Foundations of Artificial Intelligence

39. Automated Planning: Landmark Heuristics

Malte Helmert

University of Basel

May 10, 2021

Automated Planning: Overview

Chapter overview: automated planning

- 33. Introduction
- 34. Planning Formalisms
- 35.–36. Planning Heuristics: Delete Relaxation
- 37 Planning Heuristics: Abstraction
- 38.–39. Planning Heuristics: Landmarks
  - 38. Landmarks
  - 39. Landmark Heuristics

Formalism and Example

- As in the previous chapter, we consider delete-free planning tasks in normal form.
- We continue with the example from the previous chapter:

Example

actions:

- $a_1 = i \xrightarrow{3} x, y$
- $a_2 = i \xrightarrow{4} x, z$
- $a_3 = i \xrightarrow{5} y, z$
- $a_4 = x, y, z \xrightarrow{0} g$

landmark examples:

- $A = \{a_4\}$ (cost $= 0$)
- $B = \{a_1, a_2\}$ (cost $= 3$)
- $C = \{a_1, a_3\}$ (cost $= 3$)
- $D = \{a_2, a_3\}$ (cost $= 4$)

Finding Landmarks

Justification Graphs

Definition (precondition choice function)

A precondition choice function (pcf) $P : A \to V$ maps every action to one of its preconditions.

Definition (justification graph)

The justification graph for pcf P is a directed graph with labeled arcs.

- vertices: the variables V
- arcs: $P(a) \xrightarrow{a} e$ for every action a, every effect $e \in add(a)$

Example: Justification Graph

Example

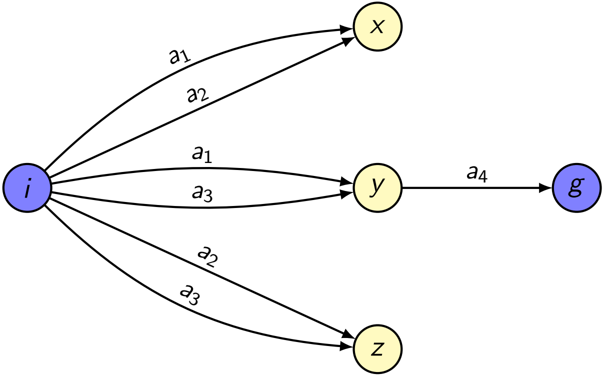
pcf P: $P(a_1) = P(a_2) = P(a_3) = i,\ P(a_4) = y$

$a_1 = i \xrightarrow{3} x, y$

$a_2 = i \xrightarrow{4} x, z$

$a_3 = i \xrightarrow{5} y, z$

$a_4 = x, y, z \xrightarrow{0} g$



Cuts

Definition (cut)

A cut in a justification graph is a subset C of its arcs such that all paths from i to g contain an arc in C.

Cuts

Definition (cut)

A cut in a justification graph is a subset C of its arcs such that all paths from i to g contain an arc in C.

Proposition (cuts are landmarks)

Let C be a cut in a justification graph for an arbitrary pcf.

Then the arc labels for C form a landmark.

Example: Cuts in Justification Graphs

Example

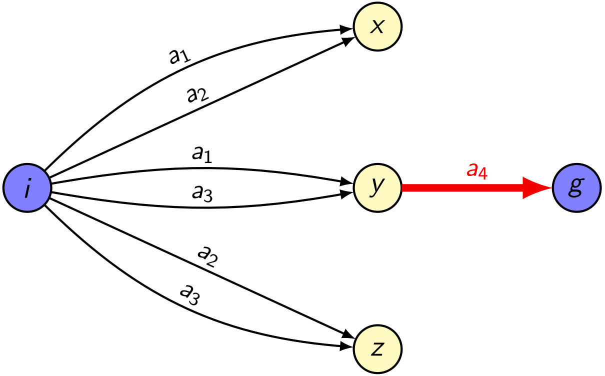
landmark $A = \{a_4\}$ (cost $= 0$)

