# Foundations of Artificial Intelligence

## 11. State-Space Search: Uniform Cost Search

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March 16, 2022









## State-Space Search: Overview

Chapter overview: state-space search

- 5.–7. Foundations
- 8.–12. Basic Algorithms
  - 8. Data Structures for Search Algorithms
  - 9. Tree Search and Graph Search
  - 10. Breadth-first Search
  - 11. Uniform Cost Search
  - 12. Depth-first Search and Iterative Deepening
- 13.–19. Heuristic Algorithms

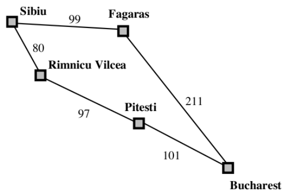




# 11.1 Introduction

# Uniform Cost Search

- breadth-first search optimal if all action costs equal
- otherwise no optimality guarantee ⇝ example:




remedy: uniform cost search

- always expand a node with minimal path cost ($n$.path_cost a.k.a. $g(n)$)
- implementation: priority queue (min-heap) for open list





# 11.2 Algorithm





# Reminder: Generic Graph Search Algorithm

reminder from Chapter 9:

**Generic Graph Search**

```
open := new OpenList
open.insert(make_root_node())
closed := new ClosedList
while not open.is_empty():
    n := open.pop()
    if closed.lookup(n.state) = none:
        closed.insert(n)
        if is_goal(n.state):
            return extract_path(n)
        for each ⟨a, s′⟩ ∈ succ(n.state):
            n′ := make_node(n, a, s′)
            open.insert(n′)
return unsolvable
```





# Uniform Cost Search

**Uniform Cost Search**

```
open := new MinHeap ordered by g
open.insert(make_root_node())
closed := new HashSet
while not open.is_empty():
    n := open.pop_min()
    if n.state ∉ closed:
        closed.insert(n.state)
        if is_goal(n.state):
            return extract_path(n)
        for each ⟨a, s′⟩ ∈ succ(n.state):
            n′ := make_node(n, a, s′)
            open.insert(n′)
return unsolvable
```

## Uniform Cost Search: Discussion

Adapting generic graph search to uniform cost search:

- here, early goal tests/early updates of the closed list not a good idea. (Why not?)
- as in BFS-Graph, a set is sufficient for the closed list
- a tree search variant is possible, but rare: has the same disadvantages as BFS-Tree and in general not even semi-complete (Why not?)

Remarks:

- identical to Dijkstra's algorithm for shortest paths
- for both: variants with/without delayed duplicate elimination





## Uniform Cost Search: Improvements

possible improvements:

- if action costs are small integers, bucket heaps often more efficient
- additional early duplicate tests for generated nodes can reduce memory requirements
  - can be beneficial or detrimental for runtime
  - must be careful to keep shorter path to duplicate state





# 11.3 Properties





## Completeness and Optimality

properties of uniform cost search:

- uniform cost search is complete (Why?)
- uniform cost search is optimal (Why?)

## Time and Space Complexity

properties of uniform cost search:

- Time complexity depends on distribution of action costs (no simple and accurate bounds).
  - Let $\varepsilon := \min_{a \in A} cost(a)$ and consider the case $\varepsilon > 0$.
  - Let $c^*$ be the optimal solution cost.
  - Let $b$ be the branching factor and consider the case $b \geq 2$.
  - Then the time complexity is at most $O(b^{\lfloor c^*/\varepsilon \rfloor + 1})$. (Why?)
  - often a very weak upper bound
- space complexity = time complexity





# 11.4 Summary





## Summary

uniform cost search: expand nodes in order of ascending path costs

- usually as a graph search
- then corresponds to Dijkstra's algorithm
- complete and optimal