Theory of Computer Science

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Exercise Sheet 3

Due: Sunday, March 19, 2017

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Exercise 3.1 (Equivalences; 2+2 marks)

(a) Transform the following formula into CNF by applying the equivalence rules shown in the lecture. For each step, only apply one equivalence rule and also specify it.

$$\phi = (\neg(A \leftrightarrow \neg B) \rightarrow C)$$

(b) Prove that the following formula is a tautology by showing that $\phi \equiv (A \vee \neg A)$ holds. Use the equivalence rules from the lectures, only apply one rule for each step and specify the applied rule.

$$\phi = (A \vee (\neg(A \wedge \neg(\neg A \wedge C)) \vee (A \wedge B)))$$

Exercise 3.2 (Formula sets; 2.5+2.5 marks)
Consider the following formula set:

$$\mathrm{KB} = \{(A \vee C), (B \vee \neg C), (C \rightarrow \neg B)\}$$

(a) Does a model $\mathcal{I}$ of KB exist which is also a model for $\phi = (C \vee \neg A)$? Prove your statement.

(b) Prove that all models $\mathcal{I}$ of KB are also models of $\phi = (A \vee B)$.

Exercise 3.3 (Predicate logic; 3 marks)
Consider the following predicate logic formula φ with the signature $\langle \{x, y\}, \{\mathrm{c}\}, \{\mathrm{f}, \mathrm{g}\}, \{\mathrm{P}\} \rangle$:

$$\varphi = (\exists x\, (\mathrm{P}(x) \wedge \neg \mathrm{P}(\mathrm{f}(x))) \wedge \forall x\, \neg(\mathrm{g}(x, y) = \mathrm{c}))$$

Specify a variable assignment α and a model $\mathcal{I} = \left\langle U, \cdot^{\mathcal{I}} \right\rangle$ of φ with $U = \{u_1, u_2, u_3\}$ and $\mathrm{g}^{\mathcal{I}}(x, y) = y$ for all $x, y \in U$. *Prove* that $\mathcal{I}, \alpha \models \varphi$.