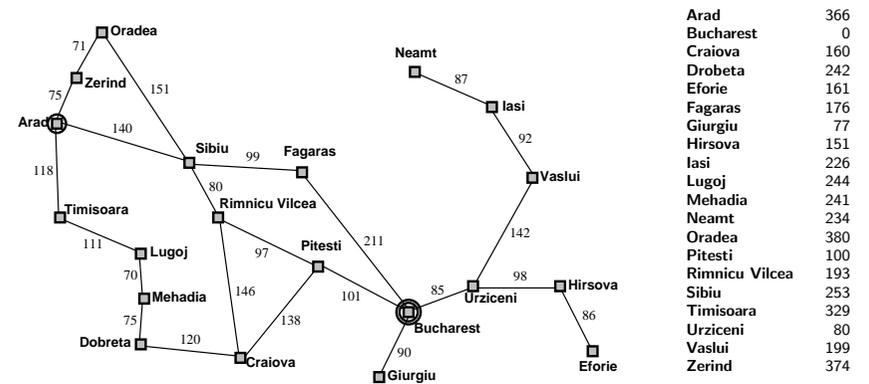
Greedy Best-first Search

Greedy Best-first Search

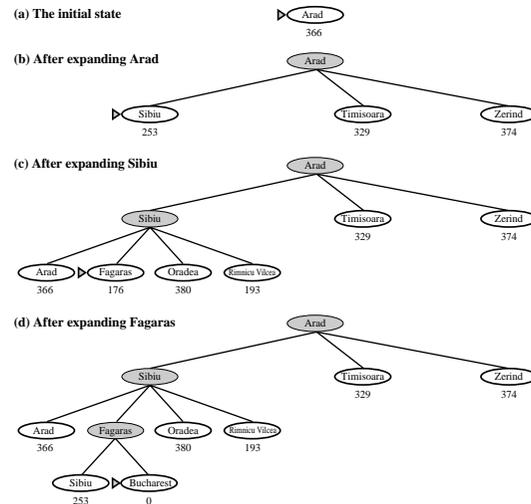
only consider the heuristic: $f(n) = h(n.state)$

Note: usually **without reopening** (for reasons of efficiency)

Example: Greedy Best-first Search for Route Planning



Example: Greedy Best-first Search for Route Planning



Greedy Best-first Search: Properties

- ▶ **complete** with **safe** heuristics (like all variants of best-first graph search)
- ▶ **suboptimal**: solutions can be **arbitrarily bad**
- ▶ often **very fast**: one of the fastest search algorithms in practice
- ▶ monotonic transformations of h (e.g. scaling, additive constants) do not affect behaviour (**Why is this interesting?**)

16.3 A*

A*

A* combine greedy best-first search with uniform cost search:

$$f(n) = g(n) + h(n.state)$$

- ▶ **trade-off** between path cost and proximity to goal
- ▶ $f(n)$ estimates overall cost of cheapest solution **from initial state via n to the goal**

A*: Citations

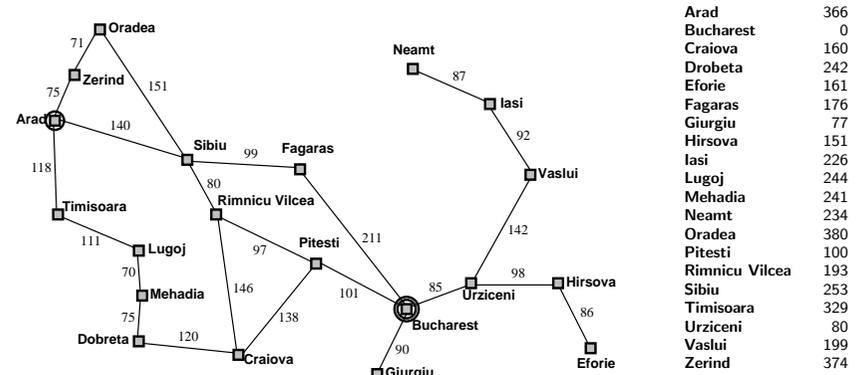
Search results for **hart nilsson raphael** (Scholar, About 11'800 results (0.06 sec))

