

Foundations of Artificial Intelligence

26. Constraint Satisfaction Problems: Path Consistency

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26.1 Beyond Arc Consistency

26.2 Path Consistency

26.3 Summary

Constraint Satisfaction Problems: Overview

Chapter overview: constraint satisfaction problems:

- ▶ 22.–23. Introduction
- ▶ 24.–26. Basic Algorithms
 - ▶ 24. Backtracking
 - ▶ 25. Arc Consistency
 - ▶ 26. Path Consistency
- ▶ 27.–28. Problem Structure

26.1 Beyond Arc Consistency

Beyond Arc Consistency: Path Consistency

idea of arc consistency:

- ▶ For every assignment to a variable u there must be a suitable assignment to every other variable v .
- ▶ If not: remove values of u for which no suitable “partner” assignment to v exists.
- ↪ tighter **unary constraint** on u

This idea can be extended to three variables (**path consistency**):

- ▶ For every joint assignment to variables u, v there must be a suitable assignment to every third variable w .
- ▶ If not: remove pairs of values of u and v for which no suitable “partner” assignment to w exists.
- ↪ tighter **binary constraint** on u and v

German: Pfadkonsistenz

Beyond Arc Consistency: i -Consistency

general concept of **i -consistency** for $i \geq 2$:

- ▶ For every joint assignment to variables v_1, \dots, v_{i-1} there must be a suitable assignment to every i -th variable v_i .
- ▶ If not: remove value tuples of v_1, \dots, v_{i-1} for which no suitable “partner” assignment for v_i exists.
- ↪ tighter **$(i-1)$ -ary constraint** on v_1, \dots, v_{i-1}
- ▶ **2-consistency = arc consistency**
- ▶ **3-consistency = path consistency (*)**

We do not consider general i -consistency further as larger values than $i = 3$ are rarely used and we restrict ourselves to binary constraints in this course.

(*) usual definitions of 3-consistency vs. path consistency differ when ternary constraints are allowed

26.2 Path Consistency

Path Consistency: Definition

Definition (path consistent)

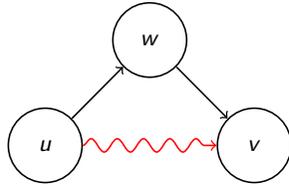
Let $\mathcal{C} = \langle V, \text{dom}, (R_{uv}) \rangle$ be a constraint network.

- Ⓐ Two different variables $u, v \in V$ are **path consistent** with respect to a third variable $w \in V$ if for all values $d_u \in \text{dom}(u), d_v \in \text{dom}(v)$ with $\langle d_u, d_v \rangle \in R_{uv}$ there is a value $d_w \in \text{dom}(w)$ with $\langle d_u, d_w \rangle \in R_{uw}$ and $\langle d_v, d_w \rangle \in R_{vw}$.
- Ⓑ The constraint network \mathcal{C} is **path consistent** if for any three variables u, v, w , the variables u and v are path consistent with respect to w .

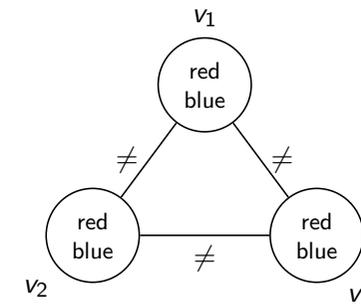
Path Consistency: Remarks

remarks:

- ▶ Even if the constraint R_{uv} is trivial, path consistency can infer nontrivial constraints between u and v .
- ▶ name “path consistency”:
path $u \rightarrow w \rightarrow v$ leads to new information on $u \rightarrow v$



Path Consistency: Example



arc consistent, but not path consistent

Processing Variable Triples: revise-3

analogous to *revise* for arc consistency:

function $\text{revise-3}(\mathcal{C}, u, v, w)$:

$\langle V, \text{dom}, (R_{uv}) \rangle := \mathcal{C}$

for each $\langle d_u, d_v \rangle \in R_{uv}$:

if there is no $d_w \in \text{dom}(w)$ with

$\langle d_u, d_w \rangle \in R_{uw}$ **and** $\langle d_v, d_w \rangle \in R_{vw}$:

remove $\langle d_u, d_v \rangle$ from R_{uv}

input: constraint network \mathcal{C} and three variables u, v, w of \mathcal{C}

effect: u, v path consistent with respect to w .

All violating pairs are removed from R_{uv} .

time complexity: $O(k^3)$ where k is maximal domain size

Enforcing Path Consistency: PC-2

analogous to *AC-3* for arc consistency:

function $\text{PC-2}(\mathcal{C})$:

$\langle V, \text{dom}, (R_{uv}) \rangle := \mathcal{C}$

$queue := \emptyset$

for each set of two variables $\{u, v\}$:

for each $w \in V \setminus \{u, v\}$:

insert $\langle u, v, w \rangle$ into $queue$

while $queue \neq \emptyset$:

remove any element $\langle u, v, w \rangle$ from $queue$

$\text{revise-3}(\mathcal{C}, u, v, w)$

if R_{uv} changed in the call to revise-3 :

for each $w' \in V \setminus \{u, v\}$:

insert $\langle w', u, v \rangle$ into $queue$

insert $\langle w', v, u \rangle$ into $queue$

PC-2: Discussion

The comments for AC-3 hold analogously.

- ▶ PC-2 enforces path consistency
- ▶ **proof idea**: invariant of the **while** loop:
if $\langle u, v, w \rangle \notin \text{queue}$, then u, v path consistent with respect to w
- ▶ time complexity $O(n^3k^5)$ for n variables and maximal domain size k (Why?)

26.3 Summary

Summary

- ▶ generalization of
arc consistency (considers **pairs** of variables)
to path consistency (considers **triples** of variables)
and i -consistency (considers **i -tuples** of variables)
- ▶ arc consistency tightens **unary** constraints
- ▶ path consistency tightens **binary** constraints
- ▶ i -consistency tightens **$(i - 1)$ -ary** constraints
- ▶ higher levels of consistency more powerful
but more expensive than arc consistency