Foundations of Artificial Intelligence

8. State-Space Search: Data Structures for Search Algorithms

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State-Space Search: Overview

Chapter overview: state-space search

- 5.–7. Foundations
- 8.–12. Basic Algorithms
  - 8. Data Structures for Search Algorithms
  - 9. Tree Search and Graph Search
  - 10. Breadth-first Search
  - 11. Uniform Cost Search
  - 12. Depth-first Search and Iterative Deepening
- 13.–19. Heuristic Algorithms

Introduction

Search Algorithms

- We now move to search algorithms.
- As everywhere in computer science, suitable data structures are a key to good performance.
  - $\rightsquigarrow$ common operations must be fast
- Well-implemented search algorithms process up to ∼30,000,000 states/second on a single CPU core.
  - $\rightsquigarrow$ bonus materials (Burns et al. paper)

this chapter: some fundamental data structures for search

Preview: Search Algorithms

- next chapter: we introduce search algorithms
- now: short preview to motivate data structures for search

Example: Search Algorithm

- Starting with initial state,
- repeatedly expand a state by generating its successors.
- Stop when a goal state is expanded
- or all reachable states have been considered.

German: expandieren, erzeugen

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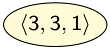

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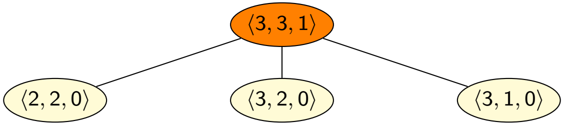

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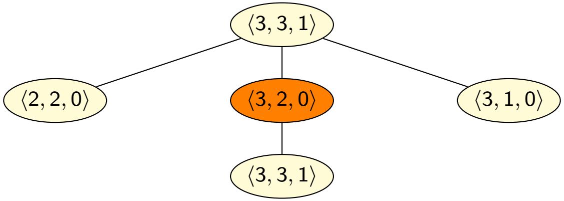

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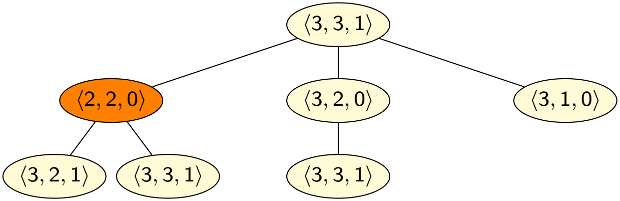
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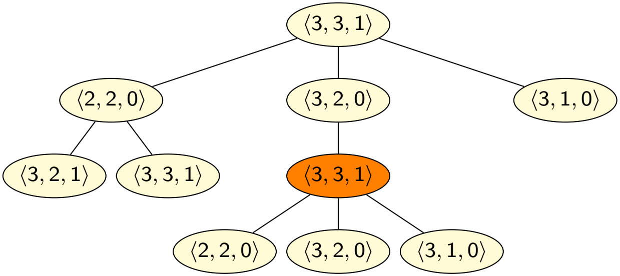

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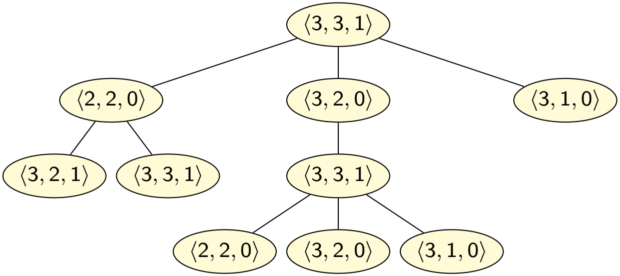


. . . and so on (expansion order depends on search algorithm used)

Fundamental Data Structures for Search

We consider three abstract data structures for search:

- search node: stores a state that has been reached, how it was reached, and at which cost
  - $\rightsquigarrow$ nodes of the example search tree
- open list: efficiently organizes leaves of search tree
  - $\rightsquigarrow$ set of leaves of example search tree
- closed list: remembers expanded states to avoid duplicated expansions of the same state
  - $\rightsquigarrow$ inner nodes of a search tree

German: Suchknoten, Open-Liste, Closed-Liste

Not all algorithms use all three data structures,
and they are sometimes implicit (e.g., in the CPU stack)

Search Nodes

Search Nodes

Search Node

A search node (node for short) stores a state
that has been reached, how it was reached, and at which cost.

Collectively they form the so-called search tree (Suchbaum).

