Theory of Computer Science

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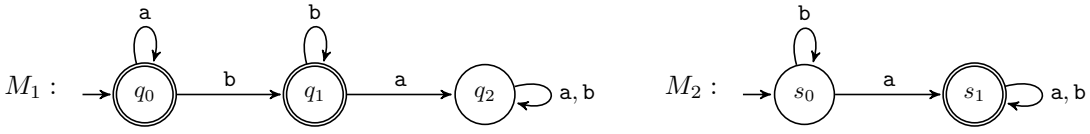
Exercise Sheet 6

Due: Sunday, April 9, 2017

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 6.1 (3 marks)

Consider the following DFAs M_1 and M_2.



Specify the product automaton that accepts $\mathcal{L}(M_1) \cap \mathcal{L}(M_2)$.

Exercise 6.2 (3 marks)

Consider the grammar $G = \langle \{a, b, c, d\}, \{S, W, X, Y, Z\}, P, S\} \rangle$ and the following rules in P:

$$(1) S \to X \quad (2) X \to Y \quad (3) Y \to \varepsilon \quad (4) Y \to \mathtt{d}$$
$$(5) X \to Z \quad (6) Z \to W\mathtt{a}X\mathtt{b}Y\mathtt{c} \quad (7) W \to \mathtt{a}W\mathtt{a} \quad (8) W \to \mathtt{b}S\mathtt{b}$$

Apply the method that is given in the proof on slides 5-7 of slide set C05 to G and provide the resulting context-free grammar G'. Give sufficient intermediate steps.

Exercise 6.3 (3 marks)

Specify a grammar G' in Chomsky normal form that generates the same language as the context-free grammar $G = \langle \Sigma, V, P, \mathrm{S} \rangle$ with $\Sigma = \{\mathtt{a}, \mathtt{b}\}$, $V = \{\mathrm{S}, \mathrm{W}, \mathrm{X}, \mathrm{Y}, \mathrm{Z}\}$ and the following rules in P:

$$\mathrm{S} \to \varepsilon \quad \mathrm{S} \to \mathrm{XW} \quad \mathrm{S} \to \mathrm{Z} \quad \mathrm{W} \to \mathrm{X} \quad \mathrm{X} \to \mathtt{aZb}$$
$$\mathrm{Y} \to \mathrm{W} \quad \mathrm{Y} \to \mathtt{b}\mathrm{Y} \quad \mathrm{Z} \to \mathtt{bb} \quad \mathrm{Z} \to \mathrm{Z}\mathtt{a} \quad \mathrm{X} \to \mathrm{Y}$$

Give sufficient intermediate steps.

Exercise 6.4 (3 marks)

Let G be a grammar in Chomsky normal form and $w \in \mathcal{L}(G)$ a non-empty word ($w \neq \varepsilon$), which is generated by G. Show that every derivation of w from the start variable of G consists of exactly $2|w| - 1$ steps.