$a_1 = i \xrightarrow{3} x, y$

$a_2 = i \xrightarrow{4} x, z$

$a_3 = i \xrightarrow{5} y, z$

$a_4 = x, y, z \xrightarrow{0} g$



Example: Cuts in Justification Graphs

Example

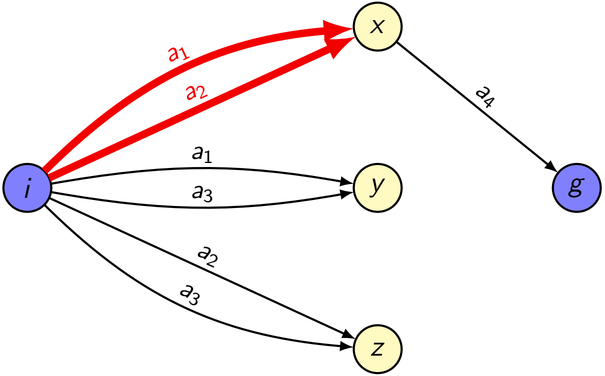
landmark $B = \{a_1, a_2\}$ (cost $= 3$)

$a_1 = i \xrightarrow{3} x, y$

$a_2 = i \xrightarrow{4} x, z$

$a_3 = i \xrightarrow{5} y, z$

$a_4 = x, y, z \xrightarrow{0} g$

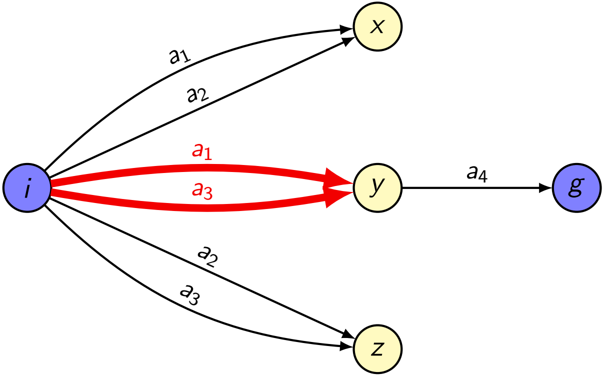


Example: Cuts in Justification Graphs

Example

landmark $C = \{a_1, a_3\}$ (cost $= 3$)

$a_1 = i \xrightarrow{3} x, y$
$a_2 = i \xrightarrow{4} x, z$
$a_3 = i \xrightarrow{5} y, z$
$a_4 = x, y, z \xrightarrow{0} g$

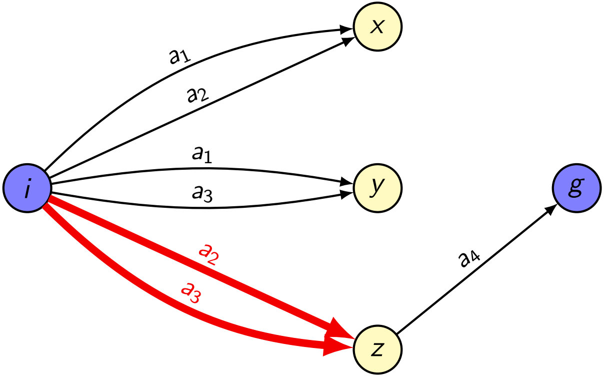


Example: Cuts in Justification Graphs

Example

landmark $D = \{a_2, a_3\}$ (cost $= 4$)

$a_1 = i \xrightarrow{3} x, y$
$a_2 = i \xrightarrow{4} x, z$
$a_3 = i \xrightarrow{5} y, z$
$a_4 = x, y, z \xrightarrow{0} g$



Power of Cuts in Justification Graphs

- Which landmarks can be computed with the cut method?

Power of Cuts in Justification Graphs

- Which landmarks can be computed with the cut method?
- all interesting ones!

Proposition (perfect hitting set heuristics)

Let $\mathcal{L}$ be the set of all "cut landmarks" of a given planning task. Then $h^{\mathrm{MHS}}(I) = h^{+}(I)$ for $\mathcal{L}$.

$\rightsquigarrow$ hitting set heuristic for $\mathcal{L}$ is perfect.

Power of Cuts in Justification Graphs

- Which landmarks can be computed with the cut method?
- all interesting ones!

Proposition (perfect hitting set heuristics)

Let $\mathcal{L}$ be the set of all "cut landmarks" of a given planning task. Then $h^{\mathrm{MHS}}(I) = h^{+}(I)$ for $\mathcal{L}$.

$\rightsquigarrow$ hitting set heuristic for $\mathcal{L}$ is perfect.

proof idea:

- Show 1:1 correspondence of hitting sets H for $\mathcal{L}$ and plans, i.e., each hitting set for $\mathcal{L}$ corresponds to a plan, and vice versa.

The LM-Cut Heuristic

LM-Cut Heuristic: Motivation

- In general, there are exponentially many pcfs, hence computing all relevant landmarks is not tractable.
- The LM-cut heuristic is a method that chooses pcfs and computes cuts in a goal-oriented way.
- A cost partitioning is computed as a side effect and is usually not optimal.
- However, the cost partitioning can be computed efficiently and is optimal for planning tasks with uniform costs (i.e., $cost(a) = 1$ for all actions).

$\rightsquigarrow$ currently one of the best admissible planning heuristics

LM-Cut Heuristic

$h^{\text{LM-cut}}$: Helmert & Domshlak (2009)

Initialize $h^{\text{LM-cut}}(I) := 0$. Then iterate:

1. Compute $h^{\max}$ values of the variables. Stop if $h^{\max}(g) = 0$.
2. Compute justification graph G for a pcf that chooses preconditions with maximal $h^{\max}$ value. (Requires a tie-breaking policy.)
3. Determine the goal zone V_g of G that consists of all vertices that have a zero-cost path to g.
4. Compute the cut L that contains the labels of all arcs $v \xrightarrow{a} v'$ such that $v \notin V_g$, $v' \in V_g$ and v can be reached from i without traversing a vertex in V_g.
   It is guaranteed that $cost(L) > 0$.
5. Increase $h^{\text{LM-cut}}(I)$ by $cost(L)$.
6. Decrease $cost(a)$ by $cost(L)$ for all $a \in L$.

Example: Computation of LM-Cut

Example

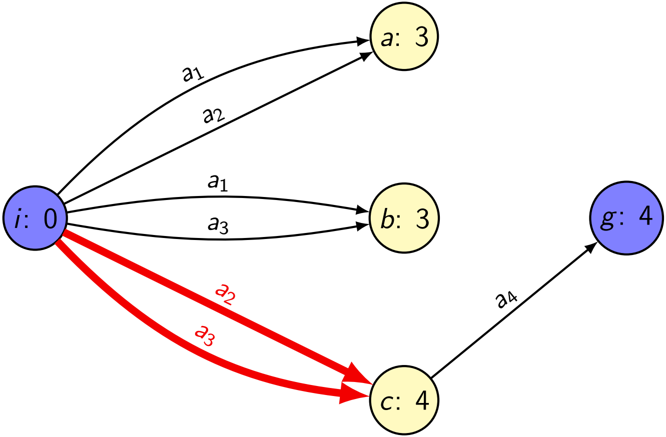
round 1: $P(a_4) = c \rightsquigarrow L = \{a_2, a_3\}$ [4]

$a_1 = i \xrightarrow{3} a, b$

$a_2 = i \xrightarrow{4} a, c$

$a_3 = i \xrightarrow{5} b, c$

$a_4 = a, b, c \xrightarrow{0} g$



Example: Computation of LM-Cut

Example

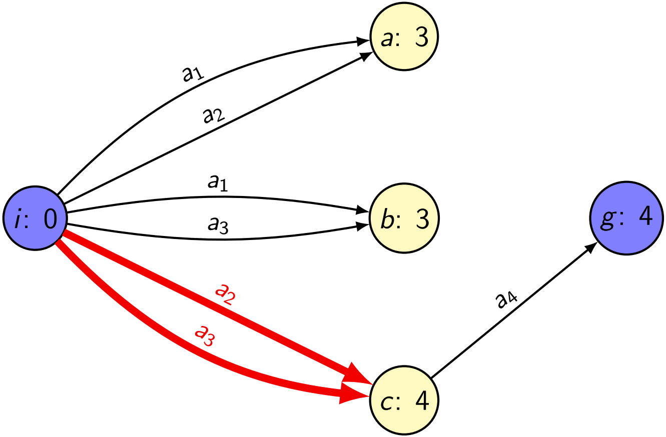
round 1: $P(a_4) = c \rightsquigarrow L = \{a_2, a_3\}$ [4] $\rightsquigarrow h^{\text{LM-cut}}(I) := 4$

