

Foundations of Artificial Intelligence

5. State-Space Search: State Spaces

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State-Space Search Problems

Classical State-Space Search Problems Informally

(Classical) state-space search problems are among the “simplest” and most important classes of AI problems.

objective of the agent:

- from a given initial state
- apply a sequence of actions
- in order to reach a goal state

performance measure: minimize total action cost

Motivating Example: 15-Puzzle

9	2	12	6
5	7	14	13
3		1	11
15	4	10	8



1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Classical Assumptions

“classical” assumptions:

- no other agents in the environment ([single-agent](#))
- always knows state of the world ([fully observable](#))
- state only changed by the agent ([static](#))
- finite number of states/actions (in particular [discrete](#))
- actions have [deterministic](#) effect on the state

↪ can all be generalized (but not in this part of the course)

For simplicity, we omit “[classical](#)” in the following.

Classification

Classification:

State-Space Search

environment:

- **static** vs. dynamic
- **deterministic** vs. non-deterministic vs. stochastic
- **fully** vs. partially vs. not **observable**
- **discrete** vs. continuous
- **single-agent** vs. multi-agent

problem solving method:

- **problem-specific** vs. general vs. learning

Search Problem Examples

- **toy problems**: combinatorial puzzles
(Rubik's Cube, 15-puzzle, towers of Hanoi, ...)
- **scheduling** of events, flights, manufacturing tasks
- **query optimization** in databases
- behavior of **NPCs** in computer games
- **code optimization** in compilers
- **verification** of soft- and hardware
- **sequence alignment** in bioinformatics
- **route planning** (e.g., Google Maps)
- ...

thousands of practical examples

State-Space Search: Overview

Chapter overview: state-space search

- 5.–7. Foundations
 - 5. State Spaces
 - 6. Representation of State Spaces
 - 7. Examples of State Spaces
- 8.–12. Basic Algorithms
- 13.–19. Heuristic Algorithms

Formalization

Formalization

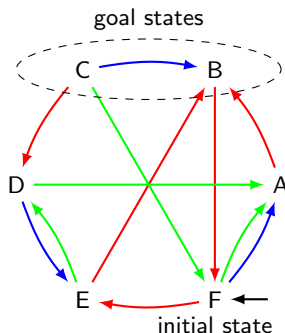
preliminary remarks:

- to cleanly study search problems we need a **formal model**
- fundamental concept: **state spaces**
- state spaces are (labeled, directed) **graphs**
- **paths** to goal states represent **solutions**
- **shortest paths** correspond to **optimal solutions**

State Spaces: Example

State spaces are often depicted as **directed graphs**.

- **states**: graph vertices
- **transitions**: labeled arcs
(here: colors instead of labels)
- **initial state**: incoming arrow
- **goal states**: marked
(here: by the dashed ellipse)
- **actions**: the arc labels
- **action costs**: described separately
(or implicitly = 1)



State Spaces

Definition (state space)

A **state space** or **transition system** is a 6-tuple

$\mathcal{S} = \langle S, A, cost, T, s_0, S_\star \rangle$ with

- S : finite set of **states**
- A : finite set of **actions**
- $cost : A \rightarrow \mathbb{R}_0^+$ **action costs**
- $T \subseteq S \times A \times S$ **transition relation**; **deterministic in $\langle s, a \rangle$**
(see next slide)
- $s_0 \in S$ **initial state**
- $S_\star \subseteq S$ set of **goal states**

German: Zustandsraum, Transitionssystem, Zustände, Aktionen, Aktionskosten, Transitions-/Übergangsrelation, deterministisch, Anfangszustand, Zielzustände

State Spaces: Transitions, Determinism

Definition (transition, deterministic)

Let $\mathcal{S} = \langle S, A, cost, T, s_0, S_\star \rangle$ be a state space.

The triples $\langle s, a, s' \rangle \in T$ are called **(state) transitions**.

We say \mathcal{S} **has the transition** $\langle s, a, s' \rangle$ if $\langle s, a, s' \rangle \in T$.

We write this as $s \xrightarrow{a} s'$, or $s \rightarrow s'$ when a does not matter.

Transitions are **deterministic** in $\langle s, a \rangle$: it is forbidden to have both $s \xrightarrow{a} s_1$ and $s \xrightarrow{a} s_2$ with $s_1 \neq s_2$.

State Spaces: Terminology

We use common terminology from graph theory.

Definition (predecessor, successor, applicable action)

Let $\mathcal{S} = \langle S, A, cost, T, s_0, S_\star \rangle$ be a state space.

Let $s, s' \in S$ be states with $s \rightarrow s'$.

- s is a **predecessor** of s'
- s' is a **successor** of s

If $s \xrightarrow{a} s'$, then action a is **applicable** in s .

German: Vorgänger, Nachfolger, anwendbar

State Spaces: Terminology

We use common terminology from graph theory.

Definition (path)

Let $\mathcal{S} = \langle S, A, cost, T, s_0, S_\star \rangle$ be a state space.

Let $s^{(0)}, \dots, s^{(n)} \in S$ be states and $\pi_1, \dots, \pi_n \in A$ be actions such that $s^{(0)} \xrightarrow{\pi_1} s^{(1)}, \dots, s^{(n-1)} \xrightarrow{\pi_n} s^{(n)}$.

- $\pi = \langle \pi_1, \dots, \pi_n \rangle$ is a **path** from $s^{(0)}$ to $s^{(n)}$
- **length** of π : $|\pi| = n$
- **cost** of π : $cost(\pi) = \sum_{i=1}^n cost(\pi_i)$

German: Pfad, Länge, Kosten

- paths may have length 0
- sometimes “path” is used for state sequence $\langle s^{(0)}, \dots, s^{(n)} \rangle$ or sequence $\langle s^{(0)}, \pi_1, s^{(1)}, \dots, s^{(n-1)}, \pi_n, s^{(n)} \rangle$

State Spaces: Terminology

more terminology:

Definition (reachable, solution, optimal)

Let $\mathcal{S} = \langle S, A, cost, T, s_0, S_\star \rangle$ be a state space.

- state s is **reachable** if a path from s_0 to s exists
- paths from $s \in S$ to some state $s_\star \in S_\star$ are **solutions for/from s**
- solutions for s_0 are called **solutions for \mathcal{S}**
- **optimal solutions** (for s) have minimal costs among all solutions (for s)

German: erreichbar, Lösung von/für s , optimale Lösung

State-Space Search

State-Space Search

State-Space Search

State-space search is the algorithmic problem of finding solutions in state spaces or proving that no solution exists.

In **optimal** state-space search, only optimal solutions may be returned.

German: Zustandsraumsuche, optimale Zustandsraumsuche

Learning Objectives for State-Space Search

Learning Objectives for the Topic of State-Space Search

- **understanding state-space search:**
What is the problem and how can we formalize it?
- **evaluate search algorithms:**
completeness, optimality, time/space complexity
- **get to know search algorithms:**
uninformed vs. informed; tree and graph search
- **evaluate heuristics for search algorithms:**
goal-awareness, safety, admissibility, consistency
- **efficient implementation** of search algorithms
- **experimental evaluation** of search algorithms
- **design and comparison of heuristics** for search algorithms

Summary

Summary

- classical state-space search problems:
find action sequence from initial state to a goal state
- performance measure: sum of action costs
- formalization via state spaces:
 - states, actions, action costs, transitions,
initial state, goal states
- terminology for transitions, paths, solutions
- definition of (optimal) state-space search