

Foundations of Artificial Intelligence

41. Board Games: Minimax Search and Evaluation Functions

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41.1 Minimax Search

41.2 Evaluation Functions

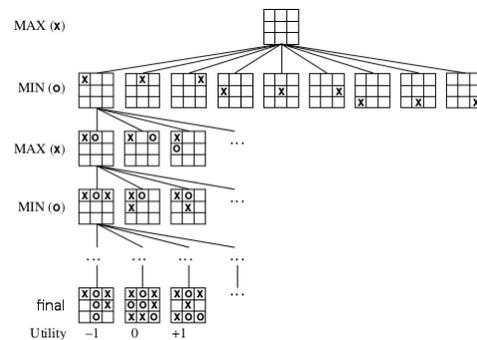
41.3 Summary

41.1 Minimax Search

Terminology for Two-Player Games

- ▶ **Players** are traditionally called **MAX** and **MIN**.
- ▶ Our objective is to compute moves for MAX (MIN is the opponent).
- ▶ MAX tries to **maximize** its utility (given by the utility function u) in the reached terminal position.
- ▶ MIN tries to **minimize** u (which in turn maximizes MINs utility).

Example: Tic-Tac-Toe

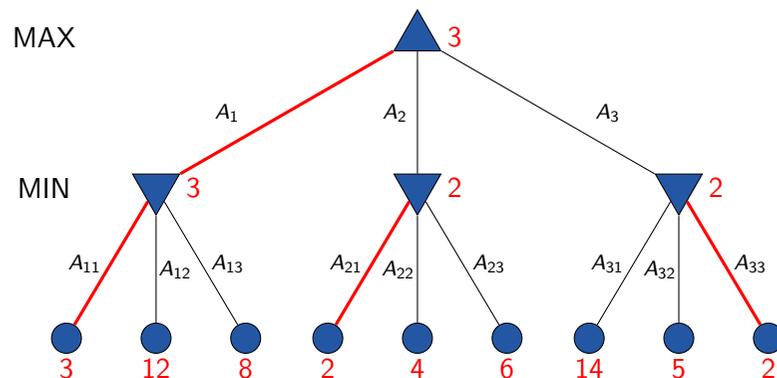


- ▶ game tree with player's turn (MAX/MIN) marked on the left
- ▶ last row: terminal positions with utility
- ▶ size of game tree?

Minimax: Computation

1. depth-first search through game tree
2. Apply utility function in terminal position.
3. Compute utility value of inner nodes from below to above through the tree:
 - ▶ MIN's turn: utility is minimum of utility values of children
 - ▶ MAX's turn: utility is maximum of utility values of children
4. move selection for MAX in root: choose a move that maximizes the computed utility value (minimax decision)

Minimax: Example



Minimax: Discussion

- ▶ Minimax is the simplest (decent) search algorithm for games
- ▶ Yields optimal strategy* (in the game theoretic sense, i.e., under the assumption that the opponent plays perfectly), but is too time consuming for complex games.
- ▶ We obtain at least the utility value computed for the root, no matter how the opponent plays.
- ▶ In case the opponent plays perfectly, we obtain exactly that value.

(*) for games where no cycles occur; otherwise things get more complicated (because the tree will have infinite size in this case).

Minimax: Pseudo-Code

```

function minimax( $p$ )
if  $p$  is terminal position:
    return  $\langle u(p), \text{none} \rangle$ 
 $best\_move := \text{none}$ 
if  $player(p) = \text{MAX}$ :
     $v := -\infty$ 
else:
     $v := \infty$ 
for each  $\langle move, p' \rangle \in succ(p)$ :
     $\langle v', best\_move' \rangle := minimax(p')$ 
    if ( $player(p) = \text{MAX}$  and  $v' > v$ ) or
        ( $player(p) = \text{MIN}$  and  $v' < v$ ):
         $v := v'$ 
         $best\_move := move$ 
return  $\langle v, best\_move \rangle$ 

```

Minimax

What if the size of the game tree is too big for minimax?
 \rightsquigarrow approximation by **evaluation function**

41.2 Evaluation Functions

Evaluation Functions

- ▶ **problem**: game tree too big
- ▶ **idea**: search only up to certain depth
- ▶ depth reached: **estimate** the utility according to **heuristic criteria** (as if terminal position had been reached)

Example (evaluation function in chess)

- ▶ **material**: pawn 1, knight 3, bishop 3, rook 5, queen 9
positive sign for pieces of MAX, negative sign for MIN
- ▶ **pawn structure, mobility, ...**

rule of thumb: advantage of 3 points \rightsquigarrow clear winning position

Accurate evaluation functions are crucial!

- ▶ High values should relate to high "winning chances" in order to make the overall approach work.
- ▶ At the same time, the evaluation should be efficiently computable in order to be able to search deeply.

Linear Evaluation Functions

Usually **weighted linear functions** are applied:

$$w_1 f_1 + w_2 f_2 + \dots + w_n f_n$$

where w_i are **weights**, and f_i are **features**.

- ▶ assumes that feature contributions are mutually **independent** (usually wrong but acceptable assumption)
- ▶ allows for efficient **incremental computation** if most features are unaffected by most moves
- ▶ Weights can be learned automatically.
- ▶ Features are (usually) provided by human experts.

How Deep Shall We Search?

- ▶ **objective**: search as deeply as possible within a given time
- ▶ **problem**: search time difficult to predict
- ▶ **solution**: **iterative deepening**
 - ▶ sequence of searches of increasing depth
 - ▶ time expires: return result of previously finished search
- ▶ **refinement**: search depth not uniform, but deeper in “turbulent” positions (i.e., with strong fluctuations of the evaluation function) \rightsquigarrow **quiescence search**
 - ▶ **example chess**: deepen the search if exchange of pieces has started, but not yet finished

41.3 Summary

Summary

- ▶ **Minimax** is a tree search algorithm that plays perfectly (in the game-theoretic sense), but its complexity is $O(b^d)$ (branching factor b , search depth d).
- ▶ In practice, the search depth must be limited \rightsquigarrow apply **evaluation functions** (usually linear combinations of features).