Foundations of Artificial Intelligence

41. Board Games: Minimax Search and Evaluation Functions

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Board Games: Overview

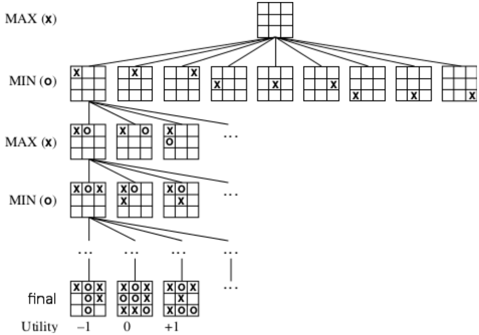
chapter overview:

Minimax Search

Terminology for Two-Player Games

- Players are traditionally called MAX and MIN.
- Our objective is to compute moves for MAX (MIN is the opponent).
- MAX tries to maximize its utility (given by the utility function u) in the reached terminal position.
- MIN tries to minimize u (which in turn maximizes MINs utility).

Example: Tic-Tac-Toe

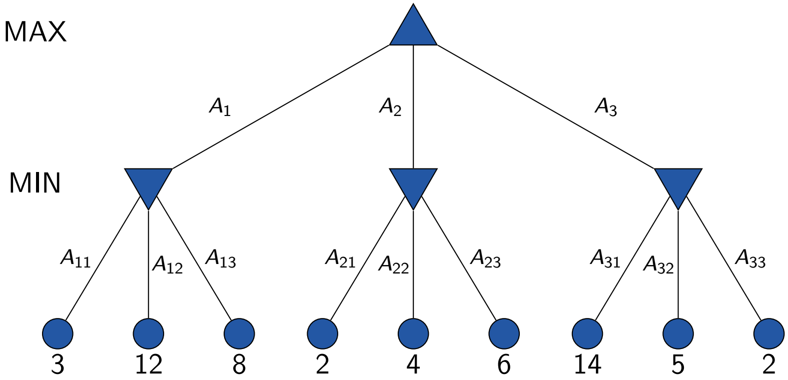


- game tree with player's turn (MAX/MIN) marked on the left
- last row: terminal positions with utility
- size of game tree?

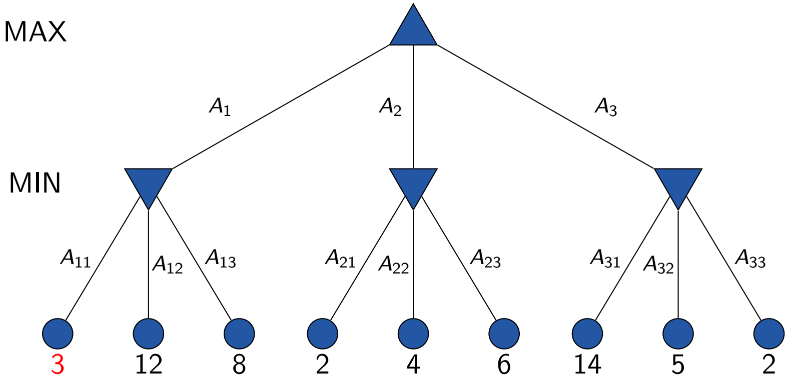
Minimax: Computation

1. depth-first search through game tree
2. Apply utility function in terminal position.
3. Compute utility value of inner nodes
   from below to above through the tree:
   - MIN's turn: utility is minimum of utility values of children
   - MAX's turn: utility is maximum of utility values of children
4. move selection for MAX in root:
   choose a move that maximizes the computed utility value
   (minimax decision)

