Theory of Computer Science

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Exercise Sheet 4

Due: Sunday, March 26, 2017

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 4.1 (2 marks)

Consider the formula φ over a signature with predicate symbols P (1-ary), Q (2-ary) and R (3-ary), the 2-ary function symbol f, the constant symbol c and the variable symbols x, y and z.

$$\varphi = (\forall x \exists y\,(\mathrm{P}(x) \to \mathrm{Q}(y, z)) \vee \neg \exists y\,(\mathrm{Q}(y, z) \wedge \forall z\,(\mathrm{R}(\mathrm{f}(y, z), c, \mathrm{f}(x, y)))))$$

Mark all occurrences of free variables in φ. *Additionaly* specify the set of free variables of φ (without proof).

Exercise 4.2 (1+3+1 marks)

Consider the following formal language over $\{\mathtt{a}, \mathtt{b}, \mathtt{c}\}$:

$$L = \{\mathtt{a}^n \mathtt{b}^m \mathtt{c}^{2n} \mid n \geq 0, m \geq 0\}$$

(a) Is ε an element of L? Justify your answer.

(b) Specify a *complete description* of a formal grammar G that generates L (i.e., $\mathcal{L}(G) = L$). A formal grammar is a four tuple $G = \langle \Sigma, V, P, S \rangle$, remember to define all components of this tuple.

(c) Which types (in the Chomsky-Hierarchy) is your formal grammar part of? You don't have to prove your answers.

Exercise 4.3 (2+2+1 marks)

Consider the grammar $G = \langle \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}, \{\mathrm{S}, \mathrm{T}, \mathrm{U}\}, P, \mathrm{S} \rangle$ with the following production rules P:

$$\begin{aligned}
\mathrm{S} &\to \mathrm{UTU} \\
\mathrm{U} &\to \mathtt{aUbbb} \\
\mathrm{U} &\to \mathtt{abbb} \\
\mathrm{T} &\to \varepsilon \\
\mathrm{T} &\to \mathrm{T}\mathtt{c}
\end{aligned}$$

(a) What is $\mathcal{L}(G)$? Describe the language as simple as possible.

(b) Specify a derivation of the word $\mathtt{abbbccaabbbbbb}$.

(c) Specify a context-free grammar that generates $\mathcal{L}(G)$.