

Foundations of Artificial Intelligence

16. State-Space Search: Greedy BFS, A^* , Weighted A^*

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State-Space Search: Overview

Chapter overview: state-space search

- 5.–7. Foundations
- 8.–12. Basic Algorithms
- 13.–19. Heuristic Algorithms
 - 13. Heuristics
 - 14. Analysis of Heuristics
 - 15. Best-first Graph Search
 - 16. Greedy Best-first Search, A*, Weighted A*
 - 17. IDA*
 - 18. Properties of A*, Part I
 - 19. Properties of A*, Part II

Introduction

What Is It About?

In this chapter we study last chapter's algorithms in more detail:

- greedy best-first search
- A*
- weighted A*

Greedy Best-first Search

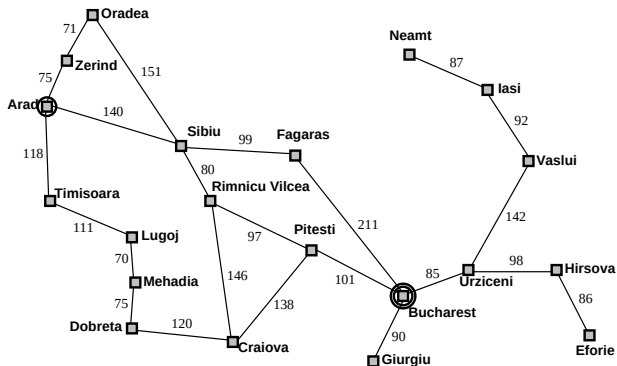
Greedy Best-first Search

Greedy Best-first Search

only consider the heuristic: $f(n) = h(n.state)$

Note: usually **without reopening** (for reasons of efficiency)

Example: Greedy Best-first Search for Route Planning



Arad	366
Bucharest	0
Craiova	160
Drobeta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Example: Greedy Best-first Search for Route Planning

(a) The initial state



Example: Greedy Best-first Search for Route Planning

(a) The initial state



(b) After expanding Arad



Example: Greedy Best-first Search for Route Planning

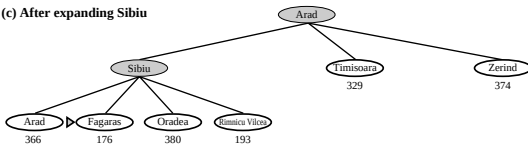
(a) The initial state



(b) After expanding Arad



(c) After expanding Sibiu



Example: Greedy Best-first Search for Route Planning

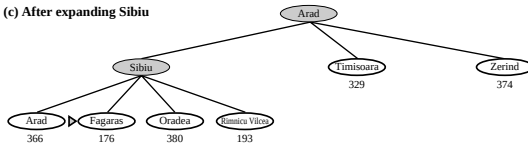
(a) The initial state



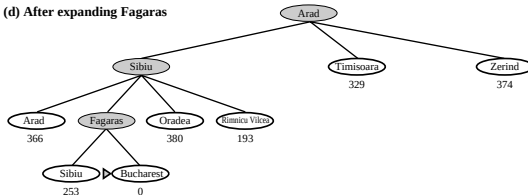
(b) After expanding Arad



(c) After expanding Sibiu



(d) After expanding Fagaras



Greedy Best-first Search: Properties

- **complete** with **safe** heuristics
(like all variants of best-first graph search)
- **suboptimal**: solutions can be **arbitrarily bad**
- often **very fast**: one of the fastest search algorithms in practice
- monotonic transformations of h (e.g. scaling, additive constants) do not affect behaviour (**Why is this interesting?**)

A^*

A*

A*

combine greedy best-first search with uniform cost search:

$$f(n) = g(n) + h(n.state)$$

- **trade-off** between path cost and proximity to goal
- $f(n)$ estimates overall cost of cheapest solution
from initial state via n to the goal

A*: Citations

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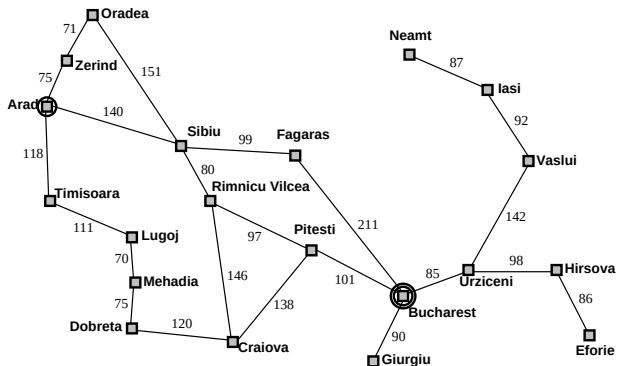
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Example: A* for Route Planning



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Example: A* for Route Planning

(a) The initial state



Example: A* for Route Planning

(a) The initial state



(b) After expanding Arad



Example: A* for Route Planning

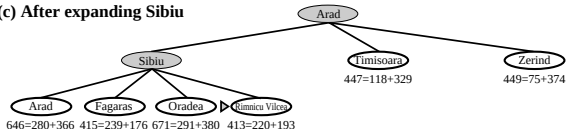
(a) The initial state



(b) After expanding Arad



(c) After expanding Sibiu



Example: A* for Route Planning

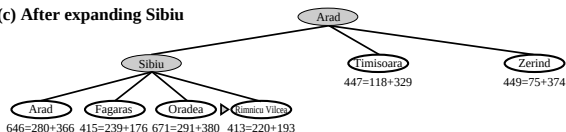
(a) The initial state



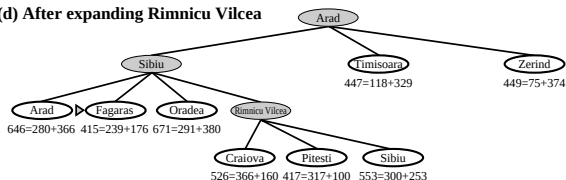
(b) After expanding Arad



(c) After expanding Sibiu

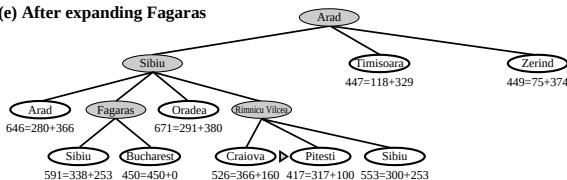


(d) After expanding Rimnicu Vilcea



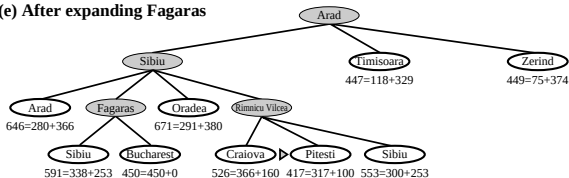
Example: A^* for Route Planning

(e) After expanding Fagaras

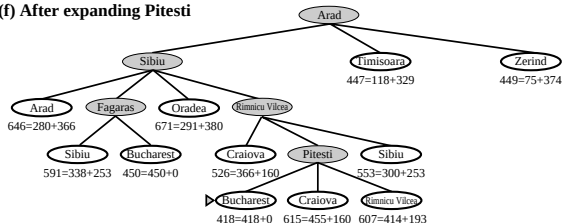


Example: A* for Route Planning

(e) After expanding Fagaras



(f) After expanding Pitesti



A*: Properties

- **complete** with **safe** heuristics
(like all variants of best-first graph search)
- **with reopening: optimal** with **admissible** heuristics
- **without reopening: optimal** with heuristics
that are **admissible** and **consistent**

↪ proofs: Chapters 18 and 19

A*: Implementation Aspects

some practical remarks on implementing A*:

- **common bug:** reopening not implemented although heuristic is not consistent
- **common bug:** duplicate test “too early” (upon generation of search nodes)
- **common bug:** goal test “too early” (upon generation of search nodes)
- all these bugs lead to loss of optimality and can remain undetected for a long time

Weighted A*

Weighted A*

Weighted A*

A* with more heavily weighted heuristic:

$$f(n) = g(n) + w \cdot h(n.\text{state}),$$

where **weight** $w \in \mathbb{R}_0^+$ with $w \geq 1$ is a freely choosable parameter

Note: $w < 1$ is conceivable, but usually not a good idea
(Why not?)

Weighted A*: Properties

weight parameter controls “greediness” of search:

- $w = 0$: like uniform cost search
- $w = 1$: like A*
- $w \rightarrow \infty$: like greedy best-first search

with $w \geq 1$ properties analogous to A*:

- h admissible:
found solution guaranteed to be at most w times
as expensive as optimum when reopening is used
- h admissible and consistent:
found solution guaranteed to be at most w times
as expensive as optimum; no reopening needed

(without proof)

Summary

Summary

best-first graph search with evaluation function f :

- $f = h$: greedy best-first search
suboptimal, often very fast
- $f = g + h$: A*
optimal if h admissible and consistent
or if h admissible and reopening is used
- $f = g + w \cdot h$: weighted A*
for $w \geq 1$ suboptimality factor at most w
under same conditions as for optimality of A*