Attributes of a Search Node

Attributes of a Search Node n

n.state	state associated with this node
n.parent	search node that generated this node (**none** for the root node)
n.action	action leading from n.parent to n (**none** for the root node)
n.path_cost	cost of path from initial state to n.state that result from following the parent references (traditionally denoted by $g(n)$)

. . . and sometimes additional attributes (e.g., depth in tree)

Search Nodes: Java

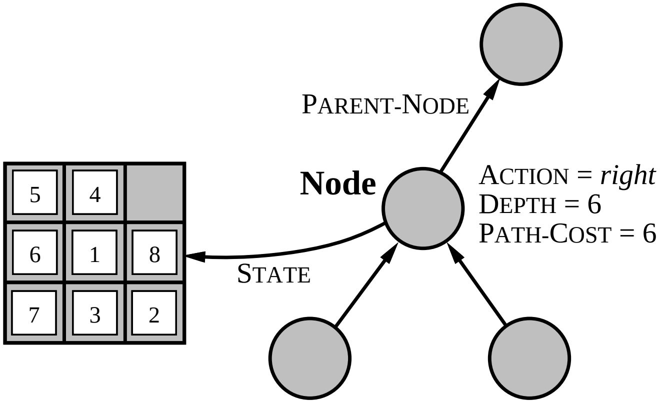
Search Nodes (Java Syntax)

```
public interface State {
}

public interface Action {
}

public class SearchNode {
    State state;
    SearchNode parent;
    Action action;
    int pathCost;
}
```

Node in a Search Tree



Implementing Search Nodes

- reasonable implementation of search nodes is easy
- advanced aspects:
  - Do we need explicit nodes at all?
  - Can we use lazy evaluation?
  - Should we manually manage memory?
  - Can we compress information?

Operations on Search Nodes: make_root_node

Generate root node of a search tree:

```
function make_root_node()
node := new SearchNode
node.state := init()
node.parent := none
node.action := none
node.path_cost := 0
return node
```

Operations on Search Nodes: make_node

Generate child node of a search node:

```
function make_node($parent$, $action$, $state$)
$node := $ new SearchNode
$node$.state $:= state$
$node$.parent $:= parent$
$node$.action $:= action$
$node$.path_cost $:= parent$.path_cost $+$ cost($action$)
return $node$
```

Operations on Search Nodes: extract_path

Extract the path to a search node:

function extract_path(*node*)

```
$path := \langle\rangle$
**while** $node$.parent $\neq$ **none**:
    $path$.append($node$.action)
    $node := node$.parent
$path$.reverse()
**return** $path$
```

Open Lists

Open Lists

Open List

The open list (also: frontier) organizes the leaves of a search tree.

It must support two operations efficiently:

- determine and remove the next node to expand
- insert a new node that is a candidate node for expansion

Remark: despite the name, it is usually a very bad idea to implement open lists as simple lists.

Open Lists: Modify Entries

- Some implementations support modifying an open list entry when a shorter path to the corresponding state is found.
- This complicates the implementation.

$\leadsto$ We do not consider such modifications
and instead use delayed duplicate elimination ($\leadsto$ later)

Interface of Open Lists

Methods of an Open List *open*

open.is_empty()	test if the open list is empty
open.pop()	removes and returns the next node to expand
open.insert(n)	inserts node n into the open list

- Different search algorithm use different strategies for the decision which node to return in *open*.pop.
- The choice of a suitable data structure depends on this strategy (e.g., stack, deque, min-heap).

Closed Lists

Closed Lists

Closed List

The closed list remembers expanded states
to avoid duplicated expansions of the same state.

It must support two operations efficiently:

- insert a node whose state is not yet in the closed list
- test if a node with a given state is in the closed list;
  if yes, return it

Remark: despite the name, it is usually a very bad idea
to implement closed lists as simple lists. (Why?)

Interface and Implementation of Closed Lists

Methods of a Closed List *closed*

closed.insert(*n*)	insert node *n* into *closed*; if a node with this state already exists in *closed*, replace it
closed.lookup(*s*)	test if a node with state *s* exists in the closed list; if yes, return it; otherwise, return **none**

- Hash tables with states as keys can serve as efficient implementations of closed lists.

Summary

Summary

- search node:
  represents states reached during search
  and associated information
- node expansion:
  generate successor nodes of a node by applying all actions
  applicable in the state belonging to the node
- open list or frontier:
  set of nodes that are currently candidates for expansion
- closed list:
  set of already expanded nodes (and their states)