A formal basis for the heuristic determination of minimum cost paths [PDF] [ieeep.org](#)
[PE Hart](#), [NJ Nilsson](#), [B Raphael](#) - IEEE transactions on Systems ..., 1968 - [ieeexplore.ieee.org](#)
 Although the problem of determining the minimum cost path through a graph arises naturally in a number of interesting applications, there has been no underlying theory to guide the ...
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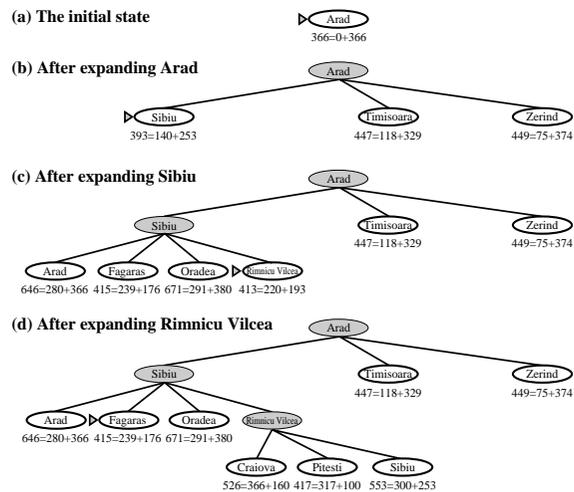
Correction to "a formal basis for the heuristic determination of minimum cost paths" [PDF] [Uni Basel](#)
[PE Hart](#), [NJ Nilsson](#), [B Raphael](#) - ACM SIGART Bulletin, 1972 - [dl.acm.org](#)
 Our paper on the use of heuristic information in graph searching defined a path-finding algorithm, A*, and proved that it had two important properties. In the notation of the paper, we ...
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Shakey: from conception to history [PDF] [aaai.org](#)
[B Kuipers](#), [EA Feigenbaum](#), [PE Hart](#), [NJ Nilsson](#) - Ai Magazine, 2017 - [ojs.aaai.org](#)
 ... One, called A* by its creators, Peter Hart, Nils Nilsson, and Bertram Raphael, had two very desirable properties. It can be rigorously proved that (a) it always finds the shortest path, and (...)
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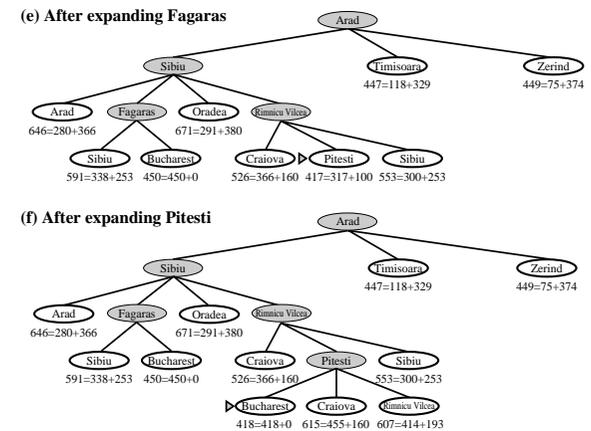
Example: A* for Route Planning



Example: A* for Route Planning



Example: A* for Route Planning



A*: Properties

- ▶ **complete** with **safe** heuristics
(like all variants of best-first graph search)
- ▶ **with reopening**: **optimal** with **admissible** heuristics
- ▶ **without reopening**: **optimal** with heuristics
that are **admissible** and **consistent**

↪ proofs: Chapters 18 and 19

A*: Implementation Aspects

some practical remarks on implementing A*:

- ▶ **common bug**: reopening not implemented
although heuristic is not consistent
- ▶ **common bug**: duplicate test “too early”
(upon generation of search nodes)
- ▶ **common bug**: goal test “too early”
(upon generation of search nodes)
- ▶ all these bugs lead to loss of optimality
and can remain undetected for a long time

16.4 Weighted A*

Weighted A*

Weighted A*

A* with more heavily weighted heuristic:

$$f(n) = g(n) + w \cdot h(n.state),$$

where **weight** $w \in \mathbb{R}_0^+$ with $w \geq 1$ is a freely choosable parameter

Note: $w < 1$ is conceivable, but usually not a good idea
(Why not?)

Weighted A*: Properties

weight parameter controls “greediness” of search:

- ▶ $w = 0$: like uniform cost search
- ▶ $w = 1$: like A*
- ▶ $w \rightarrow \infty$: like greedy best-first search

with $w \geq 1$ properties analogous to A*:

- ▶ h admissible:
found solution guaranteed to be at most w times as expensive as optimum when reopening is used
- ▶ h admissible and consistent:
found solution guaranteed to be at most w times as expensive as optimum; no reopening needed

(without proof)

16.5 Summary

Summary

best-first graph search with evaluation function f :

- ▶ $f = h$: **greedy best-first search**
suboptimal, often very fast
- ▶ $f = g + h$: **A***
optimal if h admissible and consistent
or if h admissible and **reopening** is used
- ▶ $f = g + w \cdot h$: **weighted A***
for $w \geq 1$ suboptimality factor at most w
under same conditions as for optimality of A*