Theory of Computer Science

M. Helmert
T. Keller
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University of Basel
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Exercise Sheet 10

Due: Sunday, May 21, 2017

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 10.1 (3 marks)

Show for any languages A, B and C: if $A \leq B$ and $B \leq C$, then $A \leq C$.

Exercise 10.2 (4 marks)

The *emptiness problem* Emptiness for general (type-0) grammars is defined as:

Given a general grammar G, is $\mathcal{L}(G) = \emptyset$?

Prove that Emptiness is undecidable.

Hints: you can use without proof that there is a computable function that transforms a given type-0 grammar G to a DTM M_G with $\mathcal{L}(M_G) = \mathcal{L}(G)$. Likewise, there is a computable function that transforms a given DTM M to a type-0 grammar G_M with $\mathcal{L}(M) = \mathcal{L}(G_M)$.
Use Rice's theorem in an appropriate way to show the undecidability.

Exercise 10.3 (3 marks)

The *intersection problem* Intersection for general (type-0) grammars is defined as:

Given two general grammars G_1 and G_2, is $\mathcal{L}(G_1) \cap \mathcal{L}(G_2) = \emptyset$?

Show that Intersection is undecidable, by reducing Emptiness to it.

Hint: of course you can use the fact that Emptiness is undecidable even if you did not complete exercise 12.1.

Exercise 10.4 (0.5+0.5+0.5+0.5 marks)

For which of the following languages does Rice's theorem show that the language is undecidable? For each language where Rice's theorem can be used, specify the subset of Turing-computable functions $\mathcal{S}$ for which you use the theorem.
Hint: You do not have to write down any proofs. If Rice's theorem is applicable, specify the set $\mathcal{S}$, otherwise give a short reason (1 sentence) why Rice's theorem is not applicable.

(a) $L = \{w \in \{0,1\}^* \mid M_w$ computes the binary multiplication function$\}$

(b) $L = \{w \in \{0,1\}^* \mid$ The output of M_w started on the empty tape contains `0101`$\}$

(c) $L = \{w \in \{0,1\}^* \mid M_w$ stops for at least one input after more than 10 steps with a valid output$\}$

(d) $L = \{w \in \{0,1\}^* \mid M_w$ computes a binary function over the natural numbers$\}$