Foundations of Artificial Intelligence

M. Helmert
S. Eriksson
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University of Basel
Computer Science

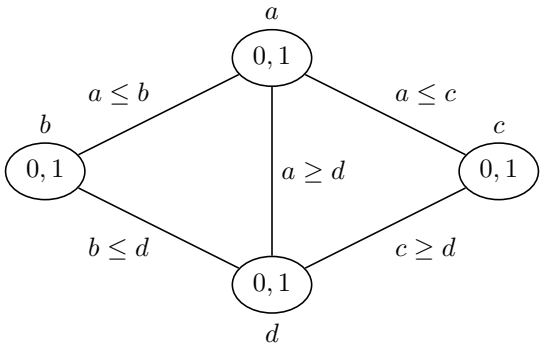
Exercise Sheet 8

Due: April 28, 2021

Exercise 8.1 (2 marks)

We define a constraint network $\mathcal{C} = \langle V, \mathrm{dom}, (R_{xy}) \rangle$ with

- $V = \{a, b, c, d\}$
- $\mathrm{dom}(x) = \{0, 1\}$ for all $x \in V$
- $R_{ab} = R_{ac} = R_{bd} = \{(0,0), (0,1), (1,1)\}$
  $R_{ad} = R_{cd} = \{(0,0), (1,0), (1,1)\}$



Apply the PC-2 algorithm that has been presented on slide 12 of chapter 26 in the print version of the lecture slides on $\mathcal{C}$. Select the variables u, v and w in each iteration of the while loop such that R_{uv} changes in the call to `revise-3`$(\mathcal{C}, u, v, w)$. Provide u, v, w and R_{uv} in each iteration. Note that you do *not* have to provide the elements that are inserted into the queue, and you may stop the algorithm as soon as there are no u, v and w such that R_{uv} changes.

Exercise 8.2 (3 marks)

Let $\mathcal{C}$ be a solvable constraint network with an acyclic constraint graph. Show that the application of the algorithm for trees as constraint graphs (slide 13 of chapter 27 of the print version of the lecture slides) leads to a solution for $\mathcal{C}$ without needing to backtrack.

Hint: First use structural induction to show that after step 3 the domain of every node v contains only values for which a consistent assignment to all variables in the subtree rooted at v can be found. Then use this to show that backtracking with forward checking will always pick a value that can be extended to a solution. Also consider that finding a solution requires that no domain ever becomes empty.

Exercise 8.3 (2+1 marks)

Consider the constraint network that is given by the graph coloring problem of a graph $G = \langle V, E \rangle$. The set of vertices V contains a vertex for each Swiss canton, and E is such that two vertices v and v' are connected iff the cantons v and v' share a border. A description of G can be downloaded from the website or on ADAM and you can vizualize it with graphviz:

```
dot -T pdf -o cantons.pdf cantons.dot
```

(a) Provide a cutset $V' \subseteq V$ for G that is as small as possible (it is not necessary to provide an explanation how you have found V'). As a reminder, a cutset of a graph is defined as a set of vertices that is such that the induced subgraph that is obtained by removing these vertices results in an acyclic graph.

Note: You get 2 marks for your solution if your cutset is optimal, 1 mark if your cutset contains exactly one more vertex than an optimal cutset and 0 marks otherwise.

(b) Assume we are interested in coloring G with 4 colors. Provide a worst-case runtime estimate of the algorithm based on cutset conditioning if your cutset from the first part of this exercise is used (i.e., compute an upper bound for the number of considered assignments). Compare your result to the estimated runtime if no cutset is used.

Exercise 8.4 (1+1 marks)

Consider the propositional formula $\varphi = \neg(A \vee \neg((A \wedge B) \vee (\neg C \wedge A)))$.

(a) Is φ satisfiable, unsatisfiable, falisfiable, valid? Briefly justify your answer for each property.

(b) Transform φ into CNF. Provide intermediate transformation steps and specify which rule(s) you applied in each step.

Submission rules:

Upload a single PDF file (ending .pdf). If you want to submit handwritten parts, include their scans in the single PDF. Put the names of all group members on top of the first page. Use page numbers or put your names on each page. Make sure your PDF has size A4 (fits the page size if printed on A4).