Foundations of Artificial Intelligence

M. Helmert
S. Eriksson
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University of Basel
Computer Science

Exercise Sheet 8

Due: April 24, 2022

Important: for submission, consult the rules at the end of the exercise. Non-adherence to the rules will lead to your submission not being corrected.

Exercise 8.1 (2 marks)

Consider the arc consistent constraint network $\langle V, \mathrm{dom}, (R_{uv})\rangle$ with

- $V = \{v_1, v_2, v_3\}$,
- $\mathrm{dom}(v_1) = \{1, \ldots, 5\}$,
  $\mathrm{dom}(v_2) = \{5, \ldots, 9\}$,
  $\mathrm{dom}(v_3) = \{-10, \ldots, -2\}$,
- $R_{v_1,v_2} = \{(x, y) \mid x + y = 10\}$,
  $R_{v_2,v_3} = \{(x, y) \mid x + y \leq 3\}$.

Apply algorithm PC-2 from slide 12 of chapter 26 (print version) to find an equivalent path consistent network. Specify for each iteration the content of the queue as well as the updated constraint if it changed. After the algorithm terminates, specify the full constraint network you obtained.

Exercise 8.2 (3 marks)

Let $\mathcal{C}$ be a solvable constraint network with an acyclic constraint graph. Show that the application of the algorithm for trees as constraint graphs (slide 13 of chapter 27 of the print version of the lecture slides) leads to a solution for $\mathcal{C}$ without needing to backtrack.
Hint: First use structural induction to show that after step 3 the domain of every node v contains only values for which a consistent assignment to all variables in the subtree rooted at v can be found. Then use this to show that backtracking with forward checking will always pick a value that can be extended to a solution. Also consider that finding a solution requires that no domain ever becomes empty.

Exercise 8.3 (2+1 marks)

Consider the constraint network that is given by the graph coloring problem of a graph $G = \langle V, E\rangle$. The set of vertices V contains a vertex for each Swiss canton, and E is such that two vertices v and v' are connected iff the cantons v and v' share a border. A description of G can be downloaded from the website or on ADAM and you can vizualize it with graphviz:

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dot -T pdf -o cantons.pdf cantons.dot
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(a) Provide a cutset $V' \subseteq V$ for G that is as small as possible (it is not necessary to provide an explanation how you have found V'). As a reminder, a cutset of a graph is defined as a set of vertices that is such that the induced subgraph that is obtained by removing these vertices results in an acyclic graph.

Note: You get 2 marks for your solution if your cutset is optimal, 1 mark if your cutset contains exactly one more vertex than an optimal cutset and 0 marks otherwise.

(b) Assume we are interested in coloring G with 4 colors. Provide a worst-case runtime estimate of the algorithm based on cutset conditioning if your cutset from the first part of this exercise is used (i.e., compute an upper bound for the number of considered assignments). Compare your result to the estimated runtime if no cutset is used.

Exercise 8.4 (1+1 mark)

We define an extension to the syntax of propositional logic by introducing biimplications. Given propositional logic formulas ψ and η, $\psi \leftrightarrow \eta$ is a propositional formula with the following semantics: for all interpretations I we have $I \models \psi \leftrightarrow \eta$ iff $I \models \psi \rightarrow \eta$ and $I \models \eta \rightarrow \psi$.
We furthermore define formula $\varphi = ((A \vee B) \rightarrow C) \wedge \neg((C \vee B) \leftrightarrow A)$ and interpretation $I = \{A \mapsto \mathbf{F}, B \mapsto \mathbf{T}, C \mapsto \mathbf{T}\}$

(a) Show that $I \models \varphi$ using the definition of $\models$.

(b) Show with a truth table whether or not φ is valid, satisfiable, falsifiable and unsatisfiable. Justify your answer in one to two sentences.

Submission rules:

- Exercise sheets must be submitted in groups of two students. Please submit a single copy of the exercises per group (only one member of the group does the submission).
- Create a single PDF file (ending .pdf) for all non-programming exercises. Use a file name that does not contain any spaces or special characters other than the underscore "_". If you want to submit handwritten solutions, include their scans in the single PDF. Make sure it is in a reasonable resolution so that it is readable, but ensure at the same time that the PDF size is not astronomically large. Put the names of all group members on top of the first page. Either use page numbers on all pages or put your names on each page. Make sure your PDF has size A4 (fits the page size if printed on A4).
- For programming exercises, only create those code textfiles required by the exercise. Put your names in a comment on top of each file. Make sure your code compiles and test it. Code that does not compile or which we cannot successfully execute will not be graded.
- For the submission: if the exercise sheet does not include programming exercises, simply upload the single PDF. If the exercise sheet includes programming exercises, upload a ZIP file (ending .zip, .tar.gz or .tgz; *not* .rar or anything else) containing the single PDF and the code textfile(s) and nothing else. Do not use directories within the ZIP, i.e., zip the files directly.
- Do not upload several versions to ADAM, i.e., if you need to resubmit, use the same file name again so that the previous submission is overwritten.