A. Student
Another Student

Creating LaTeX exercise submissions

You will find many different online tutorials on how to set up a LaTeX environment and compile files like this one. For example:

- `http://en.wikibooks.org/wiki/LaTeX/Installation` (english)
- `https://www.dpg-physik.de/dpg/gliederung/junge/rg/wuerzburg/LaTeX-InstallationsTutorial.pdf` (german)

Please ask the tutors if you have any problems with or questions about this.

The following solutions show how to use some commands you will need for the exercises and demonstrate the level of detail a complete solution should ideally have.

Aufgabe 1.1

Atomic propositions: ItsRaining, ItsCold, TheSunIsShining, BobWantsIceCream.

(a) $(\neg \text{ItsRaining} \rightarrow (\neg \text{ItsCold} \wedge \text{TheSunIsShining}))$

(b) $((\neg \text{ItsRaining} \vee \text{TheSunIsShining}) \leftrightarrow \text{BobWantsIceCream})$

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Aufgabe 1.2

(a) Since $\mathcal{I}(A) = 0$, we know that $\mathcal{I} \not\models A$ and using the semantic of conjunctions that $\mathcal{I} \not\models (A \wedge B)$. This means that $\mathcal{I} \models \neg(A \wedge B)$ (*). Replacing the abbreviation of the implication, we get $\phi \equiv (\neg(A \wedge B) \vee C)$. Using the semantic of disjunctions with (*), we conclude that $\mathcal{I} \models \phi$.

Alternatively, we can do a systematic proof:

$$\begin{aligned} \mathcal{I} \models ((A \wedge B) \to C) \ &\text{iff. } \mathcal{I} \models (\neg(A \wedge B) \vee C) \\ &\text{iff. } \mathcal{I} \models \neg(A \wedge B) \text{ or } \mathcal{I} \models C \\ &\text{iff. } [\text{not } \mathcal{I} \models (A \wedge B)] \text{ or } \mathcal{I} \models C \\ &\text{iff. } [\text{not } [\mathcal{I} \models A \text{ and } \mathcal{I} \models B]] \text{ or } \mathcal{I} \models C \\ &\text{iff. } [\text{not } [\mathcal{I}(A) = 1 \text{ and } \mathcal{I}(B) = 1]] \text{ or } \mathcal{I} \models C \\ &\text{iff. } [\text{not } <\text{false statement } >] \text{ or } \mathcal{I} \models C \\ &\text{iff. } <\text{true statement}> \text{ or } \mathcal{I} \models C \\ &\text{iff. } <\text{true statement}> \end{aligned}$$

(b) Replacing the abbreviation of the biimplication, we get

$$\phi \equiv ((A \to (B \vee C)) \wedge ((B \vee C) \to A)).$$

We consider $\phi_1 = (A \to (B \vee C))$ and $\phi_2 = ((B \vee C) \to A)$ separately for now.

We start with ϕ_1. Replacing the abbreviation for implication, shows $\phi_1 \equiv (\neg A \vee (B \vee C))$. Since $\mathcal{I}(C) = 1$, we know that $\mathcal{I} \models C$ and using the semantic for disjunctions $\mathcal{I} \models (B \vee C)$, as well. This shows (again with the semantic for disjunctions) $\mathcal{I} \models \phi_1$ (*).

Removing the abbreviation for the implication in ϕ_2, we get $\phi_2 \equiv (\neg(B \vee C) \vee A)$. Since $\mathcal{I}(A) = 1$, we know that $\mathcal{I} \models A$, which directly shows with the semantic for disjunctions that $\mathcal{I} \models \phi_2$ (**).

The results (*) and (**) show $\mathcal{I} \models \phi$ with the semantic for conjunctions.

Of course we could also have done a systematic proof here, but this would have been more complicated.

Aufgabe 1.3

(a) Truth table for ϕ:

$\mathcal{I}(A)$	$\mathcal{I}(B)$	$\mathcal{I} \models (A \rightarrow B)$	$\mathcal{I} \models (B \rightarrow A)$	$\mathcal{I} \models ((A \rightarrow B) \vee (B \rightarrow A))$
0	0	Yes	Yes	Yes
0	1	Yes	No	Yes
1	0	No	Yes	Yes
1	1	Yes	Yes	Yes

Since the result for all interpretations is “Yes”, the formula ϕ is a tautology and thus also satisfiable. For the same reason it is not falsifiable and thus also not unsatisfiable.

(b) The formula is satisfiable which can be seen for the interpretation $\mathcal{I} = \{A \mapsto 1, B \mapsto 0, C \mapsto 1, D \mapsto 1\}$: Since $\mathcal{I}(A) = 1$, we know that $\mathcal{I} \models A$ and with the semantic for disjunctions $\mathcal{I} \models (A \vee B)$. With $\mathcal{I}(C) = 1$ we can argue analogously that $\mathcal{I} \models (C \vee D)$. Together with the semantic for conjunctions we conclude that $\mathcal{I} \models \psi$.

From satisfiability we can directly conclude that ψ is not unsatisfiable.

The formula ψ is also falsifiable, which the interpretation $\mathcal{I}' = \{A \mapsto 0, B \mapsto 0, C \mapsto 1, D \mapsto 1\}$ shows: Since $\mathcal{I}'(A) = 0$ and $\mathcal{I}'(B) = 0$, we know that $\mathcal{I}' \not\models (A \vee B)$. So $\mathcal{I}'$ cannot be a model of the conjunction ψ.

Since ψ is falsifiable, it cannot be a tautology.

Aufgabe 1.4

Example automaton: