Foundations of Artificial Intelligence

17. State-Space Search: IDA*

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State-Space Search: Overview

Chapter overview: state-space search

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17.1 IDA*: Idea

IDA*

The main drawback of the presented best-first graph search algorithms is their space complexity.

Idea: use the concepts of iterative-deepening DFS

- depth-limited depth-first search with increasing limits
- instead of depth we limit f
  (in this chapter $f(n) := g(n) + h(n.\text{state})$ as in A*)

$\rightsquigarrow$ IDA* (iterative-deepening A*)

- tree search, unlike the previous best-first search algorithms

17.2 IDA*: Algorithm

Reminder: Iterative Deepening Depth-first Search

reminder: iterative deepening depth-first search

Iterative Deepening DFS

for $\textit{depth\_limit} \in \{0, 1, 2, \dots\}$:
  $\textit{solution} := \text{depth\_limited\_search}(\text{init}(), \textit{depth\_limit})$
  if $\textit{solution} \neq$ **none**:
    return *solution*

function depth_limited_search(s, *depth_limit*):

if is_goal(s):
  return $\langle\rangle$
if $\textit{depth\_limit} > 0$:
  for each $\langle a, s' \rangle \in \text{succ}(s)$:
    $\textit{solution} := \text{depth\_limited\_search}(s', \textit{depth\_limit} - 1)$
    if $\textit{solution} \neq$ **none**:
      solution.push_front(a)
      return *solution*
return none

First Attempt: IDA* Main Function

first attempt: iterative deepening A* (IDA*)

IDA* (First Attempt)

for $\textit{f\_limit} \in \{0, 1, 2, \dots\}$:
  $\textit{solution} := \text{f\_limited\_search}(\text{init}(), 0, \textit{f\_limit})$
  if $\textit{solution} \neq$ **none**:
    return *solution*

First Attempt: f-Limited Search

function f_limited_search(s, g, f_limit):
if $g + h(s) > f\_limit$:
    return none
if is_goal(s):
    return $\langle\rangle$
for each $\langle a, s'\rangle \in$ succ(s):
    solution := f_limited_search($s', g + \text{cost}(a), f\_limit$)
    if *solution* $\neq$ **none**:
        solution.push_front(a)
        return *solution*
return none

IDA* First Attempt: Discussion

- The pseudo-code can be rewritten to be even more similar to our IDDFS pseudo-code. However, this would make our next modification more complicated.
- The algorithm follows the same principles as IDDFS, but takes path costs and heuristic information into account.
- For unit-cost state spaces and the trivial heuristic $h : s \mapsto 0$ for all states s, it behaves identically to IDDFS.
- For general state spaces, there is a problem with this first attempt, however.

Growing the f Limit

- In IDDFS, we grow the limit from the smallest limit that gives a non-empty search tree (0) by 1 at a time.
- This usually leads to exponential growth of the tree between rounds, so that re-exploration work can be amortized.
- In our first attempt at IDA*, there is no guarantee that increasing the f limit by 1 will lead to a larger search tree than in the previous round.
- This problem becomes worse if we also allow non-integer (fractional) costs, where increasing the limit by 1 would be very arbitrary.

Setting the Next f Limit

idea: let the f-limited search compute the next sensible f limit

- Start with h(init()), the smallest f limit that results in a non-empty search tree.
- In every round, increase the f limit to the smallest value that ensures that in the next round at least one additional path will be considered by the search.

$\rightsquigarrow$ f_limited_search now returns two values:
  - the next f limit that would include at least one new node in the search tree (∞ if no such limit exists; **none** if a solution was found), and
  - the solution that was found (or **none**).

Final Algorithm: IDA* Main Function

final algorithm: iterative deepening A* (IDA*)

IDA*

$f\_limit = h(\text{init}())$
while $f\_limit \neq \infty$:
  $\langle f\_limit, solution \rangle := \text{f\_limited\_search}(\text{init}(), 0, f\_limit)$
  if $solution \neq$ **none**:
    return $solution$
return unsolvable

Final Algorithm: f-Limited Search

function f_limited_search(s, g, f_limit):
if $g + h(s) > f\_limit$:
  return $\langle g + h(s), \textbf{none} \rangle$
if is_goal(s):
  return $\langle \textbf{none}, \langle\rangle \rangle$
$new\_limit := \infty$
for each $\langle a, s' \rangle \in \text{succ}(s)$:
  $\langle child\_limit, solution \rangle := \text{f\_limited\_search}(s', g + \text{cost}(a), f\_limit)$
  if $solution \neq$ **none**:
    $solution$.push_front(a)
    return $\langle \textbf{none}, solution \rangle$
  $new\_limit := \min(new\_limit, child\_limit)$
return $\langle new\_limit, \textbf{none} \rangle$

Final Algorithm: f-Limited Search

function f_limited_search(s, g, f_limit):
if $g + h(s) > f\_limit$:
  return $\langle g + h(s), \textbf{none} \rangle$
if is_goal(s):
  return $\langle \textbf{none}, \langle\rangle \rangle$
$new\_limit := \infty$
for each $\langle a, s' \rangle \in \text{succ}(s)$:
  $\langle child\_limit, solution \rangle := \text{f\_limited\_search}(s', g + \text{cost}(a), f\_limit)$
  if $solution \neq$ **none**:
    $solution$.push_front(a)
    return $\langle \textbf{none}, solution \rangle$
  $new\_limit := \min(new\_limit, child\_limit)$
return $\langle new\_limit, \textbf{none} \rangle$

17.3 IDA*: Properties

IDA*: Properties

Inherits important properties of A* and depth-first search:

- semi-complete if h safe and $cost(a) > 0$ for all actions a
- optimal if h admissible
- space complexity $O(\ell b)$, where
  - ℓ: length of longest generated path (for unit cost problems: bounded by optimal solution cost)
  - b: branching factor

⇝ proofs?

IDA*: Discussion

- compared to A* potentially considerable overhead because no duplicates are detected
  - ⇝ exponentially slower in many state spaces
  - ⇝ often combined with partial duplicate elimination (cycle detection, transposition tables)
- overhead due to iterative increases of f limit often negligible, but not always
  - especially problematic if action costs vary a lot: then it can easily happen that each new f limit only considers a small number of new paths

17.4 Summary

Summary

- IDA* is a tree search variant of A* based on iterative deepening depth-first search
- main advantage: low space complexity
- disadvantage: repeated work can be significant
- most useful when there are few duplicates