Theory of Computer Science

M. Helmert
T. Keller
Spring Term 2017

University of Basel
Computer Science

Exercise Sheet 2

Due: Sunday, March 12, 2017

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file (or a printout of this file).

Note: Due to the week without lectures between March 6 and 12, you have an additional week to work on this exercise sheet.

Exercise 2.1 (Syntax; 0.5+0.5+0.5+0.5 marks)

Formalize the following statements as propositional formulas. In order to do so, also define appropriate atomic propositions. Take care to fully parenthesize all formulas.

(a) If it rains, it is not warm or it is summer.

(b) If Bob is going for a swim, then it is always summer and he does not drink tea.

(c) Bob is going for a swim exactly if he does not drink tea and it is warm or does not rain.

(d) Either Bob drinks tea or he is going for a swim (but not both).

Exercise 2.2 (Truth tables; 1+1+1+1+1 marks)

Let $A = \{\mathrm{X}, \mathrm{Y}\}$ be a set of propositional variables and $\psi = (\mathrm{X} \to \mathrm{Y})$ be a propositional formula over A.

(a) Specify the truth table for ψ (see chapter B1, slide 30).

(b) Check for each of the following properties of propositional formulas if ψ has that property. If not, specify a propositional formula φ over A with that property and then use a truth table to prove that the formula has the property.

i) ψ is satisfiable and falsifiable.

ii) ψ has exactly two models.

iii) ψ is valid and uses both variables.

iv) ψ is unsatisfiable.

Exercise 2.3 (Semantics; 2.5+2.5 marks)

Consider the propositional formula φ over $\{\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{E}, \mathrm{F}\}$:

$$\varphi = ((\mathrm{F} \vee ((\neg \mathrm{B} \leftrightarrow ((\mathrm{C} \wedge \mathrm{A}) \to \neg \mathrm{B})) \vee (\mathrm{D} \to \mathrm{E}))) \to (\mathrm{A} \to \neg \mathrm{F}))$$

(a) Specify a model $\mathcal{I}$ for φ and prove without truth table that $\mathcal{I} \models \varphi$.

(b) Specify an assignment $\mathcal{I}$ with $\mathcal{I} \not\models \varphi$ and prove that $\mathcal{I}$ has the desired property without a truth table.