Theory of Computer Science

M. Helmert
T. Keller
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University of Basel
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Exercise Sheet 8

Due: Wednesday, May 3, 2017

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Note: As there is no lecture on May 1, you have some additional days to work on this exercise sheet.

Exercise 8.1 (2 marks)

Describe (in words) a Turing machine which computes the function $f(\mathtt{a}^{2i}) = \mathtt{b}^i$ for words over the alphabet $\Sigma = \{\mathtt{a}, \mathtt{b}\}$.
Note: Use statements which have a similar level of detail as the following examples: "move the reading head to the left, until it reads an a" and "if a b is read, go into an endless loop".

Exercise 8.2 (3 marks)

Specify the transition diagram of a Turing machine which computes the *predecessor function* $pred_2$ over natural numbers (see slide 24 in slide set D1). *Additionally* describe (in words) how your Turing machine works.

Exercise 8.3 (2 marks)

Let $f : \Sigma^* \to \Sigma^*$ and $g : \Sigma^* \to \Sigma^*$ be Turing-computable partial functions for an alphabet Σ. Show that the *composition* $(f \circ g) : \Sigma^* \to \Sigma^*$ is also turing-computable.
In general the composition of two functions is defined as $(f \circ g)(x) = f(g(x))$. Specifically, the value $(f \circ g)(x)$ is undefined if $g(x)$ is undefined.

Exercise 8.4 (3 marks)

Simulate the following syntactical constructs for LOOP-programs (with obvious semantics) by using already known constructs. In addition to the base constructs of LOOP programs you may use the additional constructs introduced in chapter D2.

(a) **IF** $x_i > c$ **THEN** P **ELSE** P' **END**

(b) **IF** $x_i = x_j$ **THEN** P **END**

(c) **FOR** $x_i = 0$ **TO** c **DO** P **END**

In the above constructs P and P' are arbitrary LOOP-programs and $i, j, c \in \mathbb{N}_0$ are arbitrary natural numbers.

Exercise 8.5 (2 marks)

(a) Which binary function $f(x, y)$ is computed by the following WHILE-program?

$x_3 := x_1 + 1$
$x_3 := x_3 - x_2$
WHILE $x_3 \neq 0$ **DO**
  $x_3 := x_3 - x_2$
  $x_0 := x_0 + 1$
END

(b) Is f LOOP-computable? Justify your answer.

(c) Specify a WHILE-program which computes the modulo operation

$$g(x, y) = \begin{cases} x \bmod y, & \text{if } y > 0 \\ \text{undefined}, & \text{otherwise.} \end{cases}$$

You may use the function f from exercise (a) and the multiplication $\cdot$ in your solution.