

# Foundations of Artificial Intelligence

## 13. State-Space Search: Heuristics

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## State-Space Search: Overview

### Chapter overview: state-space search

- ▶ 5.–7. Foundations
- ▶ 8.–12. Basic Algorithms
- ▶ 13.–19. Heuristic Algorithms
  - ▶ 13. Heuristics
  - ▶ 14. Analysis of Heuristics
  - ▶ 15. Best-first Graph Search
  - ▶ 16. Greedy Best-first Search,  $A^*$ , Weighted  $A^*$
  - ▶ 17. IDA\*
  - ▶ 18. Properties of  $A^*$ , Part I
  - ▶ 19. Properties of  $A^*$ , Part II

## 13.1 Introduction

## Informed Search Algorithms

- ▶ search algorithms considered so far: **blind**  
because they do not use any aspects of the problem to solve other than its formal definition (state space)
  - ▶ **problem**: scalability  
↪ prohibitive time and space requirements already for seemingly **simple** problems
  - ▶ **idea**: try to find (problem-specific) criteria to distinguish **good** and **bad states**  
↪ **prefer good states**
- ↪ **informed** (“heuristic”) search algorithms

## 13.2 Heuristics

## Heuristics

### Definition (heuristic)

Let  $\mathcal{S}$  be a state space with states  $S$ .

A **heuristic function** or **heuristic** for  $\mathcal{S}$  is a function

$$h : S \rightarrow \mathbb{R}_0^+ \cup \{\infty\},$$

mapping each state to a non-negative number (or  $\infty$ ).

## Heuristics: Intuition

**idea**:  $h(s)$  estimates distance (= cost of cheapest path) from  $s$  to closest goal state

- ▶ heuristics can be **arbitrary** functions
- ▶ **intuition**: the closer  $h$  is to true goal distance, the more efficient the search using  $h$

Heuristics are sometimes defined for **search nodes** instead of states, but this increased generality is rarely useful. (**Why?**)

## Why “Heuristic”?

What does “heuristic” mean?

- ▶ heuristic: from ancient Greek  $\epsilon\upsilon\text{ρισκω}$  (= I find)  
 $\rightsquigarrow$  compare:  $\epsilon\upsilon\text{ρηκα!}$
- ▶ popularized by George Pólya: How to Solve It (1945)
- ▶ in computer science often used for:  
 rule of thumb, inexact algorithm
- ▶ in state-space search **technical term** for **goal distance estimator**

## Representation of Heuristics

In our black box model, heuristics are an additional element of the state space interface:

State Spaces as Black Boxes (Extended)

- ▶ `init()`
- ▶ `is_goal(s)`
- ▶ `succ(s)`
- ▶ `cost(a)`
- ▶ `h(s)`: heuristic value for state  $s$   
 result: non-negative integer or  $\infty$

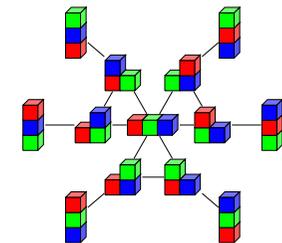
## 13.3 Examples

### Example: Blocks World

possible heuristic:

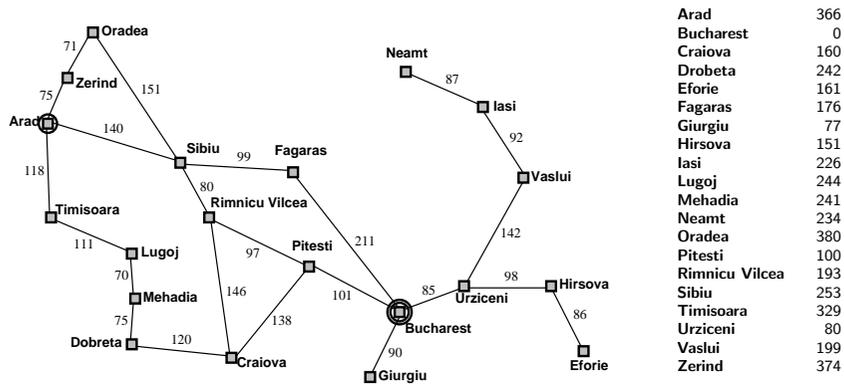
count blocks  $x$  that currently lie on  $y$   
 and must lie on  $z \neq y$  in the goal  
 (including case where  $y$  or  $z$  is the table)

How accurate is this heuristic?



## Example: Route Planning in Romania

possible heuristic: **straight-line distance to Bucharest**



## Example: Missionaries and Cannibals

Setting: **Missionaries and Cannibals**

- ▶ Six people must cross a river.
- ▶ Their rowing boat can carry one or two people across the river at a time (it is too small for three).
- ▶ Three people are missionaries, three are cannibals.
- ▶ Missionaries may never stay with a majority of cannibals.

possible heuristic: **number of people on the wrong river bank**

↪ with our formulation of states as triples  $\langle m, c, b \rangle$ :

$$h(\langle m, c, b \rangle) = m + c$$

## 13.4 Summary

### Summary

- ▶ **heuristics** estimate distance of a state to the goal
- ▶ can be used to **focus** search on **promising** states
- ↪ **soon**: search algorithms that use heuristics