

Foundations of Artificial Intelligence

16. State-Space Search: Greedy BFS, A*, Weighted A*

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16.1 Introduction

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State-Space Search: Overview

Chapter overview: state-space search

- ▶ 5.–7. Foundations
- ▶ 8.–12. Basic Algorithms
- ▶ 13.–19. Heuristic Algorithms
 - ▶ 13. Heuristics
 - ▶ 14. Analysis of Heuristics
 - ▶ 15. Best-first Graph Search
 - ▶ 16. Greedy Best-first Search, A*, Weighted A*
 - ▶ 17. IDA*
 - ▶ 18. Properties of A*, Part I
 - ▶ 19. Properties of A*, Part II

16.1 Introduction

What Is It About?

In this chapter we study last chapter's algorithms in more detail:

- ▶ greedy best-first search
- ▶ A*
- ▶ weighted A*

16.2 Greedy Best-first Search

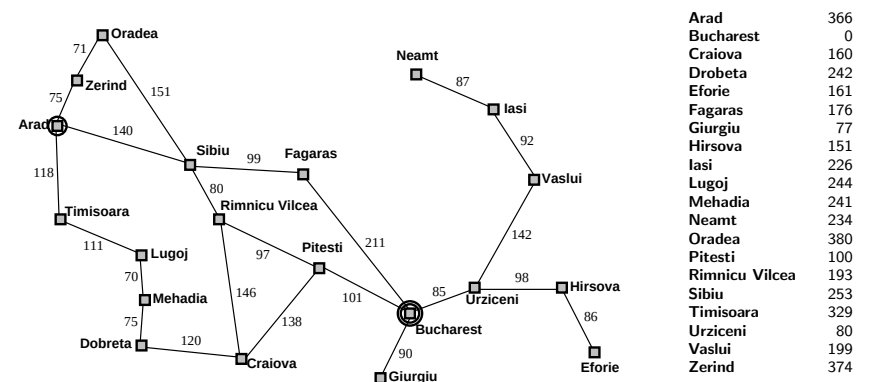
Greedy Best-first Search

Greedy Best-first Search

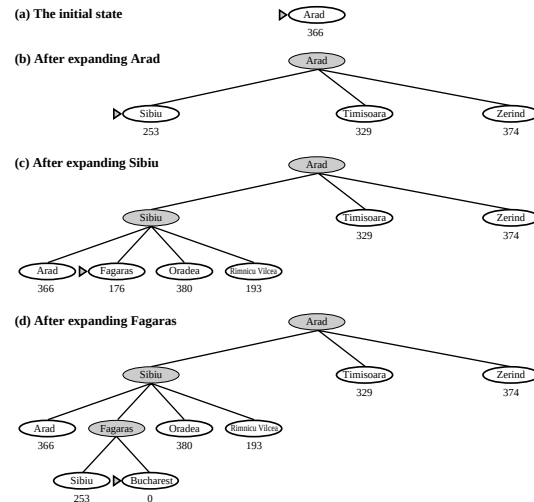
only consider the heuristic: $f(n) = h(n.state)$

Note: usually **without reopening** (for reasons of efficiency)

Example: Greedy Best-first Search for Route Planning



Example: Greedy Best-first Search for Route Planning



Greedy Best-first Search: Properties

- ▶ **complete** with **safe** heuristics (like all variants of best-first graph search)
- ▶ **suboptimal**: solutions can be **arbitrarily bad**
- ▶ often **very fast**: one of the fastest search algorithms in practice
- ▶ monotonic transformations of h (e.g. scaling, additive constants) do not affect behaviour (Why is this interesting?)

16.3 A*

A*

A*

combine greedy best-first search with uniform cost search:

$$f(n) = g(n) + h(n.state)$$

- ▶ **trade-off** between path cost and proximity to goal
- ▶ $f(n)$ estimates overall cost of cheapest solution from initial state via n to the goal

A*: Citations

hart nilsson raphael



About 8,370 results (0.10 sec)

Did you mean: [harry nilsson raphael](#)

A formal basis for the heuristic determination of minimum cost paths

[PE Hart](#), [NJ Nilsson](#), [B Raphael](#) - IEEE transactions on Systems ..., 1968 - [ieeexplore.ieee.org](#)

Although the problem of determining the minimum cost path through a graph arises naturally in a number of interesting applications, there has been no underlying theory to guide the development of efficient search procedures. Moreover, there is no adequate conceptual ...

