Theory of Computer Science

M. Helmert
T. Keller
Spring Term 2017

University of Basel
Computer Science

Exercise Sheet 12

Due: Sunday, June 4, 2017

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 12.1 (3 marks)

The following statements are all wrong. In each case, explain in 1–2 sentences why the statement is wrong and what a correct version would be.

(a) To show that a problem X is NP-complete, it suffices to show that $X \in \text{NP}$ and $X \leq_\text{p} Y$ for some NP-complete problem Y.

(b) There is a NP-complete problem X that can be solved with an efficient deterministic algorithm, even if there is none for Sat.

(c) For every NP-hard problem X: $X \leq_\text{p} \text{Sat}$.

Exercise 12.2 (2+3 marks)

Consider the decision problem HittingSet:

- *Given:* A finite set T, a set of sets $S = \{S_1, \ldots, S_n\}$ with $S_i \subseteq T$ for all $i \in \{1, \ldots, n\}$, a natural number $K \in \mathbb{N}_0$ with $K \leq |T|$.
- *Question:* Is there a set H with at most K elements that contains at least one element from each set in S?

(a) Prove that HittingSet is in NP by specifying a non-deterministic algorithm for HittingSet whose runtime is limited by a polynomial in $n|T|$.

(b) Prove that HittingSet is NP-complete. You may use without proof that the problem VertexCover (from chapter E5) is NP-complete.

Exercise 12.3 (2+2 marks)

Consider the decision problems DirHamiltonPath, DirHamiltonPathFromVertex and DirHamiltonPathToVertex:

DirHamiltonPath (see exercise 11.3):

- *Given:* directed graph $G = \langle V, E \rangle$
- *Question:* Does G contain a Hamilton path?

DirHamiltonPathFromVertex

- *Given:* directed graph $G = \langle V, E \rangle$, vertex $v_\text{start} \in V$
- *Question:* Does G contain a Hamilton path that starts in v_start?

DirHamiltonPathToVertex

- *Given:* directed graph $G = \langle V, E \rangle$, vertex $v_\text{end} \in V$

- *Question:* Does G contain a Hamilton path that ends in v_{end}?

(a) Reduce DirHamiltonPath to DirHamiltonPathFromVertex with a polynomial reduction to show

$$\textsc{DirHamiltonPath} \leq_{\text{p}} \textsc{DirHamiltonPathFromVertex}.$$

(b) Reduce DirHamiltonPathFromVertex to DirHamiltonPathToVertex with a polynomial reduction to show

$$\textsc{DirHamiltonPathFromVertex} \leq_{\text{p}} \textsc{DirHamiltonPathToVertex}.$$