$a_1 = i \xrightarrow{3} a, b$

$a_2 = i \xrightarrow{0} a, c$

$a_3 = i \xrightarrow{1} b, c$

$a_4 = a, b, c \xrightarrow{0} g$

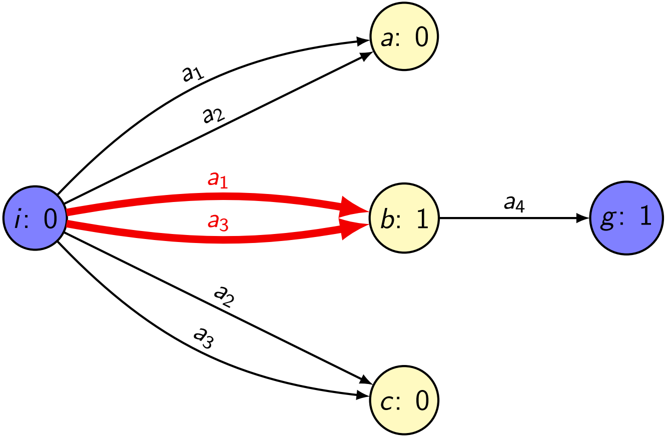


Example: Computation of LM-Cut

Example

round 2: $P(a_4) = b \rightsquigarrow L = \{a_1, a_3\}$ [1]

$a_1 = i \xrightarrow{3} a, b$
$a_2 = i \xrightarrow{0} a, c$
$a_3 = i \xrightarrow{1} b, c$
$a_4 = a, b, c \xrightarrow{0} g$



Example: Computation of LM-Cut

Example

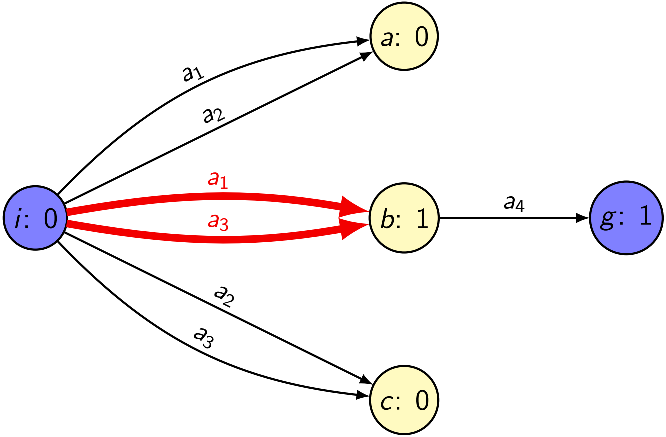
round 2: $P(a_4) = b \rightsquigarrow L = \{a_1, a_3\}$ [1] $\rightsquigarrow h^{\text{LM-cut}}(I) := 4 + 1 = 5$

$a_1 = i \xrightarrow{2} a, b$

$a_2 = i \xrightarrow{0} a, c$

$a_3 = i \xrightarrow{0} b, c$

$a_4 = a, b, c \xrightarrow{0} g$

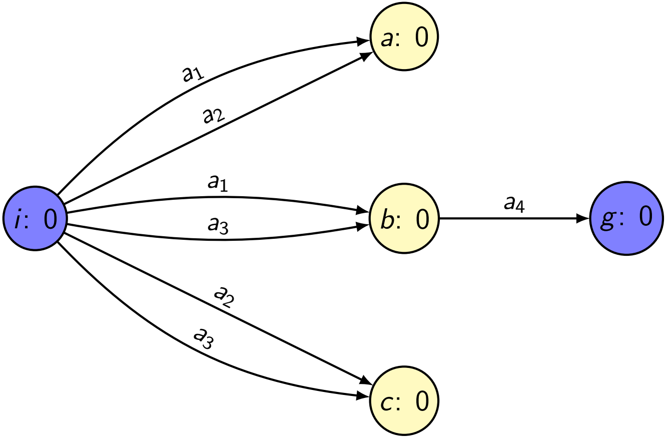


Example: Computation of LM-Cut

Example

round 3: $h^{\max}(g) = 0$ ⇝ done! ⇝ $h^{\text{LM-cut}}(I) = 5$

$a_1 = i \xrightarrow{2} a, b$
$a_2 = i \xrightarrow{0} a, c$
$a_3 = i \xrightarrow{0} b, c$
$a_4 = a, b, c \xrightarrow{0} g$



Summary

Summary

- Cuts in justification graphs
  are a general method to find landmarks.
- Hitting sets over all cut landmarks yield a perfect heuristic
  for delete-free planning tasks.
- The LM-cut heuristic is an admissible heuristic
  based on these ideas.