Theory of Computer Science

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Exercise Sheet 9

Due: Sunday, May 14, 2017

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 9.1 (2 marks)

Prove that the following definition of the binomial coefficient (choose 2) is correct, that is $binom_2(x) = \frac{x \cdot (x-1)}{2}$ for all $x \geq 2$. Also specify definitions of h and $binom_2'$ in common mathematical notation.

$$\begin{aligned} h &= compose(add, \pi_1^3, \pi_2^3) \\ binom_2' &= primitive\_recursion(null, h) \\ binom_2 &= compose(binom_2', \pi_1^1, null) \end{aligned}$$

Exercise 9.2 (1+1+1 marks)

For each of the following functions f specify a definition of μf in common mathematical notation.

(a) $f(x, y, z) = z \ominus y^x$ for all $x, y, z \in \mathbb{N}_0$

(b) $f(x, y, z) = (y \ominus x) \cdot (z \ominus x)$ for all $x, y, z \in \mathbb{N}_0$

(c) $f(x, y, z) = (y \ominus x) + (z \ominus x)$ for all $x, y, z \in \mathbb{N}_0$

Exercise 9.3 (1+1+1 marks)

Let $\Sigma = \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$. Specify total and computable functions $f : \mathbb{N}_0 \to \Sigma^*$ which recursively enumerate the following languages.

(a) $L_1 = L_A \cup L_B$ where L_A and L_B are languages over Σ that are recursively enumerated by the functions f_A and f_B.

(b) $L_2 = \{\mathtt{a}^x \mathtt{b}^y \mathtt{c}^z | x, y, z \in \mathbb{N}_0 \text{ and } x^2 + y^2 = z^2\}$

(c) $L_3 = \{w \in \Sigma^* \mid \mathtt{a} \text{ occurs in } w \text{ exactly once}\}$

Exercise 9.4 (1+1+1+1 marks)

Which of the following statements about languages A and B are true, which are false? In each case, specify a proof idea (1–2 sentences) or a counter example. You can use all results from the lecture.

(a) If A and B are semi-decidable, then $A \cup B$ is semi-decidable.

(b) If A and B are semi-decidable, then the following algorithm calculates $\chi'_{A \cap B}$:

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IF χ'_A(w) = 1 AND χ'_B(w) = 1 THEN
  RETURN 1
ELSE
  LOOP FOREVER
END
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(c) Every decidable language is accepted by a Turingmachine.

(d) Every type-0 language is decidable.