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Correction to a formal basis for the heuristic determination of minimum cost paths

[PE Hart](#), [NJ Nilsson](#), [B Raphael](#) - ACM SIGART Bulletin, 1972 - [dl.acm.org](#)Our paper on the use of heuristic information in graph searching defined a path-finding algorithm, A*, and proved that it had two important properties. In the notation of the paper, we proved that if the heuristic function $f(n)$ is a lower bound on the true minimal cost from ...☆ [Cited by 438](#) [Related articles](#) [All 9 versions](#) [✕](#)

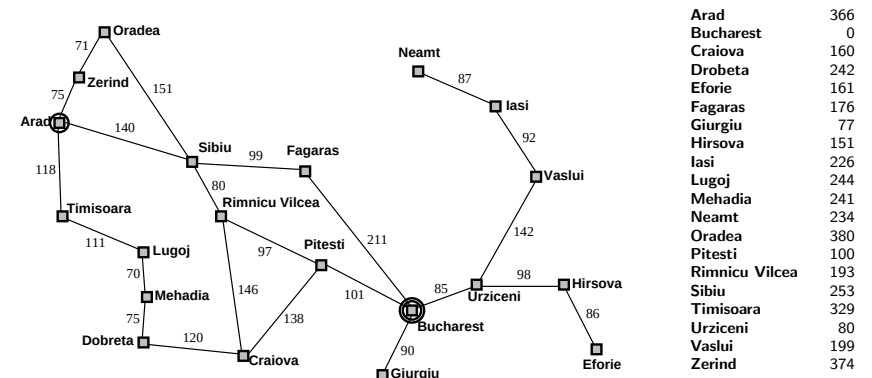
Research and applications: Artificial intelligence

[B Raphael](#), [RE Fikes](#), [LJ Chaitin](#), [PE Hart](#), [RO Duda](#)... - 1971 - [ntrs.nasa.gov](#)

A program of research in the field of artificial intelligence is presented. The research areas discussed include automatic theorem proving, representations of real-world environments, problem-solving methods, the design of a programming system for problem-solving ...

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Example: A* for Route Planning



Example: A* for Route Planning

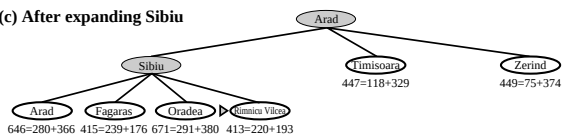
(a) The initial state



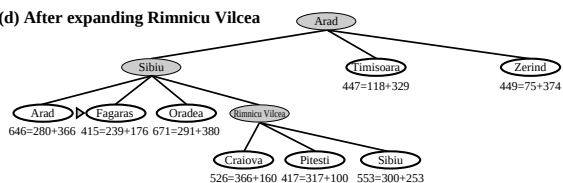
(b) After expanding Arad



(c) After expanding Sibiu

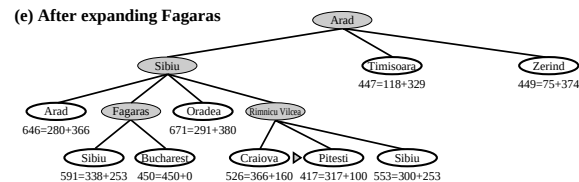


(d) After expanding Rimnicu Vilcea

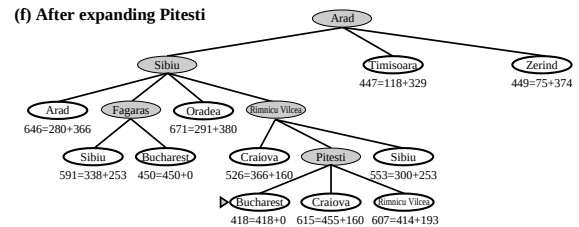


Example: A* for Route Planning

(e) After expanding Fagaras



(f) After expanding Pitesti



A*: Properties

- ▶ **complete** with **safe** heuristics
(like all variants of best-first graph search)
- ▶ **with reopening**: **optimal** with **admissible** heuristics
- ▶ **without reopening**: **optimal** with heuristics
that are **admissible** and **consistent**

⇒ proofs: Chapters 18 and 19

A*: Implementation Aspects

some practical remarks on implementing A*:

- ▶ **common bug**: reopening not implemented
although heuristic is not consistent
- ▶ **common bug**: duplicate test “too early”
(upon generation of search nodes)
- ▶ **common bug**: goal test “too early”
(upon generation of search nodes)
- ▶ all these bugs lead to loss of optimality
and can remain undetected for a long time

16.4 Weighted A*

Weighted A*

Weighted A*

A* with more heavily weighted heuristic:

$$f(n) = g(n) + w \cdot h(n.\text{state}),$$

where **weight** $w \in \mathbb{R}_0^+$ with $w \geq 1$ is a freely choosable parameter

Note: $w < 1$ is conceivable, but usually not a good idea
(Why not?)

Weighted A*: Properties

weight parameter controls “greediness” of search:

- ▶ $w = 0$: like uniform cost search
- ▶ $w = 1$: like A*
- ▶ $w \rightarrow \infty$: like greedy best-first search

with $w \geq 1$ properties analogous to A*:

- ▶ h admissible:
found solution guaranteed to be at most w times
as expensive as optimum when reopening is used
- ▶ h admissible and consistent:
found solution guaranteed to be at most w times
as expensive as optimum; no reopening needed

(without proof)

16.5 Summary

Summary

best-first graph search with evaluation function f :

- ▶ $f = h$: greedy best-first search
suboptimal, often very fast
- ▶ $f = g + h$: A*
optimal if h admissible and consistent
or if h admissible and reopening is used
- ▶ $f = g + w \cdot h$: weighted A*
for $w \geq 1$ suboptimality factor at most w
under same conditions as for optimality of A*