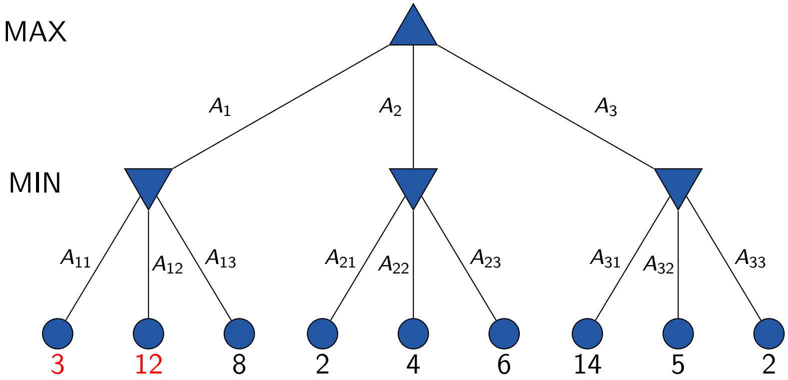
Minimax: Example



Minimax: Example



Minimax: Example



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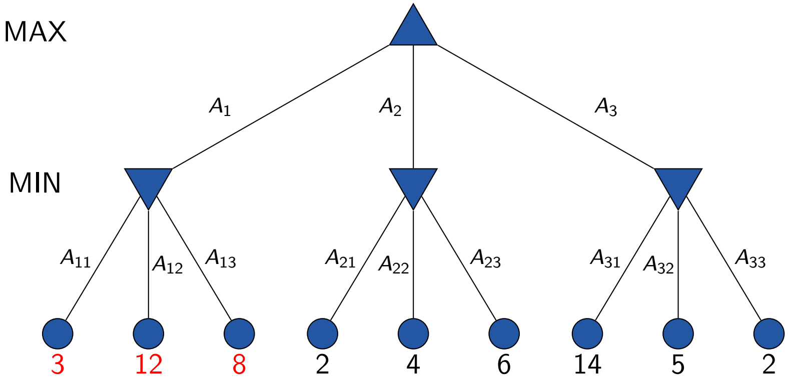

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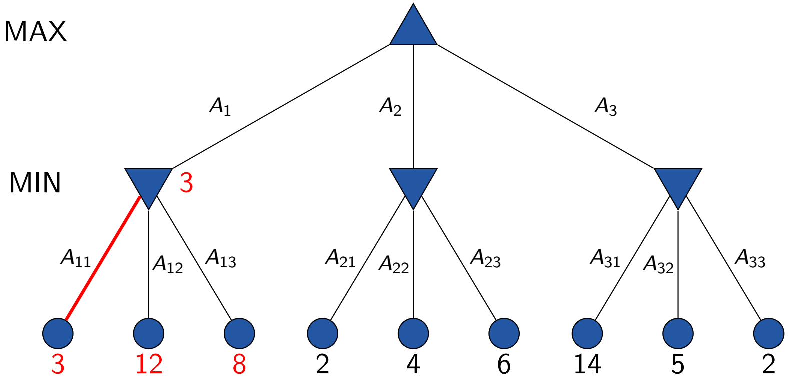

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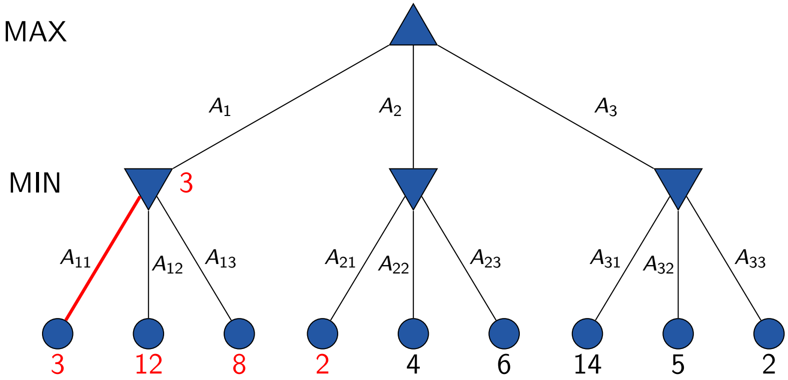

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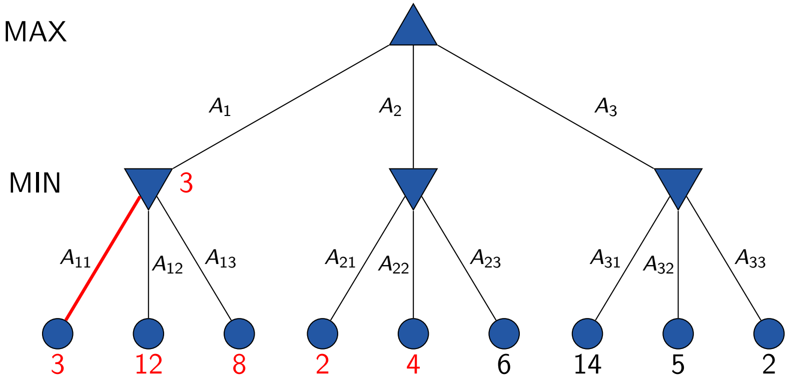

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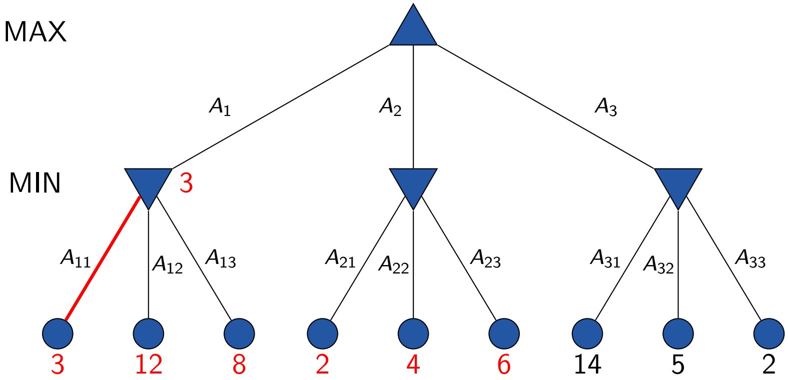

