

# Foundations of Artificial Intelligence

## 14. State-Space Search: Analysis of Heuristics

Malte Helmert and Thomas Keller

University of Basel

March 23, 2020

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March 23, 2020 — 14. State-Space Search: Analysis of Heuristics

## 14.1 Properties of Heuristics

## 14.2 Examples

## 14.3 Connections

## 14.4 Summary

## State-Space Search: Overview

### Chapter overview: state-space search

- ▶ 5.–7. Foundations
- ▶ 8.–12. Basic Algorithms
- ▶ 13.–19. Heuristic Algorithms
  - ▶ 13. Heuristics
  - ▶ 14. Analysis of Heuristics
  - ▶ 15. Best-first Graph Search
  - ▶ 16. Greedy Best-first Search,  $A^*$ , Weighted  $A^*$
  - ▶ 17. IDA\*
  - ▶ 18. Properties of  $A^*$ , Part I
  - ▶ 19. Properties of  $A^*$ , Part II

## 14.1 Properties of Heuristics

## Perfect Heuristic

### Definition (perfect heuristic)

Let  $\mathcal{S}$  be a state space with states  $S$ .

The **perfect heuristic** for  $\mathcal{S}$ , written  $h^*$ , maps each state  $s \in S$  to the cost of an **optimal solution** for  $s$ .

**remark:**  $h^*(s) = \infty$  if no solution for  $s$  exists

**German:** perfekte Heuristik

## Properties of Heuristics

### Definition (safe, goal-aware, admissible, consistent)

Let  $\mathcal{S}$  be a state space with states  $S$ .

A heuristic  $h$  for  $\mathcal{S}$  is called

- ▶ **safe** if  $h^*(s) = \infty$  for all  $s \in S$  with  $h(s) = \infty$
- ▶ **goal-aware** if  $h(s) = 0$  for all goal states  $s$
- ▶ **admissible** if  $h(s) \leq h^*(s)$  for all states  $s \in S$
- ▶ **consistent** if  $h(s) \leq \text{cost}(a) + h(s')$  for all transitions  $s \xrightarrow{a} s'$

**German:** sicher, zielerkennend, zulässig, konsistent

## 14.2 Examples

## Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

### Route Planning in Romania

straight-line distance:

- ▶ **safe**
- ▶ **goal-aware**
- ▶ **admissible**
- ▶ **consistent**

Why?

## Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

Blocks World

misplaced blocks:

- ▶ safe?
- ▶ goal-aware?
- ▶ admissible?
- ▶ consistent?

## Properties of Heuristics: Examples

Which of our three example heuristics have which properties?

Missionaries and Cannibals

people on wrong river bank:

- ▶ safe?
- ▶ goal-aware?
- ▶ admissible?
- ▶ consistent?

## 14.3 Connections

## Properties of Heuristics: Connections (1)

Theorem (admissible  $\implies$  safe + goal-aware)

*Let  $h$  be an admissible heuristic.*

*Then  $h$  is safe and goal-aware.*

Why?

## Properties of Heuristics: Connections (2)

Theorem (goal-aware + consistent  $\implies$  admissible)

*Let  $h$  be a goal-aware and consistent heuristic.*

*Then  $h$  is admissible.*

Why?

## Showing All Four Properties

How can one show most easily that a heuristic has all four properties?

## 14.4 Summary

## Summary

- ▶ **perfect heuristic  $h^*$** : true cost to the goal
- ▶ **important properties**: **safe**, **goal-aware**, **admissible**, **consistent**
- ▶ **connections** between these properties
  - ▶ **admissible  $\implies$  safe and goal-aware**
  - ▶ **goal-aware and consistent  $\implies$  admissible**