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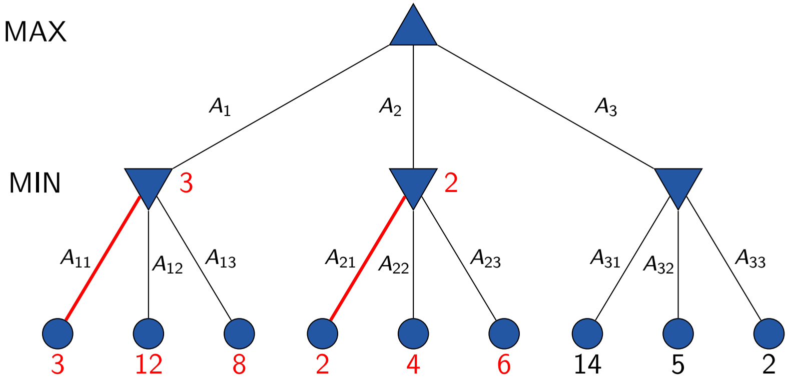

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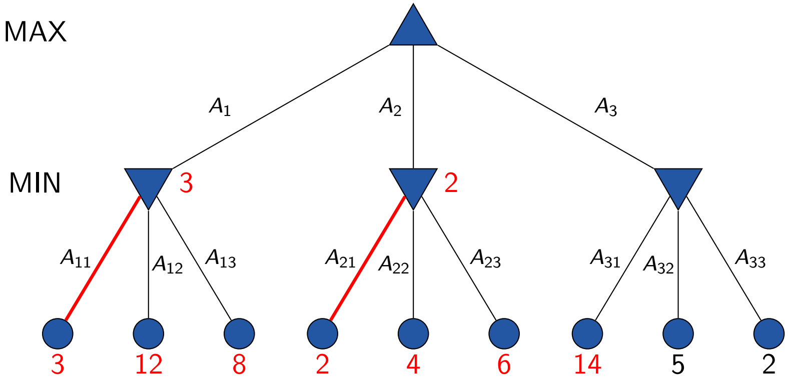

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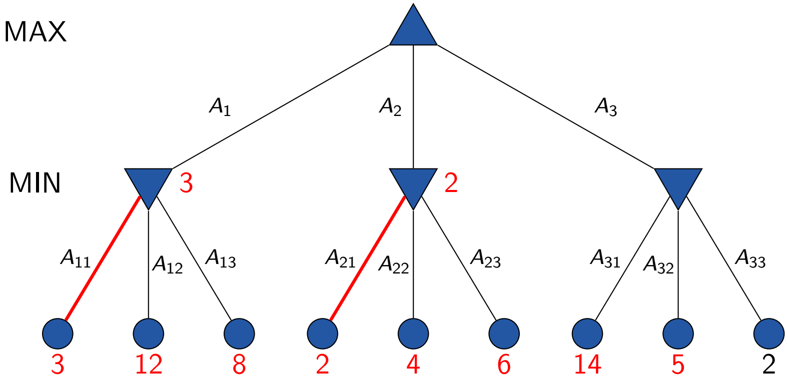

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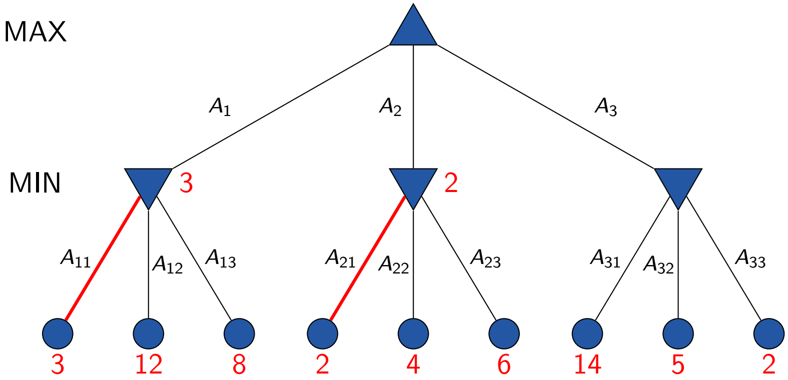

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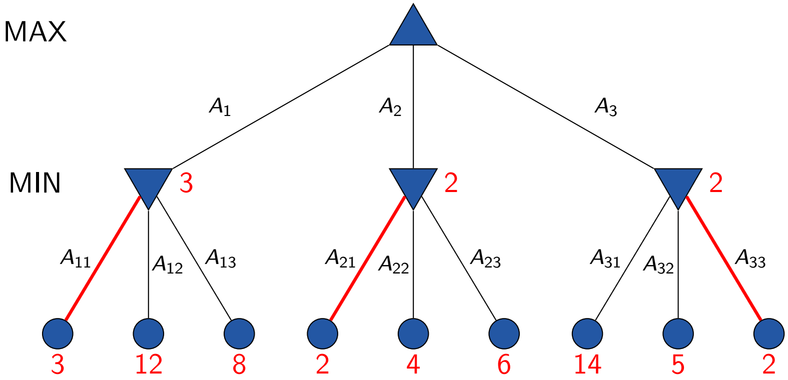

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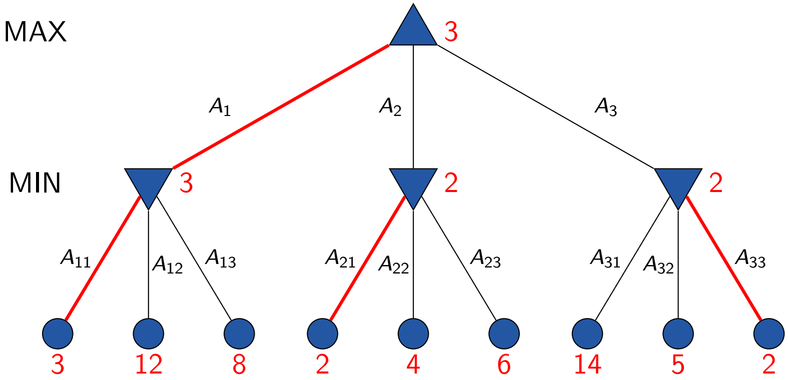

Minimax: Discussion

- Minimax is the simplest (decent) search algorithm for games
- Yields optimal strategy* (in the game-theoretic sense, i.e., under the assumption that the opponent plays perfectly), but is too time-consuming for complex games.
- We obtain at least the utility value computed for the root, no matter how the opponent plays.
- In case the opponent plays perfectly, we obtain exactly that value.

(*) for games where no cycles occur; otherwise things get more complicated (because the tree will have infinite size in this case).

Minimax: Pseudo-Code

function minimax(p)

if p is terminal position:
    return $\langle u(p), \textbf{none}\rangle$
$best\_move :=$ **none**
if $player(p) = \text{MAX}$:
    $v := -\infty$
else:
    $v := \infty$
for each $\langle move, p'\rangle \in \text{succ}(p)$:
    $\langle v', best\_move'\rangle := minimax(p')$
    if $(player(p) = \text{MAX}$ **and** $v' > v)$ **or**
      $(player(p) = \text{MIN}$ **and** $v' < v)$:
        $v := v'$
        $best\_move := move$
return $\langle v, best\_move\rangle$

Minimax

What if the size of the game tree is too big for minimax?
$\rightsquigarrow$ approximation by evaluation function

Evaluation Functions

Evaluation Functions

- problem: game tree too big
- idea: search only up to certain depth
- depth reached: estimate the utility according to heuristic criteria (as if terminal position had been reached)

Example (evaluation function in chess)

- material: pawn 1, knight 3, bishop 3, rook 5, queen 9
  positive sign for pieces of MAX, negative sign for MIN
- pawn structure, mobility, ...

rule of thumb: advantage of 3 points $\rightsquigarrow$ clear winning position

Accurate evaluation functions are crucial!

- High values should relate to high "winning chances" in order to make the overall approach work.
- At the same time, the evaluation should be efficiently computable in order to be able to search deeply.

Linear Evaluation Functions

Usually weighted linear functions are applied:

$$w_1 f_1 + w_2 f_2 + \cdots + w_n f_n$$

where w_i are weights, and f_i are features.

- assumes that feature contributions are mutually independent (usually wrong but acceptable assumption)
- allows for efficient incremental computation if most features are unaffected by most moves
- Weights can be learned automatically.
- Features are (usually) provided by human experts.

The idea dates back at least to Lolli (1763).

How Deep Shall We Search?

- objective: search as deeply as possible within a given time
- problem: search time difficult to predict
- solution: iterative deepening
  - sequence of searches of increasing depth
  - time expires: return result of previously finished search
- refinement: search depth not uniform, but deeper in "turbulent" positions (i.e., with strong fluctuations of the evaluation function) $\rightsquigarrow$ quiescence search
  - example chess: deepen the search after capturing moves

Summary

Summary

- Minimax is a tree search algorithm that plays perfectly (in the game-theoretic sense), but its complexity is $O(b^d)$ (branching factor b, search depth d).
- In practice, the search depth must be limited
  $\rightsquigarrow$ apply evaluation functions
  (usually linear combinations of features).